Ca' Foscari
University
of Venice

**Department
of Economics**

Working Paper

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Matching Frictions and
Business Creation**

ISSN: 1827/3580
No. 30/WP/2012

Technology Adoption, Job Matching Frictions and Business Creation

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This Draft: October 2012

Abstract

We study the impact of a General Purpose Technology that changes the cost structure turning fixed costs into variable ones in most sectors. A major recent example is cloud computing, whose adoption allows firms to avoid large up-front costs in IT and to rent computing capability online. We study the macroeconomic impact of the adoption of such an innovation in a DSGE model with endogenous market structures in the goods market and search and matching frictions in the labor market. To start a business, firms need to hire workers from the pool of unemployed agents and set up a stock of IT capital: the new technology allows them to rent computing capabilities reducing entry costs. Such an innovation can have a substantial impact on business and job creation.

Keywords

Endogenous entry, General purpose technology, Job creation, Search frictions, Cloud computing

JEL Codes

L1, J6, O3

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1 Introduction

In this work we study the macroeconomic impact of the diffusion of a General Purpose Technology (GPT) that changes the cost structure in multiple sectors, reducing fixed entry costs and increasing the variable costs. Standard RBC models (Kydland and Prescott, 1982) are unable to examine the impact of such a structural change because they require zero fixed costs under perfect competition. Models with monopolistic competition and positive profits (Christiano *et al.*, 2005) are also unable to do it because the cost structure does not affect the structure of the markets for goods or for labor. A recent literature started with Etro and Colciago (2010), Bilbiie *et al.* (2012) and other works, has introduced imperfect competition with endogenous entry in otherwise standard DSGE models, allowing one to examine shocks as the one described above. In a model with imperfect competition and endogenous business and job creation we can actually examine the impact of such a structural change of the cost structure at the macroeconomic level.

GPTs that affect the cost structure (rather than simply increasing productivity) are not that frequent, but they often generate radical changes in the economy. Innovations in energy provision (think of electricity) have been often of this kind, eliminating local fixed costs of energy provision and replacing them with variable costs of provision on demand. The same role has been played by fixed and mobile phones, which have turned fixed costs of communication into variable and much lower costs. Another major contemporary example on which we will focus the interpretation of our analysis is cloud computing, an Internet-based general purpose technology that is now spreading at the global level (see Fershtman and Gandal, 2012, or Etro, 2009a for an economic evaluation). Through this technology information can be stored in servers and provided on line as a service to business costumers in a pay-as-you-go manner: in a sense, we are moving toward a world where we will pay for computing capability as we pay for electricity, phone calls or any other utilities. The provision of cloud computing allows firms to avoid large up-front costs for hardware and software equipment and turn part of them into variable costs to be optimized at the margin. As a result they will be able to adjust their demand for IT technology according to their needs. The reduction in the up-front required investment will be crucial for business and job creation and for the growth of young firms which typically face stringent financial constraints and thus cannot engage in large initial investments.¹ The competition effect asso-

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¹Moreover, the new technology is likely to promote new investment in R&D for applications developed on the clouds (Borek *et al.*, 2012).

ciated with new entry is going to magnify the initial impact and create long run consequences for aggregate production and employment.

In this work we evaluate the impact of the adoption of such a GPT on output, employment and business creation. We do so resorting to a DSGE model characterized by endogenous market structures (EMSs) and job matching frictions in the spirit of Etro and Colciago (2010), Colciago and Rossi (2011) and Bilbiie *et al.* (2012). Our model is characterized by three industries: the one producing final goods, the industry producing the physical IT stock, and the industry providing IT maintenance and development services. The final good industry features many sectors, where the dynamics of the number of market competitors is endogenous. Firms face a cost of entry in the market which they decide to bear only if compensated by the expectation of future profits. They produce the final goods using labor and physical capital. The stock of capital takes the form of IT hardware which the firm has to install and maintain over time. The industry producing IT adopts physical capital as the only input, while in the industry providing IT maintenance and development services the input is labor. We define the fraction of workers employed in the latter industry as IT employment.

The labor market is characterized by frictions following the literature on job search and matching (see Pissarides, 1986, 1988, 2000; Merz, 1995; Andolfatto, 1996). In the final sector, both new firms and incumbent firms need to hire workers from the pool of unemployed agents who are looking for a job and to set up a stock of IT material before starting production. Similarly, the industry providing IT services faces labor market frictions. The role of search frictions is crucial to improve the ability of models with EMS to capture the behavior of the job market. Since a job is lost when a firm goes bankrupt, the pattern of unemployment depends on the rate of matching between vacancies and unemployed workers (which should be increasing in labor demand and decreasing in the unemployment force) and on business destruction.² We assume that employment is chosen to maximize the joint surplus of a job match and that wages are the result of Nash bargaining between firms and workers (as in the static model with endogenous markups of Blanchard and Giavazzi, 2003). In steady state, the unemployment rate must be increasing in the rate of business destruction and decreasing in all the factors that improve the matching technology. Endogenizing the matching function, Colciago and Rossi (2011) have analyzed a new channel through which structural parameters affect the labor force and, through that, the EMSs. In line with U.S. data, they find that new firms account for a relatively small share of overall employment, but they create a rel-

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²See Etro (2009,b) for a related discussion.

evant fraction of new jobs.³ Moreover, in response to a technology shock the labor share decreases on impact and overshoots its long run level, exactly what we observe in the data. Finally, the propagation on labor market variables is much stronger than in the standard search models because booms increase competition and through that wages and hiring activity. Here we examine the impact of a structural technological change on job creation taking into account these search and matching frictions.

In line with the observations made above, the model counterpart of the introduction of a GPT such as cloud computing will be a reduction in the installation and maintenance cost of IT for the individual firm. IT services, hardware and software, can be outsourced by the firm. In particular they can be obtained on demand by the provider of cloud IT services. This reduces the upfront sunk entry costs, since the firm no longer needs to build up a stock of IT before starting production. Also, it reduces maintenance costs of the IT stock. As a result of lower entry costs, there is a stronger incentive for new firms to enter into the market which promotes investment and the demand faced by incumbent producers. In turn, the creation of new firms and the growth of the existing ones promotes both competition and job creation affecting the unemployment rate. Importantly, although maintenance and development cost gets lower for the individual firm, higher IT usage promotes a long run increase, after some periods of reduction, in IT employment.

We calibrate and simulate the model to show the sizable impact of GPT on job and business creation. Notice that the empirical literature supports the view that new firms have a fundamental role for the creation of new jobs. Haltiwanger *et al.* (2010) on the basis of U.S. manufacturing data between 1972 and 1986 estimate that 25 percent of annual gross job creation is due to new establishments births. Similarly, Jaimovich and Floetotto (2008) focus on employment data at the establishment level. They estimate that the average fraction of quarterly job-gain (losses) that can be explained by the opening (closing) of establishments is about 20 percent. Therefore, our analysis of job creation derived from business creation appears well founded in the macroeconomic empirical literature.

We find that the introduction of a technology such as cloud computing can induce pervasive effects on job and business creation. In particular, the introduction of the GPT lowers the up-front entry costs in terms of IT. New firms post a large amount of vacancies to reach their desired size. This results in a persistent change in aggregate em-

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³See Moscarini and Postel-Vinay (2012) on evidence on job creation by small and new firms versus large incumbents.

ployment. A larger number of firms leads to a stronger usage of IT. Higher employment together with a higher IT stock lead in turn to a sustained increase in aggregate output. Finally, entry of new firms strengthens competition and reduces the markups, which increases real wages and magnifies the impact of the innovation. Importantly, all these effects would be neglected in the standard DGSE model used in macro-economics, where perfect competition and constant returns to scale or monopolistic competition with exogenous market structures exclude any impact of changes in the cost structure on the market structures, the level of competition, the labor market and the process of business and job creation.

The article is organized as follows. Section 2 develops the basic DSGE model with IT investment. Section 3 augments it with the introduction of cloud computing. Section 4 calibrates the model and analyzes the transition to the cloud economy. Finally Section 5 concludes.

2 The Model

The structure of the model is a DSGE model augmented with endogenous market structures (as opposed to perfectly competitive or monopolistically competitive markets) following Colciago and Etro (2010) and Etro and Colciago (2010).⁴ The economy features a continuum of atomistics sectors, or industries, on the unit interval. Each sector is characterized by different firms producing a good in different varieties, using labor and IT material as input. In turn, the sectoral goods are imperfect substitutes for each other and are aggregated into a final good. The IT material is produced by a perfectly competitive firm using physical capital as the only input. Households use the final good for consumption and investment purposes. Price competition and endogenous firms' entry is modeled at the sectoral level, where firms also face search and matching frictions in hiring workers, modeled in the tradition of the literature on job search. In the pre-cloud economy they also face maintenance costs of the IT material.

2.1 Labor Market and Job Matching

The labor market is characterized by search and matching frictions, as in Mortensen and Pissarides (1994), Merz (1995) and Andolfatto (1996).⁵ Firms producing in t need to post vacancies in order to hire new workers. Unemployed workers and vacancies combine according to a CRS match-

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⁴See Etro (2009b) for a survey of the related literature on endogenous market structures in general equilibrium.

⁵The first analysis of unemployment in a real business cycle model is due to Hansen (1985), but neglecting job search frictions.

ing function and deliver m_t new hires, or matches, in each period.⁶ The matching function is assumed a Cobb-Douglas one:

$$m_t = (\gamma_m) (v_t^{tot})^{1-\gamma} u_t^\gamma \quad (1)$$

where γ_m reflects the efficiency of the matching process, v_t^{tot} is the total number of vacancies created at time t and u_t is the unemployment rate. The probability that a firm fills a vacancy is given by $q_t = m_t/v_t^{tot}$, while the probability to find a job for an unemployed worker reads as $z_t = m_t/u_t$. Firms and individuals take both probabilities as given. Matches become productive in the same period in which they are formed. Each firm separates exogenously from a fraction $1 - \varrho$ of existing workers each period, where ϱ is the probability that a worker stays with a firm until the next period. As a result a worker may separate from a job for two reasons: either because the firm where the job is located exits from the market or because the match is destroyed. Given that population is normalized to one, the number of unemployed workers and the unemployment rate are identical. Therefore, given labor at time t as L_t , the unemployment rate is:

$$u_t = 1 - L_{t-1} \quad (2)$$

and represents also the fraction of agents searching for a job.

Given our functional form, we can express the probability of filling a vacancy as:

$$q_t = \gamma_m \left(\frac{v_t^{tot}}{u_t} \right)^{-\gamma} = \gamma_m (\theta_t)^{-\gamma} \quad (3)$$

where we define $\theta_t = v_t^{tot}/u_t$. The probability of finding a job becomes:

$$z_t = \frac{(\gamma_m) (v_t^{tot})^{1-\gamma} u_t^\gamma}{u_t} = \gamma_m (\theta_t)^{1-\gamma} \quad (4)$$

and the ratio of the two probabilities as:

$$\frac{z_t}{q_t} = \frac{\gamma_m (\theta_t)^{-\gamma}}{\gamma_m (\theta_t)^{1-\gamma}} = \theta_t$$

2.2 Households

Using the family construct of Merz (1995) we can refer to a representative household as consisting of a continuum of individuals of mass one.

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⁶For microfoundations of the matching function see Lagos (2000) and Petrongolo and Pissarides, (2001). Early models include Lucas and Prescott (1974). More recent applications include Andolfatto (1996) on business cycle models and Acemoglu and Shimer (1999) on unemployment insurance in case of risk aversion.

Members of the household insure each other against the risk of being unemployed. The representative family has lifetime utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \chi L_t \frac{h_t^{1+1/\varphi}}{1 + 1/\varphi} dj \right\} \quad \chi, \varphi \geq 0 \quad (5)$$

where $\beta \in (0, 1)$ is the discount factor, the variable h_t represents individual hours worked and C_t is the consumption of the final good. The family receives real labor income $w_t h_t L_t$, where w_t is the real wage, and profits Π_t from the ownership of firms. Unemployed individuals receive a real unemployment benefit b , hence the overall benefit for the household is $b(1 - L_t)$. This is financed through lump sum taxation by the government. The households hold the stock of physical capital, K_t , which evolves according to

$$K_t = (1 - \delta^k) K_{t-1} + I_t^k \quad (6)$$

where I_t^k is investment in capital. The household chooses how much to save in riskless bonds, physical capital and in the creation of new firms according to standard Euler and asset pricing equations. The first order condition with respect to employment, L_t , is

$$\Gamma_t = \frac{w_t h_t}{C_t} - \chi \frac{h_t^{1+1/\varphi}}{1 + 1/\varphi} - \frac{b}{C_t} + \beta E_t [(1 - \delta) \rho - z_{t+1}] \Gamma_{t+1} \quad (7)$$

where Γ_t is the marginal value to the household of having one member employed rather than unemployed and $1/C_t$ is the marginal utility of consumption. Equation (7) indicates that the household's shadow value of one additional employed member (the left hand side) has four components: first, the increase in utility generated by having an additional member employed, given by the real wage expressed in utils; second, the decrease in utility due to more hours dedicated to work, given by the marginal disutility of employment; third the foregone utility value of the unemployment benefit; fourth, the continuation utility value, given by the contribution of a current match to next period household's employment.

2.3 Technology

There are four types of firms in the economy. The producers of intermediate goods, the final good producer, the producers of IT material and the providers of maintenance services for IT. We describe them in the remainder of the section. The final good is an aggregate of a continuum of mass one of sectoral goods defined as

$$Y_t = \left[\int_0^1 \ln Y_{jt}^{\frac{\omega-1}{\omega}} dj \right]^{\frac{\omega}{\omega-1}} \quad (8)$$

where Y_{jt} denotes output of sector j and ω is the elasticity of substitution between any two different sectoral goods. The final good producers behave competitively. In each sector j , there are $N_{jt} > 1$ firms producing differentiated goods that are aggregated into a sectoral good by a CES aggregating function defined as

$$Y_{jt} = \left[\sum_{i=1}^{N_{jt}} y_{jt}(i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (9)$$

where $y_{jt}(i)$ is the production of good i in sector j , $\varepsilon > 1$ is the elasticity of substitution between sectoral goods. As in Etro and Colciago (2010), we assume a unit elasticity of substitution between goods belonging to different sectors. This allows to realistically separate limited substitutability at the aggregated level, and high substitutability at the disaggregated level. Each firm i in sector j produces a (intermediate) differentiated good with the following production function

$$y_{jt}(i) = A_t [n_{jt}(i) h_{jt}(i)]^{1-\alpha} [IT_{jt-1}(i)]^\alpha \quad (10)$$

where A_t represents technology which is common across sectors and evolves exogenously over time. The variable $n_{jt}(i)$ is firm i 's time- t workforce used for the production of the final good, and $h_{jt}(i)$ represents hours per employee. In the remainder we will refer to firms in the intermediate goods sector as producers. The variable $IT_{jt}(i)$ is the amount of IT material involved in the production process. The latter is produced by a perfectly competitive firm which uses physical capital as the only input. In each period a flow of IT, defined as ΔIT_t , is produced with technology

$$\Delta IT_t = A_t^c K_{t-1} \quad (11)$$

where K_t is the stock of capital in the economy and A_t^c is the productivity of the IT industry. Given perfect competition the price of IT material is the marginal cost of production. The latter can be obtained by profit maximization of the producer of IT services as $p_t^{IT} = r_t^k / A_t^c$. Period- t real profits of a producer are defined as

$$\pi_{jt}(i) = \rho_{jt}(i) y_{jt}(i) - w_{jt} h_{jt} n_{jt} - p_t^M M_{jt}(i) - \kappa v_{jt}(i) - p_t^{IT} I_{jt}^{IT}(i) \quad (12)$$

where $w_{jt}(i)$ is the real wage paid by firm i , $v_{jt}(i)$ represents the number of vacancies posted at time t , κ is the output cost of keeping a vacancy

open, $I_{jt-1}^{IT}(i)$ is period t investment in IT and p_t^{IT} is the price of a unit of IT in terms of the final good. Notice that $\rho_{jt}(i)$ is the real price of firm i 's output.

The term $p_t^M M_{jt}(i)$ represents maintenance and development costs of the IT stock. These services are provided by a firm which operates in perfect competition with technology $M_t = n_t^{IT} h_t$, where n_t^{IT} represents the number of workers employed in the industry. The provider of maintenance services also faces search costs in the labor market. It hires workers by posting vacancies at an output cost equal to κ , taking as given hours and the real wage determined in the bargaining process between workers and firms operating in the final good industry.⁷ Its workforce evolves according to

$$n_{jt}^{IT} = \varrho n_{t-1}^{IT} + v_t^{IT} q_t$$

The problem faced by the provider of maintenance services can thus be written as

$$\begin{aligned} & \max_{\{n_s^{IT}, v_s^{IT}\}_{s=t}^{\infty}} E_t \sum \Lambda_{t,s} (p_s^M M_s - w_s h_s n_s^{IT} - \kappa v_s^{IT}) \\ & s.t. \ n_s^{IT} = \varrho n_{s-1}^{IT} + v_s^{IT} q_s \end{aligned}$$

Profit maximization requires

$$\frac{\kappa}{q_t} = (p_t^M h_t - w_t h_t) + \varrho E_t \frac{\kappa}{q_{t+1}} \Lambda_{t,t+1}$$

this condition equates the marginal cost of hiring a worker with the marginal benefit. The latter is given by a discounted stream of firm's expected future net earnings from the marginal worker. We assume a maintenance technology such that the final good producer must acquire $\frac{m}{A_t^c}$ units of maintenance services for each unit of IT owned. As a result the individual demand of maintenance services is $\frac{m}{A_t^c} I_{jt-1}^{IT}(i)$ and we can rewrite profits of the final good producer as

$$\pi_{jt}(i) = \rho_{jt}(i) y_{jt}(i) - w_{jt} h_t n_t - p_t^M \frac{m}{A_t^c} I_{jt-1}^{IT}(i) - \kappa v_{jt}(i) - p_t^{IT} I_{jt-1}^{IT}(i)$$

The value of a final good producer is the expected discounted value of its future profits

$$V_{jt}(i) = E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} \pi_{js}(i) \quad (13)$$

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⁷In this case it is indifferent for a member of the household to work in the IT sector or in the final sector.

where $\Lambda_{t,t+1} = (1 - \delta) \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1}$ is the households' stochastic discount factor which takes into account that firms' survival probability is $1 - \delta$. Firms which do not exit from the market have a time- t individual workforce given by

$$n_{jt}(i) = \varrho n_{jt-1}(i) + v_{jt}(i) q_t \quad (14)$$

and a stock of IT equal to

$$IT_{jt}(i) = (1 - \delta^{IT}) IT_{jt-1}(i) + I_{jt}^{IT}(i) \quad (15)$$

where δ^{it} is the depreciation rate of IT material. The unit intersectoral elasticity of substitution implies that the nominal expenditure, EXP_t , is identical across sectors. Thus, the final producer's demand for each sectoral good is

$$P_{jt} Y_{jt} = P_t Y_t = EXP_t \quad (16)$$

where P_{jt} is the price index of sector j and P_t is the price of the final good at period t . Denoting with $p_{jt}(i)$ the price of good i in sector j , the demand faced by the producer of each variant is

$$y_{jt}(i) = \left(\frac{p_{jt}(i)}{P_{jt}} \right)^{-\varepsilon} Y_{jt} \quad (17)$$

where P_{jt} is defined as

$$P_{jt} = \left[\sum_{i=1}^{N_{jt}} (p_{jt}(i))^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (18)$$

Using (17) and (16) the direct demand of good i in sector j can be written as a function of aggregate expenditure,

$$y_{jt}(i) = \frac{p_{jt}^{-\varepsilon}}{P_{jt}^{1-\varepsilon}} EXP_t \quad (19)$$

The same direct demand can be inverted to obtain the inverte demand of each firm in function of the production levels of all the firms (see Etro and Colciago, 2012).

2.4 Entry

At the beginning of each period N_{jt}^e new firms enter into sector $j \in (0, 1)$, while at the end of the period a fraction $\delta \in (0, 1)$ of market participants

exits from the market for exogenous reasons. As a result, the number of firms in a sector N_{jt} , follows the equation of motion:

$$N_{jt+1} = (1 - \delta)(N_{jt} + N_{jt}^e) \quad (20)$$

where N_{jt}^e is the number of new entrants in sector j at time t . Following Bilbiie *et al.* (2012) we assume that new entrants at time t will only start producing at time $t + 1$ and that the probability of exit from the market, δ , is independent of the period of entry and identical across sectors. The assumption of an exogenous constant exit rate is adopted for tractability, but it also has empirical support. Using U.S. annual data on manufacturing, Lee and Mukoyama (2007) find that, while the entry rate is procyclical, annual exit rates are similar across booms and recessions. Below we describe the entry process and the mode of competition within in each sector in detail. Prior to entry, firms face a sunk entry cost ϕ_t^e to be paid in order to serve the market. It is made by two components:

$$\phi_t^e = \phi_t^{ad} + p_t^k (I_{jt}^{IT}(i))^{new}$$

The first term ϕ_t^{ad} represents the cost associated with regulation and barriers to entry, which is common across sectors. It is exogenous and expressed in units of the final good. The second component of the entry cost reflects instead the fact that in order to start production in the next period new firms must set up a stock of IT. This requires an amount of investment in IT given by $(I_{jt}^{IT}(i))^{new}$. If the firm exits from the market its IT stock is lost. Firms will enter into the market up to the point where their value, represented by the discounted value of their future profits, equals the sunk entry cost ϕ_t^e .

2.5 Imperfect Competition and Job Creation

We can consider different forms of competition in each period between firms. If they compete in prices, each firm i in sector j chooses its own $p_{jt}(i)$ taking as given the prices of the others and aggregate spending in the sector. If they compete in quantities, the same firm chooses its own production level $y_{jt}(i)$ taking as given the output of the other firms and aggregate spending in the sector: in this case all the prices must satisfy the inverse demand functions of the firms. Moreover, each firm i chooses also $n_{jt}(i)$, $v_{jt}(i)$ and $IT_{jt}(i)$ to maximize $\pi_t(i) + V_t(i)$.

As shown by Etro and Colciago (2010), in a symmetric equilibrium the price satisfies a mark up rule as

$$\rho_t(\varepsilon, N_t) = \mu_t m c_t \quad (21)$$

where the markup over the marginal cost μ_t is given by

$$\mu_t(\varepsilon, N_t) = \frac{\varepsilon N_t}{(\varepsilon - 1)(N_t - 1)} \quad (22)$$

under competition in quantities or

$$\mu_t(\varepsilon, N_t) = \frac{\varepsilon(N_t - 1) + 1}{(\varepsilon - 1)(N_t - 1)} \quad (23)$$

under competition in prices. In both cases, this is decreasing in the number of firms in the sector because competition is stronger when there are more firms. The FOC with respect to vacancies reads as

$$\phi_t = \frac{\kappa}{q_t} \quad (24)$$

Thus, the firm sets the value of the marginal worker, ϕ_t , equal to the expected cost of hiring the worker, $\frac{\kappa}{q_t}$. The FOC with respect to n_t delivers

$$\phi_t = \left[(1 - \alpha) mc_t A_t \left(\frac{IT_{t-1}}{n_t h_t} \right)^\alpha h_t - w_t h_t \right] + \beta \rho \frac{C_t}{C_{t+1}} E_t \phi_{t+1} \quad (25)$$

Combining the latter two equations delivers the following *Job Creation Condition* (JCC)

$$\frac{\kappa}{q_t} = (1 - \alpha) \frac{\rho_t}{\mu_t} A_t \left(\frac{IT_{t-1}}{n_t h_t} \right)^\alpha h_t - w_t h_t + \rho E_t \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} \quad (26)$$

where we used the pricing condition to substitute for $mc_t = \rho_t / \mu_t$. Since this ratio increases in the number of firms, it follows that competition leads to a rise in the marginal cost and hence in the equilibrium marginal revenue. For this reason the marginal revenue product of labor (MRP), given by $(1 - \alpha) (\rho_t / \mu_t) A_t (IT_{t-1} / n_t h_t)^\alpha h_t$, also rises with competition. Thus, stronger competition promotes the creation of vacancies and employment due to its positive effect on the MRP of labor. The firm will invest in IT up to the point where

$$p_t^{IT} = \Lambda_{t,t+1} \left[\frac{\rho_{t+1}}{\mu_{t+1}} A_{t+1} \alpha \left(\frac{IT_t}{n_{t+1} h_{t+1}} \right)^{\alpha-1} - p_{t+1}^M \frac{m}{A_{t+1}^c} \right] + \Lambda_{t,t+1} p_{t+1}^{IT} (1 - \delta^{IT}) \quad (27)$$

Increasing IT by one unit today costs p_t^{IT} . The benefit associated to the marginal unit of IT is given by the discounted marginal revenue product of IT net of maintenance costs, the first term on the right hand side, summed to the discounted value that the additional unit of IT will have tomorrow, $\Lambda_{t,t+1} p_{t+1}^{IT} (1 - \delta^{IT})$. Since IT is a stock variable, the firm is forced to look ahead when taking decisions concerning optimal investment in IT.

2.6 Bargaining over Wages and Hours

Bargaining takes place along two dimensions (Trigari, 2009): on the real wage and the hours of work. We assume Nash bargaining (see MacDonald and Solow, 1981). That is, the firm and the worker choose the wage w_t and the hours of work h_t to maximize the Nash product

$$(\phi_t)^{1-\eta} (\Gamma_t C_t)^\eta \quad (28)$$

where ϕ_t is firm value of having an additional worker, while $\Gamma_t C_t$ is the household's surplus expressed in units of consumption. The parameter η reflects the parties' relative bargaining power. The FOC with respect to the real wage is

$$\eta \phi_t = (1 - \eta) \Gamma_t C_t \quad (29)$$

Using the definition of ϕ_t in equation (25) and that of Γ_t given by equation (7), after some manipulations, yields the wage equation

$$\begin{aligned} w_t = (1 - \eta) \frac{b}{h_t} + (1 - \eta) \chi C_t \frac{h_t^{1/\varphi}}{1 + 1/\varphi} + \\ + \frac{\eta \kappa}{1 - \delta} E_t \Lambda_{t,t+1} \frac{\theta_{t+1}}{h_t} + (1 - \alpha) \eta \frac{\rho_t}{\mu_t} A_t \left(\frac{IT_{t-1}^A}{n_t N_t h_t} \right)^\alpha \end{aligned} \quad (30)$$

where $\phi_t = \kappa/q_t$, $z_t/q_t = \theta_t$, $IT_{t-1}^A = N_t IT_{t-1}$. The wage shares costs and benefits associated to the match according to the parameter η . The worker is rewarded for a fraction η of the firm's revenues and savings of hiring costs and compensated for a fraction $1 - \eta$ of the disutility he suffers from supplying labor and the foregone unemployment benefits. A distinguishing feature of our approach is that the wage depends on the degree of competition in the goods market. The direct effect of competition on the real wage is captured through the term $\eta (\rho_t/\mu_t) (1 - \alpha) A_t (IT_{t-1}^A/n_t N_t h_t)^\alpha$, which represents the share of the MRP which goes to workers. Entry leads to an increase in the ratio (ρ_t/μ_t) and hence in the MRP. Thus, everything else equal, stronger competition shifts the wage curve up. This result is similar to that in the static model by Blanchard and Giavazzi (2003), who find a positive effect of competition on the real wage. The FOC with respect to h_t yields

$$h_t = \left[\frac{(1 - \alpha)^2 \rho_t}{\chi C_t \mu_t} A_t \left(\frac{IT_{t-1}^A}{n_t N_t h_t} \right)^\alpha \right]^\varphi. \quad (31)$$

Because the firm and the worker bargain simultaneously about wages and hours, the outcome is (privately) efficient and the wage does not play an allocational role for hours. Stronger competition leads to an increase in hours bargained between the workers and firms for the same reasons for which competition positively affects the wage schedule.

2.7 Business Creation, Hiring and IT policies

Let π_t^{new} , v_t^{new} and $(I_t^{IT})^{new}$ be, respectively, the real profits, the number of vacancies posted by a new firm and investment in IT. Symmetrically, π_t , v_t and I_t^{IT} define, respectively, the individual profits and vacancies posted by an incumbent producer. New firms and incumbent firms are characterized by the same size, n_t . Thus, the optimal hiring policy of new firms, which have no initial workforce, consists in posting at time t as many vacancies as required to hire n_t workers. As a result $v_t^{new} = n_t/q_t$. Since $n_t = \varrho n_{t-1} + v_t q_t$, it has to be the case that

$$v_t^{new} = v_t + \varrho \frac{n_{t-1}}{q_t} \quad (32)$$

Hence, a new firm posts more vacancies than an incumbent producer. For this reason, and given vacancy posting is costly, the profit of new firms are lower than those of incumbent firms, in particular

$$\pi_t^{new} = \pi_t - k \frac{\varrho n_{t-1}}{q_t}$$

Consistently with U.S. empirical evidence in Haltiwanger et al. (2010) and Cooley and Quadrini (2001), a young firm creates on average more new jobs than a mature incumbent firm and distributes lower dividends.

Notice also that a new entrant must set up a stock of IT before starting production next period. Given the IT choice is symmetric across producers they have to invest during time t as much as required to reach a stock of IT identical to that held by incumbent producers at the end of time t , that is $(I_t^{IT})^{new} = IT_t$. The sunk entry cost for a new firm can thus be written as

$$\phi_t^e = \phi^{ad} + p_t^{IT} IT_t$$

In each period the level of entry is determined endogenously to equate the value of a new entrant, V_t^e , to the entry cost

$$V_t^e = \phi_t^e \quad (33)$$

Notice that perspective new entrants have lower value than producing firms because they will have (in case they do not exit from the market before starting production) to set up a workforce in their first period of activity. The difference in the value between a firm which is already producing and a perspective entrant is, in fact, the discounted value of the higher vacancy posting cost that the latter will suffer, with respect to the former, in the first period of activity. Formally

$$V_t = V_t^e + \kappa \varrho E_t \Lambda_{t,t+1} \frac{n_t}{q_{t+1}} = \phi_t^e + \kappa \varrho E_t \Lambda_{t,t+1} \frac{n_t}{q_{t+1}} \quad (34)$$

where V_t is the value of a producing firm (both new firms and incumbent firms) at time t .

3 The Introduction of a GPT

The role of a GPT which changes the cost structure is to shift some fixed costs into variable costs. In particular, the creation of cloud facilities implies that IT services, hardware and software, can be outsourced by the firm: delivery of computing and storage capacity is provided as a service that can be obtained on demand. We model this technological change as follows. We assume that after the introduction of cloud computing all producers of intermediate good will no longer own a stock of IT, but will rent it from the provider of cloud services. The existing stock of IT is transferred to the cloud-services provider. There are two main consequences spreading from this modelling device. The first one is that maintenance and depreciation costs associated to the stock of IT will be sustained by the provider of IT services. The second one is that new entrants will no longer need to build up a stock of IT before starting production. As a result, up-front sunk entry costs faced by potential entrants are strongly reduced.

Production of IT services is carried out with the same production function considered earlier, namely equation (11). However profit maximization must now take into account that the production of new IT services will no longer be sold, but rented to intermediate goods producers. We define r_t^{IT} as the rental rate of IT services. As a result the IT produced at time T , ΔIT_t , will contribute to the holdings of IT by the cloud provider. Period t profits of the cloud provider are

$$r_t^{IT} IT_{t-1}^A - p_t^M \frac{m}{A_t^c} IT_{t-1}^A - r_t^k K_{t-1}$$

The cloud-provider solves the following problem

$$\max_{\{K_{s-1}, IT_s\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \Lambda_{t,s} \left[\left(r_s^{IT} - p_s^M \frac{m}{A_s^c} \right) IT_{s-1}^A - r_s^k K_{s-1} \right] \quad (35)$$

$$s.t. \quad IT_s^A = (1 - \delta^{IT}) IT_{s-1}^A + A_s^c K_{s-1} \quad (36)$$

After defining as p_t^{IT} the Lagrange multipliers on the constraint, it can be shown that the FOCs for this problem are

$$p_t^{IT} = \frac{r_t^k}{A_t^c} \quad (37)$$

and

$$p_t^{IT} = \Lambda_{t,t+1} \left[\left(r_{t+1}^{IT} - p_{t+1}^M \frac{m}{A_{t+1}^c} \right) + (1 - \delta^{IT}) p_{t+1}^{IT} \right] \quad (38)$$

The first FOC implies that the provider of cloud services will rent capital up to the point where the rental rate of capital equals the marginal revenue product of capital. Combing the FOCs delivers

$$\frac{r_t^k}{A_t^c} = \Lambda_{t,t+1} \left[\left(r_{t+1}^{IT} - p_{t+1}^M \frac{m}{A_{t+1}^c} \right) + (1 - \delta^{IT}) \frac{r_{t+1}^k}{A_{t+1}^c} \right] \quad (39)$$

Profit maximization requires that the marginal cost of production of an additional unit of IT at time t , r_t^k/A_t^c , must be identical to the marginal revenue that the firms obtain from producing the additional unit. The latter is given by the rental rate at which the additional unit will be rented out tomorrow net of the maintenance costs, plus the continuation value, $(1 - \delta^{IT}) (r_t^k/A_t^c)$. The marginal revenue has to be discounted since the IT producers will be able to rent the additional unit produced in t just at period $t + 1$.

Sectoral goods producers will now simply demand IT up to the point where the rental cost of IT equals the marginal product of IT. Formally, the FOC with respect to IT_{t-1} is no longer equation (27), but

$$r_t^{IT} = \alpha \frac{\rho_t}{\mu_t} A_t \left(\frac{IT_{t-1}}{n_t h_t} \right)^{\alpha-1} \quad (40)$$

Notice that combining equations (40) and (39) we recover equation (27). Time t entrants will no longer need to face up-front entry costs in terms of IT. Once they are in the market and start production they will demand IT services up to the point where condition (40) is satisfied. As a result firm will enter the market up to the point where

$$\phi^e = \phi^{ad} \quad (41)$$

This means that the IT policy of a new entrant does not differ from that of an incumbent producer. As in the pre-cloud economy, goods producing firms need to set up a workforce before starting production. For this reason the difference between the hiring policy of new entrants and that of incumbent producers is the same as that spelled out above.

Considering that sectors are symmetric and have a unit mass, the sectoral number of firms and new entrants also represents their aggregate counterpart. Thus, the dynamics of the aggregate number of firms is

$$N_t = (1 - \delta) (N_t + N_t^e) \quad (42)$$

As aggregate expenditure and sectoral expenditure are identical, it follows that $EXP_t = \sum_{i=1}^{N_t} p_t y_t = N_t p_t y_t$. Considering $\rho_t = p_t/P_t$ and the individual production function we obtain

$$Y_t = \rho_t N_t y_t = \rho_t A_t (IT_{t-1}^A)^\alpha (N_t n_t h_t)^{1-\alpha} \quad (43)$$

The aggregate production function features a form of increasing returns. In this case a productivity shock impacts directly on output, but also through the firm creation channel. Total vacancies posted at period t are

$$v_t^{tot} = (1 - \delta) N_{t-1} v_t + (1 - \delta) N_{t-1}^e v_{t-1}^{new} + v_t^{IT}$$

where $(1 - \delta) N_{t-1}$ is the number of incumbent producers and $(1 - \delta) N_{t-1}^e$ is the number of new firms. Aggregating the budget constraints of households we obtain the aggregate resource constraint of the economy

$$C_t + \left(\phi^{ad} + p_t^{IT} \frac{IT_t}{N_t} \right) N_t^e + I_t^k = w_t h_t L_t + r_t^k K_{t-1} + \Pi_t \quad (44)$$

which states that the sum of consumption and investment in new entrants and capital must equal the sum between labor income and aggregate profits, Π_t , distributed to households at time t . Aggregate profits are defined as⁸

$$\Pi_t = (1 - \delta) N_{t-1} \pi_t + (1 - \delta) N_{t-1}^e \pi_t^{new} \quad (45)$$

where the first component is the total profits of the firms that were already active in the market, with

$$\pi_t = \rho_t y_t - w_t n_t h_t - \kappa v_t - p_t^M \frac{m}{A_t^c} IT_{t-1} - p_t^{IT} I_t^{IT}$$

and the second component represents the total profits of the new entrants, with

$$\pi_t^{new} = \rho_t y_t - w_t n_t h_t - \kappa v_t^{new} - p_t^M \frac{m}{A_t^c} IT_{t-1} - p_t^{IT} I_t^{IT}$$

Using $IT_{t-1}^A = N_t IT_{t-1}$ and defining $v_t^F = (1 - \delta) N_{t-1} v_t + (1 - \delta) N_{t-1}^e v_{t-1}^{new}$ as the number of vacancies created by the producers of the final good, the aggregate profits can be rearranged as follows

$$\Pi_t = Y_t - w_t N_t n_t h_t - p_t^M \frac{m}{A_t^c} IT_{t-1}^A - p_t^{IT} N_t I_t^{IT} - \kappa v_t^F$$

Notice that $p_t^M h_t = \frac{\kappa}{q_t} - \varrho \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} + w_t h_t$, therefore we can rewrite the last expression as

$$\begin{aligned} \Pi_t = & Y_t - w_t N_t n_t h_t - w_t \left(\frac{m}{A_t^c} IT_{t-1}^A \right) + \\ & - \left(\frac{\kappa}{q_t} - \varrho \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} \right) \frac{1}{h_t} \left(\frac{m}{A_t^c} IT_{t-1}^A \right) - p_t^{IT} N_t I_t^{IT} - \kappa v_t^F \end{aligned}$$

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⁸Since the producer of IT material and the provider of IT services operate in perfect competition they make no profits.

Using $M_t = n_t^{IT} h_t = (m/A_t^c) IT_{t-1}^A$ and the fact that the aggregate number of workers is $L_t = n_t N_t + n_t^{IT}$, we can finally obtain the following expression for the aggregate profits

$$\Pi_t = Y_t - w_t L_t h_t - \left(\frac{\kappa}{q_t} - \varrho \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} \right) n_t^{IT} - p_t^{IT} N_t I_t^{IT} - \kappa v_t^F$$

As a result, the clearing of the market for the final good requires

$$\begin{aligned} C_t + \left(\phi^{ad} + p_t^{IT} \frac{IT_t^A}{N_t} \right) N_t^e + I_t^k - p_t^{IT} N_t I_t^{IT} &= \\ = r_t^k K_{t-1} + Y_t - \left(\frac{\kappa}{q_t} - \varrho \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} \right) n_t^{IT} - \kappa v_t^F &= \\ = r_t^k K_{t-1} + Y_t - \frac{\kappa}{q_t} \frac{m}{A_t^c} \frac{IT_{t-1}}{h_t} + \varrho \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} \frac{m}{A_t^c} \frac{IT_{t-1}}{h_t} - \kappa v_t^F & \quad (46) \end{aligned}$$

Notice that the total IT produced at time t must be equal to the sum between investment by incumbent firms and that of new entrants, that is

$$\Delta IT_t^A = \frac{IT_t^A}{N_t} N_t^e + N_t I_t^{IT}$$

thus, using $\Delta IT_t^A = A_t^c K_{t-1}$ and $p_t^{IT} = r_t^k / A_t^c$, it follows that output can be expressed as

$$Y_t = C_t + \phi^{ad} N_t^e + I_t^k + \kappa v_t^F + \left(\frac{\kappa}{q_t} - \varrho \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} \right) \frac{m}{A_t^c} \frac{IT_{t-1}}{h_t} \quad (47)$$

Finally, the dynamics of the aggregate stock of IT reads as follows

$$IT_t^A = (1 - \delta) [(1 - \delta^{IT}) IT_{t-1}^A + \Delta IT_t^A]$$

Let us turn to the characterization of the equilibrium in the cloud economy. Here the producers of the final good rent their stock of IT from the cloud services producer. Profits of an incumbent producer are thus given by

$$\pi_t = \rho_t y_t - w_t n_t h_t - \kappa v_t - r_t^{IT} IT_{t-1}$$

while those of a new firm are

$$\pi_t^{new} = \rho_t y_t - w_t n_t h_t - \kappa v_t^{new} - r_t^{IT} IT_{t-1}$$

Aggregating as above and using $N_t IT_{t-1} = IT_{t-1}^A$ it follows

$$\Pi_t = Y_t - w_t N_t n_t h_t - \kappa v_t^F - r_t^{IT} IT_{t-1}^A$$

while the entry condition is simply $\phi^e = \phi^{ad}$. As a result the market clearing condition reads as

$$Y_t = C_t + \phi^{ad} N_t^e + I_t^k + \kappa v_t^F - (r_t^k K_{t-1} - r_t^{IT} N_t IT_{t-1}) - w_t \frac{m}{A_t^c} IT_t^A \quad (48)$$

where

$$r_t^{IT} = \alpha \frac{\rho_t}{\mu_t} A_t \left(\frac{IT_{t-1}^A}{N_t n_t h_t} \right)^{\alpha-1}$$

All the equilibrium conditions are summarized in the Appendix, where we also provide a full characterization of the steady state around which we simulate the behavior of the economy.

4 Calibration and Simulation

Calibration is conducted on a quarterly basis as in Shimer (2005) and Blanchard and Galì (2010) among others. The discount factor, β , is set to the standard value of 0.99. An exit rate of 10 percent is reported by Bilbiie *et al.* (2012) for the US, therefore we set $\delta = 0.025$. The baseline value for the entry cost is set such that the ratio of investment in new firms and physical capital is close to 15 per cent, as in Bilbiie *et al.* (2012). The implied steady state price markup is about 35 per cent. This value is within the range estimated by Oliveira Martins and Scarpetta (1999) for a large number of U.S. manufacturing sectors. With no loss of generality, the value of χ is such that steady state hours equals one. In this case the Frisch elasticity of labor supply reduces to φ , to which we assign a low value of 0.5 in line with the evidence. We assume Bertrand competition between firms and we take as the baseline value for the intersectoral elasticity of substitution $\varepsilon = 6$, as estimated by Christiano *et al.* (2005) using U.S. quarterly data between 1965 and 1995. As standard in the literature we set the steady state marginal productivity of labor, A , to 1. The same value is set for the marginal productivity of capital in the IT producing sector, A^c .

The elasticity of matches to unemployment is $\gamma = \frac{1}{2}$, within the range of the plausible values of 0.5 to 0.7 reported by Petrongolo and Pissarides (2001) in their survey of the literature on the estimation of the matching function. In the baseline parameterization we impose symmetry in bargaining and set $\eta = \frac{1}{2}$, as in the bulk of the literature. Since we consider a labor-leisure choice, the overall replacement rate is given by the sum between the unemployment insurance benefit and the disutility cost of working. We calibrate the latter to 0.95 consistently with Hagerdon and Manovskii (2008). The cost of posting a vacancy κ is obtained by equating the steady state version of the JCC and the steady state wage setting equation. Finally we set $m = 0.025$ and $\delta^{IT} = 0.025$.

These figures imply that the maintenance cost of the IT stock is about 10 percent a year, and that the IT stock of a firm fully depreciate in about ten years.

We characterize the effects of the introduction of the GPT under two alternative parameterizations of the degree of labor market flexibility. In a flexible environment like the U.S. one, we can set the separation rate φ to 0.1, as suggested by estimates provided by Hall (1995) and Davis *et al.* (1996). We then set the efficiency parameter in matching, γ_m , and the steady state job market tightness to target an average job finding rate, z , equal to 0.7 and a vacancy filling rate, q , equal to 0.9. We draw the latter value from Andolfatto (1996) and Dee Haan *et al.* (2000), while the former from Blanchard and Galí (2010). Notice that a job finding rate equal to 0.7 corresponds, approximately, to a monthly rate of 0.3. A rigid labor market such as the E.U. one can be better characterized by a separation rate equal to 0.03 in line with the estimates from the Labour Force Survey in Bell and Smith (2002) and by a job finding rate equal to 0.25 as in Thomas and Zanetti (2009). In this case we set the vacancy filling rate to 0.7 in line with estimates reported by the ECB (2002).

We can finally simulate the effect of the introduction of our GPT. We mimic the transition to the cloud as follows. We assume that IT is produced more efficiently. This is obtained featuring a one percent increase in A^c in the long run. Also we assume that bringing IT services on the cloud leads to a reduction in the units of maintenance services required for each single IT unit installed. This is formalized by assuming a five percent reduction in m . Figure 1 displays percentage deviations from the pre-cloud steady state when all existing firms adopt the cloud technology. Solid lines refer to the case of a flexible labor market, dashed lines to that of a more rigid market. Time on the horizontal axis is in quarters.⁹

The introduction of the GPT lowers the up-front entry costs in terms of IT creating large aggregate effects. The implied reduction in the entry cost is about 3 percent. This stimulates entry of new firms. Given that entry is subject to a one period time-to-build lag the total number of firms, does not change on impact, but builds up gradually, which is going to induce a gradual strengthening of competition which creates the magnification of the shock already discussed in Etro and Colciago (2010): more competition reduces the mark ups, which reduces the prices and promotes consumption and production. Moreover, the new firms post a large amount of vacancies to reach their desired size. This results in a

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⁹We use DYNARE to compute an exact transition from the pre-cloud steady state to the post-cloud steady state.

Figure 1: Transition of the main macroeconomic variables from the steady state of the pre-cloud economy to that of the cloud economy. Percentage deviations from the pre-cloud steady state on the vertical axis, quarters on the horizontal one.

persistent change in aggregate employment.

A larger number of firms leads to a stronger usage of IT. Higher employment together with a higher IT stock lead in turn to a sustained increase in aggregate output. The competition effect associated with business creation has also another consequence on the labor market: as shown by Etro and Colciago (2012), more competition reduces the mark ups and increases the real wages, which also promotes labor supply and facilitates job creation.

Notice that employment in the IT sector initially goes down, due to more efficient technology of maintenance. However, as the stock of IT raises to its new long run level, IT employment increase above its initial value. A crucial role for the increase in employment, both in the aggregate and in the IT sector, is played by the creation of new firms.¹⁰ While the previous description holds for both the flexible and the rigid characterization of the labor market, the variation in employment is, as expected, more pronounced in the market characterized by more flexibility. Interestingly this is also mirrored in the changes in the number

¹⁰One may notice that a major drawback of the diffusion of cloud computing is usually associated with job destruction in the IT sector. What we show is that such a concern appears misplaced given the substantial and quick net benefits in terms of net job creation, also in IT employment.

of firms and in the price markups, which are more relevant in the market characterized by higher worker flows.

The impact of the structural change to technology due to the introduction of the new GPT is rather pervasive, both on employment and business creation. Importantly, this effect would be neglected in the standard DGSE model used in macroeconomics, where perfect competition and constant returns to scale or monopolistic competition with exogenous market structures exclude any impact of changes in the cost structure on the market structures, the level of competition, the labor market and the process of business and job creation.

5 Conclusions

We have analyzed the diffusion of a General Purpose Technology that changes the cost structure turning fixed costs into variable ones. A major recent example is cloud computing, whose adoption allows firms to avoid large up-front costs in IT and to rent computing capability online. We have studied the macroeconomic impact of the adoption of such a technology in a DSGE model inspired to the recent literature on endogenous market structures developed by Etro and Colciago (2010), Bilbiie *et al.* (2012) and others. To start a business, firms need to hire workers and set up a stock of IT capital: the new technology allows them to rent computing capabilities reducing entry costs. The simulation has shown that such an innovation can have a substantial impact on business and job creation, both in the medium and the long run. Such an impact cannot be appreciated in standard models with perfect competition or monopolistic competition, where the cost structure is much simpler and its shocks do not affect the market structure or the job matching process. We believe that such models with endogenous market structures and job matching frictions can provide an important contribution to the understanding of the business cycle and of the impact of innovations.

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Technical Appendix

This Appendix summarizes the equilibrium relations needed to characterize the behavior of the economy and its steady state and to run the simulations described in the text. First, we write all the equilibrium conditions in a manner such that the post-GPT economy is nested in the pre-GPT economy. This allows to numerically compute the transition from the former to the latter. For this reason we use the parameter Φ , which can take either value 1 or 0. In particular, a value $\Phi = 1$ allows to portrair the pre-cloud economy, while $\Phi = 0$ refers to the post-cloud economy. The equilibrium conditions are the following

$$\begin{aligned}
Y_t &= A_t (IT_{t-1}^A)^\alpha (N_t n_t h_t)^{1-\alpha} \\
y_t &= \frac{Y_t}{\rho_t N_t} \\
Y_t &= C_t + \phi^{ad} N_t^e + I_t^k + \kappa v_t^F + \Phi \left(\frac{\kappa}{q_t} - \varrho \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} \right) \frac{m}{A_t^c} \frac{IT_{t-1}}{h_t} + \\
&\quad - (1 - \Phi) \left[(r_t^k K_{t-1} - r_t^{IT} N_t IT_{t-1}) + w_t \frac{m}{A_t^c} IT_t^A \right] \\
\rho_t &= N_t^{\frac{1}{\theta-1}} \\
p_t^{IT} &= \frac{r_t^k}{A_t^c} \\
K_t &= (1 - \delta^k) K_{t-1} + I_t^k \\
v_t^{tot} &= v_t^i + v_t^e + v_t^{IT} \\
v_t^e &= (1 - \delta) \frac{N_{t-1}^e}{N_t} \frac{N_t n_t}{q_t} \\
N_t^e &= \frac{N_{t+1}}{(1 - \delta)} - N_t \\
L_t &= N_t n_t + n_t^{IT} \\
n_t &= \rho n_{t-1} + v_t^i(i) q_t \\
n_t^{IT} &= \rho n_{t-1} + v_t q_t \\
n_t^{IT} &= \frac{m}{A_t} \frac{IT_t^A}{h_t} \\
\frac{\kappa}{q_t} &= (1 - \alpha) \frac{\rho_t}{\mu_t} A_t \left(\frac{IT_{t-1}}{n_t h_t} \right)^\alpha h_t - w_t h_t + \varrho E_t \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} \\
q_t &= \gamma_m \theta_t^{-\gamma}
\end{aligned}$$

$$w_t h_t = (1 - \eta) b + (1 - \eta) \chi C_t \frac{h_t^{1+1/\varphi}}{1 + 1/\varphi} + \\ + \frac{\eta \kappa}{1 - \delta} E_t \Lambda_{t,t+1} \theta_{t+1} + (1 - \alpha) \eta \frac{\rho_t}{\mu_t} A_t \left(\frac{IT_{t-1}^A}{N_t n_t h_t} \right)^\alpha h_t$$

$$\theta_t = \frac{v_t^{tot}}{1 - L_{t-1}}$$

$$\frac{1}{C_t} = \beta(1 + r_t) \frac{1}{C_{t+1}}$$

$$\frac{1}{C_t} = \beta(1 + r_t^k - \delta^k) \frac{1}{C_{t+1}}$$

$$V_t = (1 - \delta) \beta \frac{C_t}{C_{t+1}} (\pi_{t+1} + V_{t+1})$$

$$\phi_t^e = (1 - \delta) \beta \frac{C_t}{C_{t+1}} (\pi_{t+1}^{new} + V_{t+1})$$

$$IT_t^A = (1 - \delta \Phi) \left[(1 - \delta^{IT}) IT_{t-1}^A + \Delta IT_t \right]$$

$$p_t^{IT} = (1 - \delta) \beta \frac{C_t}{C_{t+1}} \left[r_{t+1}^{IT} - p_{t+1}^M \frac{m}{A_{t+1}^c} + p_{t+1}^{IT} (1 - \delta^{IT}) \right]$$

$$r_t^{IT} = \alpha \frac{\rho_t}{\mu_t} A_t \left(\frac{IT_{t-1}^A}{N_t n_t h_t} \right)^{\alpha-1}$$

$$\Delta IT_t = A_t^c K_{t-1}$$

$$K_t = (1 - \delta^k) K_{t-1} + I_t^k$$

$$\pi_t = \rho_t y_t - w_t n_t h_t - \kappa v_t(i) - \Phi \left(p_t^M \frac{m}{A_t^c} \frac{IT_{t-1}^A}{N_t} + p_t^{IT} I_t^{IT} \right) - (1 - \Phi) r_t^{IT} \frac{IT_{t-1}^A}{N_t}$$

$$I_t^{IT} = \frac{IT_t^A}{N_{t+1}} - (1 - \delta^{IT}) \frac{IT_{t-1}^A}{N_t}$$

$$h_t = \left[\frac{(1 - \alpha)^2}{\chi C_t} \frac{\rho_t}{\mu_t} A_t \left(\frac{IT_{t-1}^A}{N_t n_t h_t} \right)^\alpha \right]^\varphi$$

$$\Pi_t = (1 - \delta) N_{t-1} \pi_t + (1 - \delta) N_{t-1}^e \pi_t^{new}$$

$$\mu_t(\varepsilon, N_t) = \frac{\varepsilon(N_t - 1) + 1}{(\varepsilon - 1)(N_t - 1)}$$

$$\phi_t^e = \phi^{ad} + \Phi p_t^{IT} \frac{IT_t}{N_t}$$

These equations define the equilibrium of the pre-cloud economy when $\Phi = 1$ and that of the post cloud economy when $\Phi = 0$.

In what follows we focus on the steady state. We follow Bills, Chang and Kim (2011) and Costain and Reiter (2008) and set the overall replacement rate equal to the sum between the monetary unemployment benefit and the disutility of working, that is

$$\frac{b + \chi \frac{h^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \frac{1}{\lambda}}{w} = \varkappa \quad (49)$$

Also we set z, q . By definition $\theta = \frac{z}{q}$, $q = \frac{m}{v^{tot}} = \gamma_m \theta^{-\gamma}$ thus $\gamma_m = q \theta^\gamma$. The steady state version of the condition for the optimal choice of IT implies

$$\left(\frac{IT^A}{Nnh} \right) = \left[\frac{\Lambda \frac{\rho}{\mu} A \alpha}{(1 - \Lambda (1 - \delta^{IT})) p^{IT} + \Lambda p^M \frac{m}{A^c}} \right]^{\frac{1}{1-\alpha}}$$

where $p^{IT} = \frac{r^k}{A^c}$ and

$$p^M = (1 - \varrho \Lambda) \frac{\kappa}{q} \frac{1}{h} + w$$

Thus $\frac{IT^A}{Nnh}$ is a function of N, κ and w . Assuming $h = 1$, the implied value of χ is

$$\chi = \frac{(1 - \alpha)^2}{C} \frac{\rho}{\mu} A \left(\frac{IT^A}{Nnh} \right)^\alpha$$

Substituting into (49) we obtain

$$\frac{b + (1 - \alpha)^2 \frac{\rho}{\mu} A \left(\frac{IT^A}{Nnh} \right)^\alpha \frac{1}{1+\frac{1}{\varphi}}}{w} = \varkappa \quad (50)$$

where $\rho = \rho(N)$ and $\mu = \mu(N)$. Evaluating the wage schedule at the steady state where $h = 1$ leads to

$$w = (1 - \eta) b + (1 - \eta) \chi C \frac{1}{1 + 1/\varphi} + \frac{\eta \kappa}{1 - \delta} \beta \frac{\theta}{h} + (1 - \alpha) \eta \frac{\rho}{\mu} A \left(\frac{IT}{nh} \right)^\alpha$$

Substituting for χ provides

$$w = (1 - \eta) \left[b + (1 - \alpha)^2 \frac{\rho}{\mu} A \left(\frac{IT^A}{Nnh} \right)^\alpha \frac{1}{1 + 1/\varphi} \right] + \eta \kappa \beta \frac{\theta}{h} + (1 - \alpha) \eta \frac{\rho}{\mu} A \left(\frac{IT^A}{Nnh} \right)^\alpha$$

or

$$\begin{aligned} w &= (1 - \eta) \varkappa w + \eta \kappa \beta \frac{\theta}{h} + (1 - \alpha) \eta \frac{\rho}{\mu} A \left(\frac{IT}{Nnh} \right)^\alpha = \\ &= \frac{1}{1 - (1 - \eta) \varkappa} \left[\eta \kappa \beta \frac{\theta}{h} + (1 - \alpha) \eta \frac{\rho}{\mu} A \left(\frac{IT}{Nnh} \right)^\alpha \right] \end{aligned}$$

Rearranging the JCC we have

$$w = (1 - \alpha) \frac{\rho}{\mu} A \left(\frac{IT^A}{Nnh} \right)^\alpha - (1 - \varrho(1 - \delta)\beta) \frac{\kappa}{q}$$

Then we have

$$\kappa = \underbrace{\frac{1 - (1 - (1 - \eta)\kappa)^{-1} \eta}{(1 - (1 - \eta)\kappa)^{-1} \eta \beta \theta + \frac{(1 - \varrho(1 - \delta)\beta)}{q}}}_{\Theta} (1 - \alpha) \frac{\rho}{\mu} A \left(\frac{IT^A}{Nnh} \right)^\alpha$$

which provides a relationship between w , N and κ . Notice that

$$L = nN + \frac{m}{A^c} \frac{IT^A}{h}$$

Given $h = 1$ it follows that

$$\frac{L}{Nn} = 1 + \frac{m}{A^c} \frac{IT^A}{Nnh}$$

is a function of w , N and κ .

Next consider vacancies. Vacancies posted by the producer of IT services are given by

$$v^{IT} = \frac{(1 - \rho)}{q} n^{IT} = \frac{(1 - \rho)}{q} \frac{m}{A^c} \frac{IT^A}{h}$$

Vacancies posted by a mature producer are

$$v = \frac{(1 - \rho)}{q} n = \frac{(1 - \rho)}{q} \frac{Nn}{N}$$

while vacancies posted by a new producer are

$$v^e = \frac{n}{q}$$

As a result total vacancies read as

$$\begin{aligned} v^{tot} &= \frac{(1 - \rho)}{q} \frac{m}{A^c} \frac{IT^A}{h} + (1 - \delta) N \frac{(1 - \rho)}{q} \frac{Nn}{N} + (1 - \delta) \frac{N^e}{N} \frac{Nn}{q} = \\ &= \frac{(1 - \rho)}{q} \frac{m}{A^c} \frac{IT^A}{h} + (1 - \delta) N \frac{(1 - \rho)}{q} \frac{Nn}{N} + \delta \frac{Nn}{q} \end{aligned}$$

Using $v^{tot} = \theta(1 - L)$ in the above condition one can derive

$$L = \frac{\theta}{\theta + \frac{1}{q} \left[(1 - \rho) \frac{m}{A^c} \frac{IT^A}{hNn} + (1 - \delta) (1 - \rho) + \delta \right] \frac{Nn}{L}},$$

which delivers total employment L as a function of w , N and κ .

Then consider again the JCC

$$\begin{aligned} w &= (1 - \alpha) \frac{\rho}{\mu} A \left(\frac{IT}{nh} \right)^\alpha - (1 - \varrho(1 - \delta)\beta) \frac{\kappa}{q} = \\ &= \left[(1 - \alpha) \frac{\rho}{\mu} A - \frac{(1 - \varrho(1 - \delta)\beta)}{q} \Theta (1 - \alpha) \frac{\rho}{\mu} A \right] \left(\frac{IT^A}{Nnh} \right)^\alpha \end{aligned}$$

which provides again a relationship between the three variables mentioned earlier. Then IT^A can be written as

$$IT^A = \left(\frac{IT^A}{Nn} \right) \left(\frac{L}{Nn} \right)^{-1} L$$

and

$$Nn = \left(\frac{L}{Nn} \right)^{-1} L, \quad y = A \left(\frac{IT}{nN} \right)^\alpha \frac{nN}{N} \quad \text{and} \quad Y = \rho N y = \rho A \left(\frac{IT}{nN} \right)^\alpha nN$$

The euler equation for capital implies

$$r^k = \left(\frac{1}{\beta} - 1 + \delta^k \right)$$

As a result we can determine

$$p^{IT} = \frac{r^k}{A^c}$$

At the steady state $\Delta IT^A = A^c K$. From the accumulation equation of IT, we obtain

$$\Delta IT^A = \left[\frac{1 - (1 - \delta)(1 - \delta^{IT})}{(1 - \delta)} \right] IT^A$$

Combing the latter two equations we obtain the steady state level of capital

$$K = \left[\frac{1 - (1 - \delta)(1 - \delta^{IT})}{(1 - \delta)} \right] \frac{IT^A}{A^c}$$

as a function of N and w .

Next we determine the steady state labor share, the total cost of posting vacancies as a fraction of output and the profit share. Using the JCC (together with $h = 1$) delivers

$$w = (1 - \alpha) \frac{\rho}{\mu} A \left(\frac{IT^A}{Nnh} \right)^\alpha - (1 - \varrho\Lambda) \frac{\kappa}{q}$$

as a result

$$\frac{wL}{Y} = (1 - \alpha) \frac{\rho}{\mu} A \left(\frac{IT^A}{Nn} \right)^\alpha \frac{L}{Y} - (1 - \varrho\Lambda) \frac{\kappa}{q} \frac{L}{Y}$$

Since

$$\frac{Y}{L} = \rho A \left(\frac{IT^A}{L} \right)^\alpha \left(\frac{Nn}{L} \right)^{1-\alpha} = \rho A \left(\frac{IT^A}{Nn} \right)^\alpha (Nn) L^{\alpha-1} L^{-\alpha} = \rho A \left(\frac{IT^A}{Nn} \right)^\alpha \frac{Nn}{L}$$

it follows

$$\frac{L}{Y} = \frac{1}{\rho A} \left(\frac{IT^A}{Nn} \right)^{-\alpha} \frac{L}{Nn}$$

Also notice that

$$\kappa \frac{L}{Y} = \frac{\Theta(1 - \alpha)}{\mu} \frac{L}{Nn}$$

Putting everything together we obtain

$$\begin{aligned} \frac{wL}{Y} &= (1 - \alpha) \frac{\rho}{\mu} A \left(\frac{IT^A}{Nn} \right)^\alpha \frac{L}{Y} - \frac{(1 - \varrho\Lambda)}{q} \kappa \frac{L}{Y} \\ &= (1 - \alpha) \frac{\rho}{\mu} A \left(\frac{IT^A}{Nn} \right)^\alpha \frac{L}{Y} - \frac{(1 - \varrho\Lambda)}{q} \frac{\Theta(1 - \alpha)}{\mu} \frac{L}{Nn} \\ &= (1 - \alpha) \frac{1}{\mu} \left[1 - \Theta \frac{(1 - \varrho\Lambda)}{q} \right] \frac{L}{Nn} \end{aligned}$$

The total cost of posting vacancies as a share of output is

$$\begin{aligned} \frac{\kappa v^{tot}}{Y} &= \frac{\kappa \theta (1 - L)}{Y} = \Theta(1 - \alpha) \frac{\rho}{\mu} A \left(\frac{IT^A}{Nnh} \right)^\alpha \theta \frac{(1 - L)}{Y} \\ &= \Theta \frac{(1 - \alpha)}{\mu} \rho A \left(\frac{IT^A}{Nnh} \right)^\alpha \theta \frac{(1 - L)}{Y} \frac{Nnh}{Nnh} \\ &= \Theta \frac{(1 - \alpha)}{\mu} \theta \frac{(1 - L)}{Nnh} \end{aligned}$$

Notice that

$$\begin{aligned} Y &= C + \phi^{ad} N^e + I^k + \kappa v^F + \frac{\kappa}{q} (1 - \varrho\Lambda) \frac{m}{A^c} \frac{IT^A}{h} = \\ &= C + \phi^{ad} N^e + I^k + \kappa v^F + \frac{\kappa}{q} (1 - \varrho\Lambda) \frac{m}{A^c} IT^A \end{aligned}$$

since $h = 1$. Moreover it holds that

$$C + \phi^{ad} N^e + I^k = whL + r^k K + \Pi - p^{IT} \frac{IT^A}{N} N^e \quad (51)$$

Combining the latter two equations we obtain

$$\Pi = Y + p^{IT} \frac{IT^A}{N} N^e - \kappa v^F - \frac{\kappa}{q} (1 - \varrho \Lambda) \frac{m}{A^c} IT^A - (whL + r^k K)$$

and the profit share, $\frac{\Pi}{Y}$, reads as

$$\frac{\Pi}{Y} = 1 + p^{IT} \frac{IT^A}{Y} \frac{N^e}{N} - \frac{\kappa v^F}{Y} - \frac{\kappa}{q} (1 - \varrho \Lambda) \frac{m}{A^c} \frac{IT^A}{Y} - \frac{whL}{Y} - \frac{r^k K}{Y}$$

where

$$\frac{\kappa v^F}{Y} = \kappa \frac{v^{tot} - \frac{(1-\rho)}{q} \frac{m}{A^c} \frac{IT^A}{h}}{Y} = \kappa \frac{v^{tot}}{Y} - \kappa \frac{1-\rho}{q} \frac{m}{A^c} \frac{IT^A}{hY}$$

and

$$\frac{IT^A}{Y} = \frac{IT^A}{\rho A \left(\frac{IT^A}{Nnm} \right)^\alpha Nnh} = \frac{1}{\rho A} \left(\frac{IT^A}{Nhn} \right)^{1-\alpha}$$

Given $K = \left[\frac{1-(1-\delta)(1-\delta^{IT})}{(1-\delta)} \right] \frac{IT^A}{A^c}$ it follows

$$\frac{K}{Y} = \left[\frac{1 - (1-\delta)(1-\delta^{IT})}{(1-\delta)} \right] \frac{1}{A^c} \frac{IT^A}{Y}$$

To determine the ratio $\frac{C}{Y}$ we must find the ratio $\frac{I}{Y}$. Notice that aggregate investment at time t is given by the sum of investment in capital and in new firms

$$I = I^k + \left(\phi^{ad} + \frac{IT^A}{N} \right) N^e$$

Then

$$\frac{I}{Y} = \frac{I^k}{Y} + \frac{\phi^{ad} N^e}{Y} + \frac{IT^A}{Y} \frac{N^e}{N}$$

Moreover, we have $I^k = \delta^k K$ which implies

$$\frac{I^k}{Y} = \delta^k \frac{K}{Y}$$

Given that

$$C + \phi^{ad} N^e + I^k = whL + r^k K + \Pi - p^{IT} \frac{IT^A}{N} N^e$$

dividing by Y and using $\delta/(1-\delta) = N^e/N$ we have

$$\frac{C}{Y} = \frac{whL}{Y} + r^k \left(\frac{K}{Y} - \frac{1}{A^c} \frac{IT^A}{Y} \right) + \frac{\Pi}{Y} \frac{\delta}{1-\delta} - \left(\frac{\phi^{ad} N^e}{Y} + \frac{I^k}{Y} \right)$$

Notice also that

$$\frac{K}{Y} = \left[\frac{1 - (1 - \delta)(1 - \delta^{IT})}{(1 - \delta)A^c} \right] \frac{IT^A}{Y}$$

which implies

$$\frac{C}{Y} = \frac{wLh}{Y} + r^k \left(\frac{1 - (1 - \delta)(1 - \delta^{IT})}{(1 - \delta)} - 1 \right) \frac{IT^A}{Y} \frac{1}{A^c} + \frac{\Pi}{Y} \frac{\delta}{1 - \delta} - \left(\frac{\phi^{ad} N^e}{Y} + \frac{I^k}{Y} \right)$$

The Euler equation for the value of an incumbent leads to

$$V = \frac{\Lambda}{1 - \Lambda} \pi \quad (52)$$

while that for the value of a new entrant

$$\frac{V^e}{\Lambda} = \left[\left(\pi - \kappa \frac{\varrho n}{q} \right) + V \right] \quad (53)$$

Since it has to be the case that $V^e = \phi^e$ it follows

$$\frac{\phi^{ad} + p^{IT} \frac{IT^A}{N}}{\Lambda} = \left[\left(\pi - \kappa \frac{\varrho n}{q} \right) + V \right] \quad (54)$$

and

$$\begin{aligned} \pi &= \left(\phi^{ad} + p^{IT} \frac{IT^A}{N} \right) \frac{1}{\Lambda} + \left(\kappa \frac{\varrho n}{q} - V \right) = \\ &= \left(\phi^{ad} + p^{IT} \frac{IT^A}{N} \right) \frac{1}{\Lambda} + \kappa \frac{\varrho n}{q} - \frac{\Lambda}{1 - \Lambda} \pi \end{aligned}$$

where we substituted the definition of the value of the incumbent. This implies

$$\pi = \left(\phi^{ad} + p^{IT} \frac{IT^A}{N} \right) \frac{1 - \Lambda}{\Lambda} + (1 - \Lambda) \kappa \frac{\varrho n}{q}$$

Multiplying both sides by $\frac{N}{Y}$ delivers

$$\begin{aligned} \frac{N}{Y} \pi &= \frac{N}{Y} \left(\phi^{ad} + p^{IT} \frac{IT^A}{N} \right) \frac{1 - \Lambda}{\Lambda} + (1 - \Lambda) \kappa \frac{\varrho n N}{q Y} = \\ &= \frac{N^e \phi^{ad}}{Y} \frac{N}{N^e} \frac{1 - \Lambda}{\Lambda} + p^{IT} \frac{IT^A}{Y} \frac{1 - \Lambda}{\Lambda} + \frac{(1 - \Lambda) \kappa \varrho n N}{q Y} \end{aligned}$$

Since $\delta/(1 - \delta) = N^e/N$ we get

$$\frac{N}{Y} \pi = \frac{\phi^{ad}}{Y} \frac{1 - \Lambda}{\Lambda} N + p^{IT} \frac{IT^A}{Y} \frac{1 - \Lambda}{\Lambda} + \frac{(1 - \Lambda) \kappa \varrho n N}{q Y}$$

Next given

$$\frac{Y}{Nn} = \rho Ah \left(\frac{IT^A}{Nnh} \right)^\alpha, \quad \frac{Nn}{Y} = \frac{1}{\rho Ah} \left(\frac{IT^A}{Nnh} \right)^{-\alpha} \quad \text{and} \quad \frac{IT^A}{Y} = \frac{1}{\rho A} \left(\frac{IT^A}{Nhn} \right)^{1-\alpha}$$

we obtain

$$\frac{N}{Y}\pi = \frac{\phi^{ad}}{Y} \frac{1-\Lambda}{\Lambda} N + p^{IT} \frac{1}{\rho A} \left(\frac{IT^A}{Nhn} \right)^{1-\alpha} \frac{1-\Lambda}{\Lambda} + \frac{(1-\Lambda)\kappa\varrho}{q} \frac{1}{\rho Ah} \left(\frac{IT^A}{Nnh} \right)^{-\alpha} \quad (55)$$

Notice that it also holds that

$$\pi = \rho y - wnh - \kappa v - p^M \frac{m}{A^c} IT - p^{IT} I^{IT}$$

Individual vacancies of an incumbent are given by $v = (1 - \varrho)n/q$. Remembering that $p^M = (1 - \varrho\Lambda)\kappa/qh + w$ we can derive

$$\pi = \rho y - w \frac{nNh}{N} - \frac{\kappa(1-\varrho)nN}{q} - (1-\varrho\Lambda) \frac{\kappa}{q} \frac{1}{h} \frac{m}{A^c} \frac{IT^A}{N} - w \frac{m}{A^c} \frac{IT^A}{N} - p^{IT} I^{IT}$$

while

$$I^{IT} = \frac{IT^A}{N} - (1 - \delta^{IT}) \frac{IT^A}{N} = (1 - (1 - \delta^{IT})) \frac{IT^A}{N}$$

and $p^{IT} = r^k/A^c$. Therefore we have

$$\begin{aligned} \pi = & \rho y - w \frac{nNh}{N} - \frac{\kappa(1-\varrho)nN}{q} - (1-\varrho\Lambda) \frac{\kappa}{q} \frac{1}{h} \frac{m}{A^c} \frac{IT^A}{N} + \\ & - w \frac{m}{A^c} \frac{IT^A}{N} - p^{IT} (1 - (1 - \delta^{IT})) \frac{IT^A}{N} \end{aligned}$$

and it follows that

$$\frac{N}{Y}\pi = \left[\rho A \left(\frac{IT^A}{Nnh} \right)^\alpha - w \left(1 + \frac{m}{A^c} \frac{IT^A}{Nnh} \right) - \frac{\kappa}{qh} \left((1-\varrho) + (1-\varrho\Lambda) \frac{m}{A^c} \frac{IT^A}{Nnh} \right) - p^{IT} (1 - (1 - \delta^{IT})) \frac{IT^A}{Nnh} \right] \frac{Nnh}{Y} \quad (56)$$

where

$$\frac{Y}{Nnh} = \rho A \left(\frac{IT^A}{Nnh} \right)^\alpha$$

Equating (55) and (56) we obtain the relationship between N , w and κ that we were after. Considering the latter equation together with

$$w = \left[(1-\alpha) \frac{\rho}{\mu} A - \frac{(1-\varrho(1-\delta)\beta)}{q} \Theta (1-\alpha) \frac{\rho}{\mu} A \right] \left(\frac{IT^A}{Nnh} \right)^\alpha$$

and

$$\kappa = \Theta (1-\alpha) \frac{\rho}{\mu} A \left(\frac{IT^A}{Nnh} \right)^\alpha$$

provides a system of three equations which can be solved for N , w and κ .