Ca' Foscari
University
of Venice

**Department
of Economics**

Working Paper

**Massimiliano Caporin and
Loriana Pelizzon**

**Market volatility, optimal
portfolios and naive asset
allocations**

ISSN: 1827/3580
No. 08/WP/2012

Market volatility, optimal portfolios and naive asset allocations

Massimiliano Caporin
Università di Padova

Loriana Pelizzon
Università Ca' Foscari Venezia

First Draft: 15 giugno 2012

Abstract

This paper investigates the impact of a financial turmoil on the performances of traditional, and naive, asset allocation strategies. We compare over a long time span (lasting for the last 60 years) the 1/N portfolio with mean-variance optimal portfolio strategies. Our analyses consider several datasets, and different approaches for the estimation of expected returns, starting from simple historical moments to the use of predictable variables, mean reversion or both. By employing rolling estimation approaches and robust Sharpe ratio testing we determine if during different market volatility states calibrated portfolios perform better than optimally determined allocations.

Keywords

Mean reversion, strategy preference, 1/N, predictability, testing Sharpe equivalence.

JEL Codes

G11

Address for correspondence:

Loriana Pelizzon
Department of Economics
Ca' Foscari University of Venice
Cannaregio 873, Fondamenta S.Giobbe
30121 Venezia - Italy
Phone: (+39) 041 234164
Fax: (+39) 041 2349176
e-mail: [email](mailto:loriana.pelizzon@unive.it)

This Working Paper is published under the auspices of the Department of Economics of the Ca' Foscari University of Venice. opinions expressed herein are those of the authors and not those of the Department. The Working Paper series is designed to divulge preliminary or incomplete work, circulated to favour discussion and comments. Citation of this paper should consider its provisional character.

1 Introduction

In the last 30 years we saw a long debate on the performance of various asset allocation strategies when the investor cares only about the mean and the variance of static portfolio returns. In a recent paper, De Miguel, Garlappi and Uppal. (2009) later on (DGU), show that the $1/N$ asset-allocation rule typically has a higher out-of-sample Sharpe Ratio, a higher certainty-equivalent value, and a lower turnover than several static and dynamic optimal asset allocation policies for different datasets.

As already highlighted by other authors (Hodges and Brealey (1978), Michaud (1989), Best and Grauer (1991), Litterman (2003), Jagannathan and Ma (2003)) this result is mostly due to the estimation error problem. This problem could be related, as stressed by Merton (1980), to the difficulties in estimating the expected returns precisely; or to the uncertain estimates of the variance-covariance matrix (see Green and Hollifield (1992) and Jagannathan and Ma (2003)).

This issue has been addressed in the literature by the use of Bayesian approaches ((Barry (1974) and Bawa, Brown, and Klein (1979), Jobson and Korkie (1980), Jorion (1985, 1986), Pastor (2000), Pastor and Stambaugh (2000)), non Bayesian approaches based on moment restrictions and optimal combination of portfolios (Goldfarb and Iyengar (2003), Garlappi, Uppal, and Wang (2006), Kan and Zhou (2005), MacKinlay and Pastor (2000), Best and Grauer (1992), Chan, Karceski, and Lakonishok (1999), and Ledoit and Wolf (2004)) and portfolio rules that impose constraints on portfolio weights (Frost and Savarino (1988), Chopra (1993), Jagannathan and Ma (2003) and De Miguel et al. (2009)).

However, the analysis of DGU shows that thirteen strategies that try to overcome the estimation problem based on Bayesian approaches, moment restrictions and optimal combination of portfolios, and portfolio rules that impose constraints on portfolio weights are not able out of sample to perform statistically better than the $1/N$ strategy.

As in DGU the objective of this paper is to understand the conditions under which mean-variance optimal portfolio models can be expected to perform better than the $1/N$ even in the presence of estimation risk. The perspective that we investigate in this paper is the role of predictability and mean-reversion in allowing mean variance optimal

portfolio models to perform better than the $1/N$ in the short term. We also add a further purpose, by verifying if the introduction of predictability or mean-reversion has an impact on strategy preference during market turmoil.

For this purpose, we investigate whether improving the estimation of the expected returns through the use of predictable variables, mean reversion or both, mean-variance optimal portfolio strategies are able to perform statistically better than the $1/N$ portfolio. We concentrate on four mean-variance allocation strategies and on 5 equity datasets. We considered three models of mean forecast: (i) historical moment, (ii) mean forecast based on different predictor variables: the dividend yield, the short term rate, the term spread, and the credit spread. We consider each separately or all of them jointly (we call this model "predictability"); and (iii) VAR(1) that capture mean reversion. The performances are evaluated using the Sharpe Ratio, and inference in the Sharpe Ratio follows the approach of Ledoit and Wolf (2008).

Our analysis shows that, even for the short term horizon, exploiting the mean reversion and predictability feature present in some of the datasets allow mean variance strategies to perform statistically better than the $1/N$ strategy. Differently from DGU, we also analyse the preference relation across strategies on a rolling approach (the paper by DGU can be considered a full sample static comparison). We investigate whether this result is strictly related to the ability of our prediction models to explain the returns of the single assets. The analysis of the R^2 shows that the ability of the model to explain the dynamic of the single asset return is very poor. However, by performing a rolling window analysis of the shape ratio we detected periods where the mean variance strategies perform statistically better and others where they are not statistically different. The interesting fact is that mean-variance strategies are not statistically different than the $1/N$ during crisis periods. There are potentially two different reasons for this result. First, the predictability power of the predictor variables disappears during these periods as well as mean reversion effects. Second, data are more noisy, volatility is higher as well as the estimation errors. In general we have that the data generating process that characterizes the dynamic of financial assets returns changes during crisis periods. On the other side we may have that simply the $1/N$ weights are not so different from the op-

timal weights that arise from the mean variance strategies. In order to try to investigate this result we determined the implicit mean that, given the variance covariance matrix we estimate, make the $1/N$ portfolio the optimal mean variance portfolio. We compared these means with those of our forecasts. We find that when the differences among these parameters are large, the mean-variance strategy is performing statistically better than the $1/N$, in the other cases the difference is not statistically significant. The practical implications of our results is that: when the mean reversion model is producing a forecast for the mean that is significantly different than the one implied by the $1/N$ model, it is worth to implement the optimization strategy, in the other cases it is preferable to adopt the $1/N$ strategy.

The paper is organized as follows. Section 2 presents the models we use to forecast the mean and the variance-covariance matrix. Section 3 describes the investment sets considered. Section 4 shows the performance evaluation method used. Section 5 presents the results from the full sample and Section 6 those from the rolling window analysis. Section 7 concludes.

2 Mean and variance forecasts

We determine optimal portfolios on a monthly basis using several strategies. Most of the strategies we implement determine next month optimal portfolios using the one months ahead forecasts of the assets means and variances.

We denote by X_t the n -dimensional vector of monthly log-returns for the assets we monitor, by $E_t[\cdot]$ and $Var_t[\cdot]$ the expectation and variance for the relevant quantity made at time t , and with m we indicate the length of the estimation window used. In our analysis we use $m = 60$ for the main results while the case $m = 120$ is considered for a robustness check.

We compute next month mean and variance forecasts using four different approaches:

(1) sample means and sample variances over the last m months computed as $\mu_{t+1} = \frac{1}{m} \sum_{j=t-m+1}^t X_j$ and $\Sigma_{t+1} = \frac{1}{m} \sum_{j=t-m+1}^t (X_j - E_t[X_{t+1}])^2$;

(2) mean forecast from a regression model with explanatory variables, and variance forecasts from mean model residuals: $\mu_{t+1} = \hat{\alpha} + \hat{\beta}Z_t$ and $\Sigma_{t+1} = \frac{1}{m} \sum_{j=t-m+2}^t (X_j - \hat{\alpha} - \hat{\beta}Z_{j-1})^2$,

where α and β have been estimated from a linear regression of X_t on the k -dimensional vector of predicting variables Z_t ;

(3) VAR(1) model (mean reversion only) for the mean forecasts, and variances coming from model residuals: $\mu_{t+1} = E_t[X_{t+1}] = \hat{\alpha} + \hat{\beta}X_t$ and $\Sigma_{t+1} = \frac{1}{m} \sum_{j=t-m+2}^t \left(X_j - \hat{\alpha} - \hat{\beta}X_{j-1}\right)^2$, where α and β have been estimated using a least squares method with robust standard errors;

(4) VARX(1) model (mean reversion and predictability) for the mean forecasts, and variances coming from model residuals: $\mu_{t+1} = E_t[X_{t+1}] = \hat{\alpha} + \hat{\beta}Z_t + \hat{\delta}X_t$ and $\Sigma_{t+1} = \frac{1}{m} \sum_{j=t-m+2}^t \left(X_j - \hat{\alpha} - \hat{\beta}Z_{j-1} - \hat{\delta}X_{j-1}\right)^2$, where α , β and δ have been estimated using a least squares method with robust standard errors.

2.1 Asset Allocation Strategies.

Following Campbell and Viceira (1999, 2001), we use data expressed in log-returns to extract expectations about next period mean and variances, while we focus on an allocation problem where the strategy evaluation makes use of percent returns. Given that we use a mean-variance utility function, or a certainty equivalent satisfaction index (see Meucci, 2005), we can optimize portfolios solving a problem expressed in log-returns but correcting the objective function. The correction makes the results approximately equal to those we could obtain by solving the allocation problem in percent returns. Following this approach, the maximization problem depends on expected portfolio mean and variances with the following expressions

$$\begin{aligned}\tilde{\mu}_{t+1} &= \omega' \mu_{t+1} + \frac{1}{2} \omega' (\text{diag}(\Sigma_{t+1}) - \Sigma_{t+1}) \omega \\ \tilde{\Sigma}_{t+1} &= \omega' \Sigma_{t+1} \omega\end{aligned}\tag{1}$$

Portfolio mean and variances are then used within a set of alternative allocation problems.

The first set of strategies that we consider comes from the classical Mean Variance optimization (MV). We maximize a mean variance utility function with risk-aversion

coefficient set to 1, 2, 3, 5, 8, 15, 20 and 50. The strategy labels are MV-(A), where (A) identifies the risk aversion level. All these strategies do not allow for short selling and weights are determined solving the following maximization problem:

$$\begin{aligned} \max_{\omega} \quad & \tilde{\mu}_{t+1} - \frac{R_A}{2} \tilde{\Sigma}_{t+1} \\ \text{s.t.} \quad & \omega' \mathbf{1} = 1, \quad \omega \geq \mathbf{0} \end{aligned} \tag{2}$$

The second strategy set considers variations of the Global Minimum Variance portfolio. The first is the general Global Minimum Variance portfolio (denoted as GMV and with strategy label GMV) where the strategy weights are determined solving the following problem:

$$\begin{aligned} \min_{\omega} \quad & \tilde{\Sigma}_{t+1} \\ \text{s.t.} \quad & \omega' \mathbf{1} = 1 \end{aligned} \tag{3}$$

The second is the Global Minimum Variance with No Short Selling (GMVB), where the strategy weights are determined solving the following problem:

$$\begin{aligned} \min_{\omega} \quad & \tilde{\Sigma}_{t+1} \\ \text{s.t.} \quad & \omega' \mathbf{1} = 1, \quad \omega \geq \mathbf{0} \end{aligned}$$

The third and fourth are the Global Minimum Variance with no Short Selling and Upper Bound over weights set at $q\%$ with q equal respectively to 50% and 33%, (GMVB50 and GMVB33):

$$\begin{aligned} \min_{\omega} \quad & \tilde{\Sigma}_{t+1} \\ \text{s.t.} \quad & \omega' \mathbf{1} = 1, \quad 0 \leq \omega \leq \mathbf{q}. \end{aligned} \tag{4}$$

3 Investment sets

In the empirical comparison of the allocation strategies we considered the following datasets from the website of Kenneth French. The first dataset is characterized by five Industry portfolios in the US market: Consumer, Manufacture, Hi-tech, Healthcare and Others. The second dataset is composed by six asset portfolios sorted by Size and Book-to-Market. The portfolios, which are constructed at the end of each June, are the intersections of 2 portfolios formed on size (market equity, ME) and 3 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoint for year t is the median NYSE market equity at the end of June of year t (Small, Big). BE/ME for June of year t is the book equity for the last fiscal year end in $t - 1$ divided by ME for December of $t - 1$. The BE/ME breakpoints are the 30th and 70th NYSE percentiles (Value, Neutral, Growth). The third is composed by six asset portfolios sorted by Size and Short term reversal. The portfolios, which are constructed monthly, are the intersections of 2 portfolios formed on size (market equity, ME) and 3 portfolios formed on prior (1-1) return. The monthly size breakpoint is the median NYSE market equity (Small, Big). The monthly prior (1-1) return breakpoints are 30th and 70th NYSE percentiles (High, Medium and Low). The fourth is characterized by six asset portfolios sorted by Size and Momentum. The portfolios are the intersections of 2 portfolios formed on size (market equity) and 3 portfolios formed on prior (2-12) return. The monthly size breakpoint is the median NYSE market equity (Small, Big). The monthly prior (2-12) return breakpoints are 30th and 70th NYSE percentiles (High, Medium and Low). The last dataset is composed by six asset portfolios sorted by Size and Long term reversal. The portfolios, which are constructed monthly, are the intersections of 2 portfolios formed on size (market equity, ME) and 3 portfolios formed on prior (13-60) return. The monthly size breakpoint is the median NYSE market equity. The monthly prior (13-60) return breakpoints are 30th and 70th NYSE percentiles.

The portfolios constructed each month include NYSE, AMEX, and NASDAQ stocks with prior return data. All these dataset have been collected in the range April 1953 to December 2010 (693 observations).

In the robustness check of our results, we make use of the same datasets previously described but with a finer market decomposition. The six assets portfolios have been generalized to the 25 assets case by considering intersections between five categories for company Size, Book-to-Market, Short-term reversal, Momentum, and Long-term reversal. In this case the breakpoints have been fixed according to Fama and French to the equally spaced 20% quantiles. Furthermore, the industry portfolio has been generalized to the 10-assets sectoral decomposition: Consumer Non-durables, Consumer Durables, Manufacturing, Energy (oil, gas and coal extraction and production), Hi-tech, Telecommunications, Retail, Healthcare, Utilities, Others.

We then consider the following predicting variables: the dividend yield over the US stock market index (composite index of NYSE, NASDAQ and AMEX) obtained by difference from the value weighted index with and without dividends; the credit spread determined from Moody's AAA and BAA corporate issues; the term spread between the 10 years Treasury Bond and the 3 months Treasury Bills yields to maturity; the short term rate defined as the Treasury Bills yield. Finally, in order to translate the returns into real terms, we used as a proxy of the inflation the CPI index all consumer all goods obtained from the Federal Reserve on-line database (FRED).

All datasets include equity indices computed on a value weighted base at the monthly frequency. The Industry portfolios and the Size and Book-to Market datasets are similar to those considered by DGU. The main differences are: (i) the longer sample period and (ii) the different combination of the asset portfolios (we consider both the two and five group decompositions proposed by Fama and French) and (iii) the use of a rolling approach for comparing across time the strategy preference.

The other portfolios are included in the analysis for completeness and given the objective of the paper, i.e. that exploiting the mean reversion and predictability feature present in some of the datasets allow mean variance strategies to perform statistically better than the $1/N$ strategy. Furthermore, the introduction of Momentum and Short-term reversal is interesting given the purpose of evaluating strategy preference during large market movements (market turmoil), since those portfolios would have a faster reaction compared to portfolios based on Industries or on Long-term reversal.

4 Performance evaluation

The performance evaluation methodology we use is the same as the one adopted by DGU. First we choose a window for estimating the parameter based on 60 months (in the robustness section we consider also a window of 120 months). Second, we estimate the parameters over the estimation window for the particular mean and variance forecast model we consider, let indicate m indicate a particular mean and variance forecast method with $m = 1, 2, 3$. Third, using the estimated parameters we calculate the weights of the asset allocation strategy considered. Let k to indicate one of the 13 strategies considered. Fourth, we calculate the return $R_t^{m,k}$ from holding the portfolio with these weights over the next period, t , that is, out-of-sample. Fifth, we repeat this procedure for the next period, using a "rolling-window" approach, i.e. by including the data for the new date and dropping the data for the earliest. We repeat this procedure till the end of the dataset is reached. At the end for each strategy and each mean and variance forecast methodology we have the time series of the T returns determined out of sample that we represent with the vector $\underline{R^{m,k}}$. The initialization of the procedure creates a reduction in the sample dimension (60 months are needed in order to obtain the first out-of-sample forecast). All strategies and models provide an evaluation sample with length $T=633$ (when dealing with the 120 months window, the sample size reduces to $T=573$ observations).

The performance evaluation is based on two different analysis: (i) the T out of sample observation and (ii) on a rolling window of the out of sample portfolios returns with a size of M months.

In the first case, from the out of sample return vector $\underline{R^{m,k}}$ we calculate the mean, $\mu_T^{m,k}$ and the standard deviation $\sigma_T^{m,k}$ for all the T out of sample returns. The corresponding out of sample Sharpe Ratio is determined as $SR_T = \frac{\mu_T^{m,k}}{\sigma_T^{m,k}}$. In the second case we determined a time series of the out of sample sharpe ratio based on a window of M months. The corresponding out of sample Sharpe Ratios are determined as $SR_M = \frac{\mu_M^{m,k}}{\sigma_M^{m,k}}$. The inference on the Sharpe Ratio follows the approach of Ledoit and Wolf (2008). Given two Sharpe Ratios we test the null hypothesis of equivalence computing

the test-statistic p-values by a block-bootstrap approach. The bootstrap p-values are based on 1000 simulations and a block-size of 6 months.

5 Results from the full sample analysis

In this section we present the Sharpe Ratio and the p-values of the test of the null hypothesis that the Sharpe Ratio of the 1/N and those of the asset allocation strategies are the same. The analysis is performed on the full sample (therefore on $T = 633$ or $T = 573$).

The first result is that, in most of the cases the Sharpe Ratio based on the Mean-Variance strategy for the different Size-based datasets is larger than the one of the 1/N strategy when we focus on Mean-Variance. Differently, the results of constrained global minimum variance strategies are very close to the performances of the 1/N case. The results are opposite for the industry dataset where mean-variance strategies provide lower performances compared to the equally weighted allocation. The results are confirmed if we focus on 60 or 120 estimation windows, or if we modify the risk aversion value. However, when we analyse for a given dataset if the Sharpe Ratios of a given strategy is statistically different than the 1/N strategy, we have that for the Industry database the null hypothesis is accepted for both the 60 and 120 months estimation window. Differently, for the other datasets, the results are influenced by the estimation strategy. If we consider sample moments, we have limited evidences (at the 5% confidence level) for Mean-Variance strategies beating the 1/N for the Size and Momentum and Size and Short-Term reversal datasets (the result is valid for 60 and 120 months estimation period, and somewhat stronger for Size and Momentum with the longer estimation period). Results for the model with predictability are similar, and include also some cases (still at the 5% confidence level) also for the Size and Book-to-Market dataset. However, there is a much stronger evidence for a preference of Mean-Variance strategies compared to the equally weighted ones when we focus on the model with Mean-Reversion. Such a result is particularly evident for Size and Momentum and Size and Short-term reversal datasets. Again this is confirmed by varying the estimation sample size. Finally, if we include

both mean-reversion and predictability, only for the datasets including Momentum and Short-term reversal we note a statistical preference for Mean-Variance strategies with respect to the $1/N$.

This indicates that mean variance strategies are preferred when the asset allocation is based on Size, Momentum and Short-term reversal, i.e. in presence of mean reversion. This suggests that the predictability effect is less relevant, and also point out that the introduction of a larger number of parameters creates an increase of the estimation error that might affect the benefits coming from the mean-reversion component of the model. In conclusion is preferable to exploit the mean-reversion effect and that the use of predictor variables has the negative effect of increasing the estimation error.

All the previous results are confirmed if we consider the Fama-French datasets composed by 25 portfolios instead of 6 portfolios. However, overall, we have as in DGU that there is not a strategy that among all the different datasets is always performing better than the $1/N$ with many cases of strategy equivalence. Moreover, the predictability component plays only a limited role, while the possible presence of mean-reversion seems to be more relevant.

The results on the mean-reversion is however quite surprising and we investigate the reason for this. We investigate first the relevance of the mean reversion in the full sample estimation of the models. If we consider the Size and Book-to-Market sample we have that the portfolio-specific R^2 is very low: it ranges from 3.50% to 5.93%. If we consider the Industry dataset, the R^2 is quite similar, it ranges from 2.52% till 6.31%. If we consider the R^2 evaluated on rolling samples of 60 or 120 months, the relevance of predictability and mean-reversion sensibly change over time. Figure 1 reports an example. Furthermore, if we analyse the number of coefficients that are statistically significant in the mean-reversion specification, we find that they largely depend on the period considered. In fact, there are sample periods where almost one half of the coefficients are statistically significant and others where this number is zero. These results indicate that there are sample periods where there is a possibility of exploiting the mean reversion feature, even if in the full sample this feature is not so relevant. Moreover, when such

a feature is stronger, the model estimation error reduced and optimal mean-variance strategies over-perform the $1/N$ naive strategy.

In summary, one key aspect of our result is that not only the number of observation in the sample period considered is relevant, as stressed by DGU, but also the period considered.

[FIGURE 1]

6 Rolling performance evaluation and market volatility

In order to identify the reasons of the previous result, we repeat the Sharpe Ratio test for a rolling window of 5 years and 10 years. For each sample we performed the robust test of Ledoit and Wolf (2008) for the equivalence of the Sharpe Ratios of the $1/N$ strategy and of an optimized strategy. Using a rolling approach on 60 months, we have a time series of $T=525$ evaluations points. In this case our analysis is implicitly biased toward accepting the null hypothesis. In fact, the estimation errors increase when the number of observations are lower.

As we expect, the strategy preference (or equivalence) is influenced by several factors: the indices used, the model adopted, and the estimation window size. As illustrative example, Figure 2 reports the p-values for the test of Ledoit and Wolf (2008). P-values lower than 10% suggest a preference for the optimized strategies, while p-values larger than 90% suggest a preference for the equally weighted portfolio. Table 1 summarizes the frequency of strategy preference across datasets, strategies and estimation window size. In general, we observe that in most cases there is a preference for either the $1/N$ or the optimized strategy is less than 50% of the estimates. However, if we compare results across the optimized strategies, we note the following elements: in most cases the frequency of preference for optimized strategies is larger than that of the equally weighted portfolio; the introduction of mean-reversion sensibly increases the frequency of preference for Mean-Variance strategies and is only marginally affecting the preference for Global Minimum Variance strategies; as a consequence of this second element, the

preference for the equally weighted portfolio decreases when mean-reversion is included in the model. Moreover, results also differ across datasets. The preference for optimized strategies is larger for the Size and Book-to-Market and for the Size and Momentum cases. In the Size and Short-term reversal, results are quite different from the other when considering the Global Minimum Variance portfolio. We motivate this finding by the fact that the French indices are created taking into account the previous month return, thus something strongly related to the mean-reversion. Therefore, it is not surprising that the $1/N$ is here quite frequently preferred.

[FIGURE 2]

In order to further relate the strategy preference to the entire equity market volatility, we evaluate the strategy preference relations when the equity market is in the high-volatility state. We determined the occurrence of high-volatility by mean of a simple two-state Markov Switching model (with switching mean and variance) fitted on the Fama-French market factor from January 1960 to December 2010. The identified high volatility periods are the following: from March to December 1962; from February 1969 to September 1970; from March 1973 to January 1976; from June 1978 to November 1982; from May 1986 to January 1988; from January 1990 to February 1991; from April 1998 to April 2003, from November 2007 to December 2010. The identified periods are reasonable and correspond to known high-volatility occurrences in the equity market. We thus report in Table 1 the percentage of strategy preference over the last high-volatility period and over all the other high-volatility periods. Results indicate that, during the recent financial crisis, the large market volatility strongly affected the model estimation leading to a large estimation error, that is to a very large uncertainty about the mean and variance forecasts. Therefore, in most cases the performances of the $1/N$ strategy and of the optimized strategies turn out to be statistically equivalent. This is due both to the increase of the estimation error, potentially harming the performances of the optimized strategies, as well as to the large market volatility, that influences the asymptotic properties of the test statistic.

If we compare the strategy preference over the last high volatility period to that observed over previous high volatility occurrences, results are mixed but the most common

outcome is related to an increase in the preference for 1/N portfolios of for an increase in the equivalence across portfolio performances. Such a result points at the effect of model estimation error that induce poor performances on optimized strategies. As a result, during turbulent market periods, equally weighted portfolios seem to be a good compromise between the need for diversification and the search of performance.

7 Conclusions

Our analysis shows that exploiting the mean reversion and predictability feature present in some of the datasets allow mean variance strategies to perform statistically better than the 1/N strategy. Our investigation on the R^2 shows that this result is not strictly related to significant ability of the model to explain the dynamic of the single asset return. The key determinant is the presence of prediction that implies weights largely different than the 1/N weights. When these differences are not so large, optimal mean-variance strategies do not perform statistically better than the naive strategy based of equally weighted portfolios. The rolling window analysis of the shape ratio shows that there are periods where the mean variance strategies perform statistically better and others where they are not statistically different. This means that there are only some periods where mean reversion really matters for asset allocation strategies. The practical implications of our results is that when the mean reversion model is producing a forecast for the mean that is significantly different than the one implied by the 1/N model, it is worth to implement the optimization strategy, in the other cases it is preferable to perform the 1/N strategy. Sadly, crisis periods are those where usually mean variance optimal strategies, given the large estimation errors, present optimal weights that are very similar to the 1/N naive strategy.

8 References

Barry, C. B. (1974) Portfolio Analysis under Uncertain Means, Variances, and Covariances. *Journal of Finance*, 29: 515-522.

Bawa, V. S., S. Brown, and R. Klein (1979) Estimation Risk and Optimal Portfolio Choice, North Holland, Amsterdam.

Best, M. J., and R. R. Grauer (1991) On the Sensitivity of Mean-Variance-Efficient Portfolios to Changes in Asset Means: Some Analytical and Computational Results. *Review of Financial Studies*, 4: 315-242.

Best, M. J., and R. R. Grauer (1992) Positively Weighted Minimum-Variance Portfolios and the Structure of Asset Expected Returns. *Journal of Financial and Quantitative Analysis*, 27: 513-537.

Campbell, J.Y. and Viceira, L.M. (1999) Consumption and portfolio decisions when expected returns are time varying. *Quarterly Journal of Economics*, 114: 433-495.

Campbell, J.Y. and Viceira, L.M. (2001) Who Should Buy Long-Term Bonds? *American Economic Review* 91: 99-127.

Chan, L. K. C., J. Karceski, and J. Lakonishok (1999) On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model. *Review of Financial Studies*, 12: 937-974.

Chopra, V. K. (1993) Improving Optimization. *Journal of Investing*, 8: 51-59.

De Miguel V., L. Garlappi, F. Nogales, R. Uppal (2009) A Generalized Approach to Portfolio Optimization: Improving Performance by Constraining Portfolio Norms. *Management Science*, 55: 798-812.

De Miguel V., L. Garlappi and R. Uppal. (2009) Optimal versus Naive Diversification: How Inefficient Is the 1/N Portfolio Strategy? *The Review of Financial Studies*, 22: 1915-1953.

Frost, P. A., and J. E. Savarino (1986) An Empirical Bayes Approach to Efficient Portfolio Selection. *Journal of Financial and Quantitative Analysis*, 21: 293-305.

Garlappi L., R. Uppal, and T. Wang (2007) Portfolio Selection with Parameter and Model Uncertainty: A Multi-Prior Approach. *The Review of Financial Studies*, 20: 41-81.

Goldfarb D. and G. Iyengar (2003) Robust Portfolio Selection Problems. *Mathematics of Operational Research* 28: 1-38.

Green, R., and B. Hollifield (1992) When will mean-variance efficient portfolios be well diversified. *Journal of Finance*, 47: 1785-1809.

Hodges, S. D., and R. A. Brealey (1978) Portfolio Selection in a Dynamic and Uncertain World, in James H. Lorie, and R. A. Brealey (ed.), *Modern Developments in Investment Management*, Dryden Press, Hinsdale, Illinois, 2nd edn.

Jagannathan, R., and T. Ma (2003) Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps. *Journal of Finance*, 58: 1651-1684.

Jobson, J. D., and R. Korkie (1980) Estimation for Markowitz Efficient Portfolios. *Journal of the American Statistical Association*, 75: 544-554.

Jorion, P. (1985) International Portfolio Diversification with Estimation Risk. *Journal of Business*, 58: 259-278.

Jorion, P. (1986) Bayes-Stein Estimation for Portfolio Analysis. *Journal of Financial and Quantitative Analysis*, 21: 279-292.

Kan, R., and G. Zhou (2005) Optimal estimation for economic gains: Portfolio choice with parameter uncertainty. Working paper, University of Toronto.

Ledoit, O., and M. Wolf (2004b) Honey, I Shrunk the Sample Covariance Matrix. *Journal of Portfolio Management*, 30: 110-119.

Ledoit, O., and M. Wolf (2008) Robust performance hypothesis testing with the Sharpe Ratio. *Journal of Empirical Finance*, 15: 850-859.

Litterman, R. (2003) *Modern Investment Management: An Equilibrium Approach*, Wiley.

MacKinlay, A. C., and L. Pastor (2000) Asset Pricing Models: Implications for Expected Returns and Portfolio Selection. *Review of Financial Studies*, 13: 883-916.

Merton, R. C. (1980) On Estimating the Expected Return on the Market: An Exploratory Investigation. *Journal of Financial Economics*, 8: 323-361.

Michaud, R. O. (1989) The Markowitz Optimization Enigma: Is Optimized Optimal. *Financial Analysts Journal*, 45: 31-42.

Pastor, L. (2000) Portfolio Selection and Asset Pricing Models. *Journal of Finance*, 55:179-223.

Pastor, L., and R. F. Stambaugh (2000) Comparing Asset Pricing Models: An Investment Perspective. *Journal of Financial Economics*, 56: 335-381.

Figure 1: Rolling R-squared

The figure reports the rolling R-squared for selected datasets referred to the Mean-Reversion model. The estimation window has a size of 60 months.

Figure 2: Robust Sharpe testing across forecasts

The figure reports the P-value for testing the equivalence of the mean-variance strategy without short selling and of the equally weighted portfolio. We consider the Size and Momentum portfolios, the risk aversion has been set to 3 and the mean and variance forecasts have been set equal to the sample moments, to the model with predictability and to the one with mean reversion. Results are based on a 60 months rolling window.