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**Financial Literacy, Information
Acquisition and
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Financial Literacy, Information Acquisition and Asset Pricing Implications *

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Abstract

This paper analyses the information acquisition process in a simple asset pricing model with heterogeneous beliefs about future prices. This is with a view to investigating the effects of financial literacy on market volatility. I posit that financial literacy affects the cost of acquiring information regarding asset payoff and show that the effect on market volatility depends on the uncertainty of fundamentals. The main intuition is that lower information cost for less literate households leads them to acquire more private information and to trade more actively. Having more private information revealed by market prices affects market volatility positively. On the other hand, with low uncertainty in the fundamentals, the positive informational effect is offset by the negative effect of having more traders with less precise beliefs, who trade more conservatively. Two policy exercises derive the implications of more market transparency and the provision of financial education financed through a proportional tax on excess returns.

Keywords: Asset pricing, Financial literacy, Transparency, Heterogeneous agents.

JEL classification: D82, G12, G14, G18.

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1 Introduction

Due to the changes that have occurred in the welfare state and in the ageing path of the industrialized countries, the burden and the risk of financial choices has increasingly shifted onto individuals. Thus, the role played by financial literacy in the financial decision processes of investors is attracting the attention of policy makers.¹

Lack of abilities or skills in managing lifetime financial wealth, such as a misunderstanding of financial matters or underestimating current opportunities and future needs, becomes a crucial issue in the policy makers' agenda, especially during times of crisis. Having identified the "vulnerable population", the policy makers focus their efforts on financial education programs and on market regulation.² Within the former, they intervene directly on the abilities of the individuals, making them more financially literate. The latter approach improves the transparency of the financial markets, encouraging a better understanding on the part of the individuals.

However, the effects of these policies are still under debate, in particular with regard to changes in financial behaviour. While partial equilibrium models provide a rationale for financial education programs (Jappelli and Padula 2011), the empirical literature shows mixed evidence about individual benefits (Lusardi and Mitchell 2009, Christelis, Jappelli, and Padula 2010, Lusardi and Tufano 2009). The poor financial performance of households, such as low equity market participation or a low level of portfolio diversification, is explained by a stream of household finance literature that is focussed mainly on the individual's abilities to process information: overconfidence (Odean 1998), awareness (Guiso and Jappelli 2005), limited cognitive abilities (Christelis, Jappelli, and Padula 2010). We need structural models to capture the feed-back effects on the individual's behaviour, in particular the informational externalities, and to derive asset pricing implications.

Thus, my approach takes into account feedback effects within a general equilibrium framework. I adopt a framework based on strategic substitutability in information acquisition to analyse the effect of policies aimed at improving individual financial understanding. I model explicitly how agents acquire information and how financial literacy affects this process, fol-

¹Financial literacy concerns the knowledge and understanding of financial concepts. It involves the ability to manage personal finances and confidence regarding the choices made in a complex financial environment. For a review of financial literacy definitions, see (Hung, Parker, and Yoong 2009), Remund (2010) and Huston (2010).

²According to the definition provided by OECD-INFE, the "vulnerable populations" include groups likely to face economic challenges and less likely to fully participate in the financial mainstream. These groups may include, but are not limited to: youth and young adults, the unemployed and under-employed, low-income consumers, those with little or no savings, other consumers outside of or partially outside of the workforce (for example, people with disabilities), and other vulnerable socio-demographic groups such as women and racial and ethnic minorities. See OECD website for a list of financial education programs: <http://www.financial-education.org/home.html>

lowing the literature on costly information acquisition.³ Rational agents purchase the precision of the signal to reduce uncertainty regarding their investment in a risky asset. Within the same framework, Peress (2004) explains an increasing portfolio share in risky assets on the part of the wealthier households through decreasing absolute risk aversion and the availability of costly information: he points out that information generates increasing returns, leading the wealthier to seek further information. Nieuwerburgh and Veldkamp (2010) explain under-diversification bias by the strategic substitutability of information in a multi-asset model: they point out that investors want to learn about assets that others are not learning about. In equilibrium, the ex-ante identical investors choose to observe different signals and hold different assets.

I assume heterogeneous information acquisition costs. I describe a situation where the agents read the same financial report and extract a private signal about the value of a company. The more expert they are, the lower the costs they have, the more precise the signal will be. Their ability to understand financial information is exogenous.⁴ Policy makers can intervene to improve the information processing which affects the degree of heterogeneity, i.e. reduce the gap between the more and the less literate agents. The source of heterogeneity is relevant because similar policies are not easily implementable if we take into account heterogeneity in risk aversion or other subjective characteristics.

I perform a policy exercise involving a reduction of the information acquisition cost, pursued through the implementation of better transparency rules concerning financial prospects. Such a policy should improve the abilities of the less literate agents, without affecting the more literate ones. I investigate the policy implications on the volatility of the market. As a proxy, I use market price variance. The impact of the policy on market volatility can be either positive or negative, depending on the uncertainty of the market fundamentals. Even if the aggregate informativeness always increases, market volatility monotonically decreases with high uncertainty in the fundamentals and it monotonically increases with low uncertainty. The main intuition is that lower information cost for the less literate households leads them to acquire more private information and to trade more actively. Therefore, more private information revealed by market prices affects positively market volatility. On the other hand, with low uncertainty in the fundamentals, the positive informational effect is offset by the negative effect of having more traders with less precise beliefs, who trade more conservatively. The model does not allow a proper welfare analysis because the consumption preferences of noise traders are not modelled.⁵ However, I discuss the policy implications on the expected

³See Grossman and Stiglitz (1980), Verrecchia (1982), Peress (2004), Garcia and Vanden (2009), Nieuwerburgh and Veldkamp (2010).

⁴Padula and Pettinicchi (2012) allow the agents to choose optimally their degree of financial literacy, when they face cognitive costs in attending a financial education course.

⁵Bhattacharya and Nicodano (2001) model noise trading with uncertainty on other traders' intertemporal pref-

utility of the agents. The impact of the policy on the illiterates is not monotonic with respect to the mass of literates: improving transparency increases the expected utility of the illiterates only when there are sufficient literates in the market.

Furthermore, I perform another policy exercise where policy makers provide financial education programs. Starting with homogeneous agents, the policy makers offer a course for a proportion of the population to improve their ability. The educational program is financed through general taxation with a distortionary tax on market capital gains. Increasing the quality of the programs affects positively the expected utility of those agents who become literates. On the other hand, the illiterates are rendered worse off by the presence of the literates and the distortionary tax amplifies the reduction of their expected utility. Extending the size of the program affects negatively the expected utility of both the literates and the illiterates

The paper is organized as follow: in Section 2, I set up the model. In Section 3, I characterize the equilibrium and I discuss the implications of the model. Section 4 performs two policy exercises. The last section concludes and points to further research steps. All the proofs are collected in the Appendices.

2 Model

In this model, the agents face two choices: in the first period, they have to choose if and how much information they want to purchase. In the second period, if and how to trade in the market. There are two primitive assets available for trading. A riskless asset that is perfectly elastically supplied and pays a rate of return r ($R = 1 + r$). A risky asset with price p that pays a payoff π with $\pi \sim N(\mu_\pi, \tau_\pi^{-1})$.⁶ Short selling is allowed and a tax rate t affects the capital gains of the traders.

The per capita supply of the risky asset is $\theta \sim N(\mu_\theta, \tau_\theta^{-1})$, i.e. noise trading.⁷ The fundamentals, π and θ , are mutually independent random variables and their joint distribution is common knowledge.

erences and perform a welfare analysis using numerical simulation.

⁶The assumption of unbounded normally distributed payoffs allows the elaboration of a closed form solution. Alternative distributional assumptions on payoff shocks are used by Barlevy and Veronesi (2007) and Breon-Drish (2012).

⁷The introduction of an exogenous aggregate risk avoids the Grossman-Stiglitz paradox. With extra noise, market price are not fully revealing, therefore agents still have some incentives to purchase information. In the literature, different interpretations characterize this assumption: presence of irrational noise traders, random endowment shocks (Biais, Bossaerts, and Spatt 2010) or other types of individual shocks such as liquidity needs (Wang 1994).

2.1 Agents and policy makers

Agents differ in their ability to acquire information. Heterogeneity is expressed by c that affects the costly information acquisition process. Without loss of generality, I assume two types of agents. Type L (literate agents) with low cost and type H (illiterate agents) with high cost of acquiring financial information. $J = L \cup H$ is the set of all agents. Both types of agents maximize the same concave utility function of their final wealth. For tractability, I assume a CARA utility function with absolute risk aversion coefficient ρ : $U(W) = -\frac{1}{\rho}e^{-\rho W}$.⁸ The population of agents has mass one and there are sufficient agents for each type, so that the law of large numbers applies.⁹ The proportion of literate agents is denoted by λ and both types face a tax rate t on their capital gain.

2.2 Information structure

Once π is realised but not revealed, each agent can purchase an unbiased signal s about the risky asset payoff:

$$s = \begin{cases} \pi + \sqrt{\frac{1}{x}}\epsilon & \text{if } x > 0 \\ \{\emptyset\} & \text{if } x = 0 \end{cases},$$

where ϵ is a white noise, independent of π , θ , and across agents. Agents can purchase private signal precision x paying an opportunity cost $C(x, c)$.

Formally, the cost of acquiring an amount x of precision is modelled by a continuous and twice differentiable function $C(x, c)$ over $x \in \mathbb{R}^+$ that is convex: $C_x > 0$, $C_{xx} \geq 0$ and $C_c > 0$, $C_{cc} \geq 0$. It is continuous at $x = 0$: $C(0, c) = 0$, $\forall c$. Furthermore, $\lim_{x \rightarrow +\infty} C(x, c) = +\infty$. The two properties imply that a totally uninformative signal is costless and a fully revealing signal is infinitely expensive. Moreover, the marginal cost C_x is increasing in c ($C_{xc} > 0$): acquiring information at the margin is more costly for less literate agents. These assumptions ensure the existence of a solution for the information choice. To illustrate the main intuition, I provide a simplified example to derive the numerical outcomes.

Example. The cost function is: $C(x, c) = c(x^2 + x)$ with $c = \{c_L, c_H\}$ and $c_L < c_H$.

To solve the model, I focus on a partially revealing noisy rational expectation equilibrium.

⁸The assumed shape of the utility function fully satisfies the participation principle: given a positive equity premium, all the agents invest money in the risky asset, regardless of the degree of risk aversion or the riskiness of the asset. Empirical studies show that the participation principle fails in reality (the stock-holding puzzle): limited market participation and heterogeneous portfolio behaviours characterize real financial markets (Haliassos and Bertaut (1995), Guiso, Haliassos, and Jappelli (2003)). In Pettinicchi (2012) I introduce a participation cost to model the limited market participation.

⁹Within the group agents differ only in the realization of their private signal, if they purchase one. Moreover, I posit that they do not realize they can act strategically, affecting market price through their informative choice and their asset demand.

After having privately observed unbiased signals, the agents transfer their private information to the market price through their asset demand. Subsequently, the market price reveals some information about the true value of the risky asset and the agents use it as an informative signal. However, the market price is not fully revealing given the noise trading. Following the literature, I look for an equilibrium in which the market price is a linear function of the fundamentals: $pR = a + b\pi - d\theta$ where the coefficients a , b , and d are determined in equilibrium, assuming full rationality of the agents.¹⁰

I denote the agent j 's information set as $\mathcal{F}_j = \{s_j, p\}$, where s_j denotes the agent j 's private signal and it is informative only if the agent j acquires some information precision.

2.3 Timing

There are two periods. In period 1, the planning period, the agent can purchase a private signal s , choosing its precision x . In period 2, the trading period, after having observed her private signal s and the market price p , the agent trades in a competitive market, by choosing her portfolio share α . Hereafter, the timeline of the model (Fig. 1).

Figure 1: Timeline

3 Equilibrium

I solve the model by backward induction. I compute the optimal choices of the agents. In addition, I compute the tax rate that satisfies the budget constraint of the policy makers.

In the trading period, the agent faces a portfolio allocation problem where she needs to decide the share of the portfolio to be invested in the risky asset, in order to maximize her expected utility. At this point in time, the precision x is already purchased and the initial wealth is reduced by the amount C spent for acquiring the private signal. The agent observes

¹⁰The exponential-normal framework allows for closed form solutions under special assumptions: normal joint distribution of the stochastic variables and normal distribution of consumption, conditional on the informative set that implies a well defined expected utility function. Bernardo and Judd (2000) explores numerical solutions to overpass the limits of the framework, allowing for general tastes, returns and informational asymmetries.

a private and a public signal (the market price) and computes posterior beliefs about the asset payoff: $E[\pi|\mathcal{F}]$ and $Var[\pi|\mathcal{F}]$.

In the planning period, the agent chooses how much private signal precision x he wants and pays the monetary cost $C(x, c)$ that is affected by the type specific parameter $c \in \{c_L, c_H\}$, the ability of acquiring financial information.

I describe the individual problem in each period and provide the definition of an equilibrium within the family of the noisy rational expectation equilibria. Proposition 1 claims the existence and the uniqueness of the equilibrium. Then, I discuss the results of the model.

3.1 The trading period

In this period, each agent maximizes the utility of the final wealth, optimally allocating the financial portfolio:

$$\max_{\alpha} E[U(W_2)|\mathcal{F}],$$

subject to the budget constraint:

$$W_2 = (W_1 - C)R^p,$$

where R^p is the portfolio return:

$$R^p = \alpha \left(\frac{\pi - pR}{p} \right) (1 - t) + R.$$

The tax rate affects only the capital gains of the risky asset and the sunk cost C is given by the information choice that is made in the previous period.

The optimal share invested in risky assets differs between agents, depending on the signal observed and the precision purchased. In the trading period, all the information choices are already made and each trading agent transfers some of the information purchased to the market price through the risky asset demand. Consequently, the market price partially reveals the private information. When the rational agents form their posterior beliefs and formulate their asset demands, they take into account the aggregate informativeness and transform the market price into an unbiased public signal. The indirect utility for agent j 's portfolio problem is $E[U(W_2^*)|\mathcal{F}_j]$, which I note as $v(s_j, p; \Theta)$.¹¹

3.2 The planning period

In the planning period, each agent maximizes the indirect utility with respect to information choice, the precision of the private signal. To simplify the notation, I drop the subscript j and

¹¹ $\Theta = \{c_L, c_H, R, W_1, \rho, \mu_\pi, \mu_\theta, \tau_\pi, \tau_\theta, \lambda, t\}$.

write:

$$\max_{x \geq 0} E[v(s, p; \Theta)],$$

subject to:

$$W_1 \geq C(x, c),$$

where the expected utility is computed over the joint probability distribution of s and p . Note that signal precision x affects, by assumption, only the distribution of the private signal s . The individual optimal precision $x^*(\Theta)$ depends directly on the same type-specific information cost and indirectly on other type-specific information costs through aggregate informativeness. In equilibrium, the condition must hold such that the aggregate informativeness I is given by all the private information optimally acquired:

$$I = \int_{j \in J} x_j dG(j).$$

3.3 The equilibrium

A rational expectations equilibrium is given by an asset demand function α_j , an information demand function x_j for all agents and a price function p of π and θ such that:

1. $x_j = x_j^*(\Theta)$ and $\alpha_j = \alpha^*(s_j, p; \Theta)$ solve the maximization problems.
2. p clears the market for the risky asset:

$$\int_{j \in J} \alpha_j \frac{W_1 - C_j}{p} dG(j) = \theta.$$

In noisy rational expectations equilibrium models, the agents make self-fulfilling conjectures about prices and the equilibrium is defined as the set of allocations such that the agents maximize their utilities, the markets clear and the individual optimal choices are consistent with the aggregate variables.

The following proposition claims the existence and the uniqueness of an equilibrium within the family of noisy rational expectation equilibria.

Proposition 1. *There exists a unique noisy rational expectation equilibrium with linear price function.*

Proof. See the appendix. □

Here I provide a sketch of the proof and hereafter I discuss the main features of the equilibrium. For given aggregate informativeness, I derive the market equilibrium, computing the optimal asset demands and the equilibrium market price function. In addition, I compute

the optimal information choices, checking if the individual choices are consistent with the assumed aggregate informativeness.

The information choice

The optimal information choice x^* is the maximum between zero and the solution of the following equation:

$$2\rho RC_x(x^*, c)k^* = 1, \quad (1)$$

where k^* is the precision of the individual's posterior beliefs after having observed the private signal and the market price.¹²

The cost of acquiring financial information affects directly the choice of whether to be informed. The agents with an information acquisition cost greater than the endogenous threshold $\bar{c}(\Theta)$ optimally choose to remain uninformed. Therefore, the threshold identifies the lowest level over which it is worthwhile to be informed and it is given implicitly by the following formula:

$$C_x(\bar{c}, 0) = \frac{1}{2\rho R(\tau_\pi + \frac{I^2}{\rho^2(1-t)^2}\tau_\theta)}.$$

Let us consider a policy aimed at making the illiterate agents more informed and, therefore, more active traders. The policy makers can intervene improving their information abilities, through the implementation of better transparency rules that do not affect the abilities of the literates. The model allows us to measure the policy makers' effort to make the policy effective and capable of changing the behaviour of the illiterate agents, i.e. from being uninformed to being informed. I focus on the gap between the endogenous threshold $\bar{c}(\Theta)$ and c_H , and I use it as a measure of the effort that policy makers must make in order that their program be effective. c_L is constant and c_H decreases (Table 1). In the first column, I report the optimal information choice of the illiterates. In the last column, positive values of the gap indicate that the illiterates are optimally uninformed and the greater values imply a greater increase in market transparency aimed at making the illiterates informed traders.

Even if the policy does not affect directly the information cost of the literates, the change in the aggregate informativeness implies relatively less information acquired by them. This outcome is due to strategic substitutability between private and public information: the more information the market price reveals, the less incentives the agents have to acquire information. Differentiating equation (1) with respect to I , I can show that the optimal information

¹² k^* is the sum of τ_π , the precision of the prior beliefs, x^* , the precision of the private signal and τ_ξ , the precision of the public signal ξ derived by the market price. See the Appendix A for the derivation of ξ (3) and τ_ξ (6).

choice is a non increasing function of the aggregate informativeness. Formally, I have:

$$\frac{dx}{dI} = -\frac{2C_x}{C_{xx}k + C_x} \frac{I\tau_\theta}{\rho^2(1-t)^2} \leq 0.$$

Therefore, policy aimed at improving the abilities of the illiterates affects only indirectly the information choice of the literates: more aggregate informativeness induces them to reduce their acquired information, $\frac{dx_L}{dc_H} \geq 0$. On the other hand, the overall effect on the information choice of the illiterates is positive: lower information acquisition cost leads them to acquire more information, $\frac{dx_L}{dc_H} \leq 0$.¹³

The portfolio choice

The optimal portfolio share is the standard solution for the maximization problem of an agent with a CARA utility function. The agent optimally chooses to trade when he believes that the expected excess return is positive. For each agent $j \in J$, the optimal portfolio share α_j depends on the precision of the posterior belief, k_j^* , and on the expected excess return conditional on the agent's informative set, $E[\pi|\mathcal{F}_j] - pR$:

$$\alpha_j^* = \frac{p k_j^*}{\rho(W_1 - C_j)(1-t)} (E[\pi|s_j, p] - pR),$$

and the following condition holds $\alpha_j^* (E[\pi|s_j, p] - pR) > 0$. To highlight the role played by information in portfolio choice, I rewrite the optimal portfolio share:

$$\alpha_j^* = \frac{W_1}{W_1 - C_j} \alpha^*|_{x=0} + \frac{p}{\rho(1-t)(W_1 - C_j)} [x_j(s_j - pR)],$$

where the first term is the optimal share of an uninformed agent who follows only market feelings, i.e. public knowledge embodied in prior beliefs and market price, herding on what the market partially reveals.¹⁴ The second bracketed term is the risky asset's premium as predicted by the private signal. It is the extra portfolio share of the informed agent, who uses the private information to balance what the market suggests: her trading position is consistent with her private signal. In the case that he gets a private signal realization different from the market feelings, he would prefer to bet against the market, in order to speculate on her private knowledge. Furthermore, the more precise the private signal he has, the more

¹³See the appendix B.

¹⁴When $I = 0$, no agents purchase private information. There are only uninformed and noise traders in the markets. The market price merely reflects the noisy supply and the agents hold risky assets in order to offset it. When $I \rightarrow \infty$, $(1 - \tau_\xi \frac{1}{I + \tau_\xi})$ goes to zero and agent j does not purchase any risky assets: $\alpha_j^* = 0$. The market price fully reveals the value of the fundamentals, therefore there are no reasons to trade.

aggressively he would be inclined to trade.

The market price variance

The equilibrium market price is a linear function of the fundamentals: $pR = a + b\pi - d\theta$, where the coefficients a , b , and d are determined by (5):

$$a = \frac{\mu_\pi \tau_\pi + \frac{I\mu_\theta \tau_\theta}{\rho(1-t)}}{\tau_\pi + I + \tau_\xi}, \quad b = \frac{I + \tau_\xi}{\tau_\pi + I + \tau_\xi}, \quad d = \frac{\rho + \frac{I\tau_\theta}{\rho(1-t)}}{\tau_\pi + I + \tau_\xi}.$$

The equilibrium market price depends on the aggregate information I and I work out the relationship through the beliefs of the "average agent". It does not mean there exists a real agent with such beliefs. It means a fictitious agent with private signal realization $s_j = \pi$ and private signal precision equal to I , as in a shared-information economy where the central planner can observe all the private signals taking the sample mean: $\int_j s_j dG(j) = \pi$, with precision $\int_j x_j dG(j) = I$.

The average agent has posterior precision:

$$K = \tau_\pi + I + \tau_\xi,$$

and posterior mean:

$$\overline{E}[\pi|\pi, p] = K^{-1} [\mu_\pi \tau_\pi + \pi I + \xi \tau_\xi].$$

After substituting a , b , and d and few algebra steps, the equilibrium market price can be rewritten as:

$$pR = \overline{E}[\pi|s, p] - \frac{\rho(1-t)\theta}{K}.$$

It is driven by two components: the posterior belief of the average agent and the discount on the price demanded by the traders to compensate for the uncertainty due to the noisy supply.

The market price is random because it depends on the realization of the fundamentals. Given the assumed probability distribution, the equilibrium market price is a Gaussian with mean: $\mu_{pR} = \mu_\pi - \frac{\rho(1-t)}{K} \mu_\theta$ and variance:

$$\sigma_{pR}^2 = \frac{1}{K^2} \left\{ (\tau_\xi + I)^2 \frac{1}{\tau_\pi} + \left[\frac{I\tau_\theta}{\rho(1-t)} + \rho(1-t) \right]^2 \frac{1}{\tau_\theta} \right\}.$$

Hereafter, I check the impact on the market price variance of changes in exogenous factors, such as the uncertainty of the fundamentals and the agents' risk aversion.

Lower uncertainty about asset payoff and asset supply decreases the market price vari-

ance, with a wider range for the former (Table 2(a)) than for the latter (Table 2(b)). The market price reflects the uncertainty in the fundamentals, but the impact of the noise in the asset supply is mitigated by the aggregate informativeness, the risk aversion and the tax rate.

Risk aversion does not monotonically affect volatility. Either with high or low risk aversion, the market price variance is higher than when the agents are moderately risk averse (Table 3). This result is driven by the impact of their risk aversion on the coefficients of the fundamentals in the price function. Less risk averse agents (low ρ) trade heavily and transfer more of their private information to the market price. The market price is more sensible to asset payoff shocks (high b) and is less sensible to noisy supply shocks (low d), given the ability of the traders to absorb the shocks of the noisy supply.

Increasing risk aversion leads agents to acquire more private information and to reduce their trading. Therefore, the aggregate informativeness increases but the sensitivity of the market price to the asset payoff shocks decreases. Moreover, the reduced trading of the agents implies that the market dries up and the sensitivity of the market price to the noisy supply shock increases but at a lower rate with respect to the decrease of the sensitivity to the asset payoff shocks. The total effect is that higher risk aversion leads to a lower market price variance.

This mechanism works up to a point, according to which an increase in risk aversion has the agents reduce their acquisition of private information, given the lower value attached to it. Therefore, the aggregate informativeness decreases and reinforces the decrease of the sensitivity of the market price to asset payoff shocks. Moreover, the direct effect of the reduced trading and the indirect effect of lower aggregate informativeness increases the sensitivity of the market price to noisy supply shocks. The effect of the risk aversion on the market price variance changes and increasing risk aversion leads to higher volatility in the market price.

I do not go deeply into the explanation of the mechanism underlying the effect of the risk aversion on the volatility of the market price. I focus on the effects of exogenous variables that are under the control of policy makers: the tax rate through fiscal policies, the cost of acquiring information, improving the transparency rules of financial markets, and boosting the number of literate agents by providing financial education programs.

The role of the fiscal policy will be discussed in the next section, while, hereafter, we focus on an improvement of the market transparency that lowers c_H . I show the following theorem:

Theorem 3.1. *The impact of the policy on the market volatility is cross-partial non-monotonic, even if the policy always increases the aggregate informativeness.*

As a proxy of market volatility, I use market price variance. The policy affects the information choice of the illiterates, and indirectly also that of the literates, through the aggregate informativeness. I break down the effect to work out the region of the parameter space where

the impact of the policy is well determined. Formally, the policy effect on market stability is given by:

$$\frac{d\sigma_{pR}^2}{dc_H} = \frac{d\sigma_{pR}^2}{dI} \frac{dI}{dc_H}, \quad (2)$$

where $\frac{d\sigma_{pR}^2}{dI}$ is the impact of the aggregate informativeness on market price variance and $\frac{dI}{dc_H}$ is the impact of the policy on aggregate informativeness.¹⁵

The latter, $\frac{dI}{dc_H}$, is always non-positive. The result is given by two effects: the first one will tend to drive up aggregate informativeness. The policy allows illiterate agents to acquire information at a lower cost and, therefore, they optimally acquire more information. The second effect would drive down aggregate informativeness: higher private information acquired by the illiterates implies higher aggregate informativeness, leading both types to reduce their acquisition of private information. The first effect dominates the second and the policy leads overall to higher aggregate informativeness. Only where it is optimal for the illiterates to remain uninformed, will the policy have no impact on the aggregate informativeness.

The former term, $\frac{d\sigma_{pR}^2}{dI}$, is cross-partial non-monotonic, i.e. the derivative changes the sign across different values of the fundamentals uncertainty, τ_π and τ_θ . In order to study it, I collapse the behaviour of the two types into the fictitious average agent described above. The impact of the aggregate informativeness on market price variance is negative when the market price variance is greater than a threshold given by two terms. The first one is the prior variance of the asset payoff, weighted by the sensitivity of the market price to asset payoff shocks, i.e. $\frac{b}{\tau_\pi}$. The second term is the inverse of the effect of aggregate informativeness on the variance of the average agent's belief, weighted by the sensitivity of the market price to noisy supply shocks, i.e. $\frac{d}{\rho(1-t)} \frac{1}{dK/dI}$. Namely, I have:

$$\frac{d\sigma_{pR}^2}{dI} < 0 \iff \sigma_{pR}^2 > \frac{b}{\tau_\pi} + \frac{d}{\rho(1-t)} \frac{1}{dK/dI},$$

where $\frac{dK}{dI}$ is the monotonic change in the posterior precision of the average agent's belief when the aggregate informativeness increases.

Higher aggregate informativeness decreases the market price variance if the latter is sufficiently higher than the prior variance of the asset payoff $\frac{b}{\tau_\pi}$, depending on the sensitivity of the market price to market shocks and on the changes in the posterior variance of the average agent's belief with respect to aggregate informativeness. If K is not much responsive to aggregate informativeness, for market volatility to decrease as aggregate informativeness increases, it must hold that the gap between the market price variance and the prior variance of the asset payoff ($\sigma_{pR}^2 - \frac{b}{\tau_\pi}$), is high enough, i.e., the contribution of the noisy supply to the

¹⁵See the appendix B, Equation (10).

overall market volatility is small.¹⁶

I summarize the impact of the policy on market volatility with the following formula:

$$\frac{d\sigma_{pR}^2}{dc_H} > 0 \iff \sigma_{pR}^2 > \frac{b}{\tau_\pi} + \frac{d}{\rho(1-t)} \frac{1}{dK/dI}.$$

Policy makers can manipulate the information acquisition cost of the illiterates and this intervention affects aggregate informativeness through the agents' behaviour. The behaviour of the fictitious average agent helps to illustrate the effect of the policy on market stability, which depends on the gap between the market price variance and the prior variance of the asset payoff, and on the sensitivity of the posterior belief variance with respect to the aggregate informativeness.

The main intuition is that implementing a policy aimed at lowering the information costs for illiterate agents leads them to acquire more private information and to trade more actively. Thus, more private information revealed by market prices affects positively market volatility. On the other hand, with low uncertainty in the fundamentals, the positive informational effect is offset by the negative effect of having more traders with less precise beliefs, who trade more conservatively.

I report the market price variance for increasing values of the financial information acquisition costs of the illiterates (Table 4) and I plot the variance of the market price and the threshold for different values of fundamentals uncertainty: $\tau_\pi, \tau_\theta = [.3, .5, 1, 2]$ (see Fig. 2). When the market price variance is above the threshold, the market price variance increases with the inequality of the financial information acquisition costs between the two types of agents. In the opposite case it decreases with the inequality.

3.4 The expected excess return

The incentive to trade is the expected excess return: $f = E[\pi|\mathcal{F}] - pR$. At the planning period, it is a random variable formed by equilibrium market price and posterior beliefs about the payoff. The latter is itself a random variable normally distributed with mean and variance:

$$E[E[\pi|\mathcal{F}]] = \mu_\pi, \quad Var[E[\pi|\mathcal{F}]] = \frac{1}{\tau_\pi} - \frac{1}{k}.$$

The expectation is the prior mean of the risky asset and it is the same for both types. The variance of the posterior beliefs differs between types: it is higher for the literates compared to the illiterates. The literates acquire more information and they end up, on average, with

¹⁶Note that the market volatility reflects both the volatility of the asset and of the noisy supply, given the assumed market structure.

posterior beliefs close to the true value. The illiterate agents rely less on their private signal and their posterior beliefs are, on average, closer to the market price.

Thus, I compute the distribution of the expected excess return: $f \sim N(\mu_f, \sigma_f^2)$:

$$\mu_f = \rho\mu_\theta K^{-1}, \quad \sigma_f^2 = \frac{d^2}{\tau_\theta} + \frac{1}{K} \left(\frac{\tau_\pi}{K} - \frac{K}{k} \right).$$

The expected gain from being a trader is given by the mean of the noisy supply multiplied by the risk aversion and the posterior beliefs precision of the average agent. It derives from the opportunity to deploy informational advantages against those agents who need to trade for exogenous reasons. It decreases with the aggregate informativeness.

The variance of the expected excess return differs between types. There is a common part $\frac{d^2}{\tau_\theta}$ that refers to the volatility of the noisy supply multiplied by the square of the sensitivity of the market price to the noisy supply shocks, i.e. the market depth. Lower volatility of the noisy supply and higher liquidity of the market imply lower volatility of the expected excess return. The second part, $\frac{1}{K} \left(\frac{\tau_\pi}{K} - \frac{K}{k} \right)$, shows the informational advantage or disadvantage of one type with respect to the average agent. However, the magnitude decreases as the aggregate informativeness increases.

4 Policy implications

In this section, I use the model to perform two policy exercises: the first one involves increasing market transparency with and without literate agents. The policy can be performed without any cost for the policy makers, requiring clearer financial reports for the retail investors or collecting relevant information in a single website. The second exercise handles the effects of financial education programs that make a proportion of the population literate. The cost of the policy which is proportional to the size and to the quality of the program is financed through general taxation on the market excess return.

4.1 Transparency

I distinguish between two scenarios for understanding the role played by the literate traders. Let us start when there are only illiterate agents. Implementing better transparency rules implies lower cost of acquiring information for all the agents in the market. They will then acquire more information and their beliefs will be more concentrated around the true value of the risky asset. The market price will be more revealing and their expected excess return will be lower. When $\lambda = 0$, the expected utility of the illiterates is decreasing in the market transparency, showing that improving the abilities of the agents reduces the benefits that they

can derive from the informational advantage with respect to noise traders (Table 6).

Once I introduce heterogeneity in the abilities of acquiring information, the illiterate agents face traders with greater informational advantages. I work out how the mass of literates affects policy implications. With few literates ($\lambda = 0.1$), the expected utility of the illiterates is still decreasing with the market transparency (Table 6) while, with a higher proportion of literates ($\lambda = 0.25$), more market transparency implies higher expected utility for the illiterates (Table 6).

The intuition behind this result is that the market price, as a public signal, affects the beliefs of the literates, reinforcing or mitigating their private information. The aggregate informativeness is a linear combination of the private information of the two types. If the illiterates contribute more to the aggregate informativeness (λ is low), the market price induces the literates to trade more conservatively and to exploit less their informational advantage. The expected utility of the illiterates is higher than what they would obtain were they as literate as the other type. Reducing the inequality between the two types decreases the expected utility of the illiterates, reducing at the same time the gap between their expected utility and the utility of the other type. If the illiterates contribute less to the aggregate informativeness (λ is high), the market price reinforces the beliefs of the literate, making them more aggressive traders. The illiterates face higher informational disadvantage and their expected utility is lower than what they would obtain were they literate. Improving the market transparency allows the illiterates to reduce the informational disadvantage and to increase their expected utility.

4.2 Financial education

The model allows us to understand the effect of a financial education program aimed at making more literate a proportion of the population that is assumed to be initially homogeneous. I compare the expected utility of the agents before and after the introduction of the financial education program. The provided financial course is assumed to be fully effective: i.e. it improves the abilities of all the agents who attend it.¹⁷ The size of the program, i.e. the number of agents who become more literate, is set by the policy makers, as well as the quality of the course. The program is financed through general taxation and the policy makers choose the tax rate t on the excess return of the traders.¹⁸ The amount to be financed depends on the

¹⁷This assumption rule out the case where the educational program is not able to affect the abilities of the agents, leaving their behaviour unchanged. Mixed empirical evidence highlights the need for further debate about this issue Gale and Levine (2011).

¹⁸The distortionary tax rate drives the results of this section, lowering the expected utilities. Notwithstanding, I think it is relevant to take into account the cost of similar policies to balance the informational effects due to the improved abilities. An alternative taxation scheme could affect only the excess return of the literates. It would be a kind of fee to be paid to attend the financial course. However, the choice should be modelled to take into

size of the program, λ , and its quality, the distance between the agents' current abilities and their acquired abilities after having attended the program ($c_H - c_L$). The budget constraint is satisfied in expectation terms over the joint distribution of the fundamentals:

$$E[t \int_j \max\{CG_j, 0\} dG(j)] = \lambda(c_H - c_L),$$

with $CG = \alpha_j^*(W_1 - C_j^*) \frac{\pi - pR}{p}$.¹⁹ In the model, λ not only captures the cost of the policy, but also affects the incentive to become informed and to trade actively. On the other hand, the ability acquired by the agents who attend the financial education program, c_L , affects their information choice and the information choice of the illiterates, through the aggregate informativeness. To evaluate the impact of the policy, I discuss the effects along two dimensions: i) the proportion of the literates and ii) the inequality in financial information acquisition cost.

When λ is 0.25 and the inequality in the financial information acquisition costs increase, i.e. higher quality of the financial education program due to better qualified teachers, the results show an increasing tax rate with the inequality (Table 7). Increasing the ability of the agents who attend the financial education program implies higher aggregate informativeness due to their information choice. The feed-back effect decreases the expected fiscal revenue, so that the policy makers need to set a higher tax rate to finance policy expenditures. The expected utility of the literates increases due to their informational advantage, while that for the illiterate decreases, driving down the aggregate welfare. I report the expected utility of the two types and a linear combination to capture a measure of the aggregate welfare WU , a weighted sum of the individuals' expected utility (Fig. 3). The right panel shows the non monotonic behaviour of the expected utility of the literates: when the size of the program is big enough, above a given threshold, the benefit of having programs of higher quality first increases, then reduces the expected utility of the literates.

When c_L is set to 0.01 and the proportion of the literates increases, the results show higher tax rates with more literates (Table 8). The impact of the policy on aggregate informativeness depends on the changes in λ and on the information choices of the two types. Higher λ implies a higher contribution of the literates to aggregate informativeness and, at the same time, lower information acquired, either due to strategic substitutability or to the lower return of the information acquisition. The expected utility of both types decreases with an increasing proportion of the literates (Fig. 4). I report different degrees for the quality of the program. Higher inequality in the financial information costs amplifies the gap between the expected utilities of the two types.

account the incentives to be literate. Padula and Pettinicchi (2012) provide insights about this issue.

¹⁹See Appendix C for the computations and the numerical results. Simulations with different policy cost functions are available upon request.

5 Conclusion

This paper adopts a noisy rational expectations equilibrium with endogenous information acquisition to analyse heterogeneity in financial literacy and its impact on market volatility. The model provides rationale for the implementation of better transparency rules, aimed at improving individual financial literacy. Moreover, I evaluate the policy implications of the provision of financial education, financed through general taxation.

In the model, I posit that financial literacy affects individual information acquisition costs. The individuals choose how much information to acquire and the share of wealth to invest in risky assets. There are two types of individuals, literate and illiterate, and they trade against noise traders. I point out that the more financially literate individuals are, the more information they acquire and the more information the market price reveals. Within a general equilibrium framework, I derive the market volatility and the implications of policies aimed at improving the financial abilities of the agents.

For a policy that improves the transparency of the financial markets, the policy makers should take into account a change in the direction of the impact on the market volatility. The effect depends on the uncertainty in the market fundamentals. The policy implication is to reduce the information acquisition cost only when there is high uncertainty in the market fundamentals. The main intuition is that lower information cost for the less literate households leads them to acquire more private information and to trade more actively. Therefore, more private information revealed by market prices affects the market volatility positively. On the other hand, with low uncertainty in the fundamentals, the positive informational effect is offset by the negative effect of having more traders with less precise beliefs, who trade more conservatively. The policy affects the expected utility of the illiterates and the impact is not monotonic in the mass of literates: improving transparency increases the expected utility of the illiterates when there are enough literates trading in the markets. Thus, the policy reduces the informational disadvantage of the illiterates that is not so large when the mass of literates is small.

Furthermore, the model allows us to evaluate the provision of financial education, financed through general taxation. The policy makers choose the size and the quality of the program, but only the latter affects positively the expected utility of those agents who become literates. On the other hand, the illiterates are worse off for the presence of the literates and the distortionary tax amplifies the reduction of their expected utility.

Further research will take into account participation costs in order to explain limited market participation and let the individual's amount of financial literacy be endogenously chosen by the agents. I will perform a welfare analysis, introducing uncertainty in the proportion of the literates.

Appendix A - Proof of proposition 1

In order to prove the existence and the uniqueness of the equilibrium, I need to prove the existence and the uniqueness of an equilibrium market price that linearly depends on fundamentals, π and θ , and clears the market. In equilibrium, all the agents maximize their utility. I compute optimal asset demands and the equilibrium market price for given aggregate informativeness I . In the following step of the proof, I derive optimal information choices and I work out the equilibrium in the information market, proving the existence and the uniqueness of the solution.

Lemma 1 (Individual asset demands and Market price). *For given aggregate informativeness I , the optimal portfolio share for agent $j \in J$ is given by:*

$$\alpha_j^* = \frac{k_j p}{\rho[W_1 - C(x_j, c_j)](1-t)}(E[\pi|\mathcal{F}_j] - pR),$$

and the equilibrium market price is $pR = a + b\pi - d\theta$, where the coefficients a , b , and d are determined in (5).

Proof. The proof is given in five steps. In the first step, I guess a price linear function and derive the informationally equivalent public signal ξ from the price function. In the second step, I compute the mean and the variance of posterior beliefs, given two unbiased signals, ξ and s . In the third step, I derive optimal asset demands. In the fourth step, I derive market clearing conditions and, in the last step, I impose rationality and determine the coefficients of the guessed linear price function.

Remind that the joint distribution of the payoff, the supply and the signals is:

$$\begin{pmatrix} \theta \\ \pi \\ s_j \\ \vdots \\ s_{j'} \\ pR \end{pmatrix} \sim N \left\{ \begin{pmatrix} \mu_\theta \\ \mu_\pi \\ \mu_\pi \\ \vdots \\ \mu_\pi \\ a + b\mu_\pi - d\mu_\theta \end{pmatrix}, \begin{pmatrix} \frac{1}{\tau_\theta} & 0 & 0 & \cdots & d0 & \frac{1}{\tau_\theta} \\ 0 & \frac{1}{\tau_\pi} & \frac{1}{\tau_\pi} & \cdots & \frac{1}{\tau_\pi} & b\frac{1}{\tau_\pi} \\ 0 & \frac{1}{\tau_\pi} & \frac{1}{\tau_\pi} + \frac{1}{x} & \cdots & 0 & b\frac{1}{\tau_\pi} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & \frac{1}{\tau_\pi} & 0 & \cdots & \frac{1}{\tau_\pi} + \frac{1}{x} & b\frac{1}{\tau_\pi} \\ d\frac{1}{\tau_\theta} & b\frac{1}{\tau_\pi} & b\frac{1}{\tau_\pi} & \cdots & b\frac{1}{\tau_\pi} & b^2\frac{1}{\tau_\pi} + d^2\frac{1}{\tau_\theta} \end{pmatrix} \right\}.$$

Step 1 The agents guess a price function that is linear in π (future payoff) and θ (noisy supply):

$$pR = a + b \left[\lambda \int_{j \in L} s_j dG(j) + (1 - \lambda) \int_{j \in H} s_j dG(j) \right] - d\theta.$$

Applying the law of large number within each group of traders, $\int_j \epsilon_j dG(j) = 0$ with probability one. Therefore, I can rewrite the price function as:

$$pR = a + b\pi - d\theta.$$

The agents use the observed private signals to update their prior beliefs $\pi \sim N(\mu_\pi, \tau_\pi^{-1})$. The private signal is unbiased by construction, $s|\pi \sim N(\pi, x^{-1})$, and conditionally independent from the prior belief, μ_π : $E[(\mu_\pi - \pi)(s - \pi)] = 0$. Rational agents use the price as a public signal. It is not unbiased: $E[pR|\pi] = a - d\mu_\theta + b\pi$. To apply Bayesian updating, the agents transform the price in an informationally equivalent variable ξ :

$$\xi = \frac{pR - a + d\mu_\theta}{b} = \pi - \frac{d}{b}(\theta - \mu_\theta), \quad (3)$$

that is unbiased, $\xi|\pi \sim N(\pi, \frac{d^2}{b^2} \frac{1}{\tau_\theta})$.

Step 2 Each agent j observes $\mathcal{F}_j = \{s_j, p\} \equiv \{s_j, \xi\}$ and updates her prior beliefs with the two Gaussian signals. Using the formula for the multivariate normal distribution (Degroot (2004), p. 55), the posterior expectation is given by:

$$\begin{aligned} E[\pi|s_j, p] &= \mu_\pi + \frac{1}{k} \left\{ x_j (s_j - E[s_j]) + \frac{b^2}{d^2} \tau_\theta (\xi - E[\xi]) \right\} \\ &= \frac{1}{k_j} \left(\tau_\pi \mu_\pi + x_j s_j + \frac{b^2}{d^2} \tau_\theta \xi \right), \end{aligned}$$

where the precision of the posterior belief k_j is given by the sum of precisions of the prior, of the private signal and of the public signal:

$$k_j = \frac{1}{\text{Var}[\pi|s_j, p]} = \tau_\pi + x_j + \frac{b^2}{d^2} \tau_\theta.$$

Step 3 Maximizing the CARA utility function with respect to the control variable α , each agent solves the following:²⁰

$$\max_{\alpha} E \left[-\frac{1}{\rho} e^{-\rho W_2(\alpha)} | s, p \right] = \max_{\alpha} -\frac{1}{\rho} e^{-\rho \{E[W_2(\alpha)|s, p] - \frac{\rho}{2} \text{Var}[W_2(\alpha)|s, p]\}}. \quad (4)$$

²⁰I use the log-normal distribution properties and I drop subscript j to simplify notation. It will be restored during the aggregation of individual asset demands.

The final wealth can be rewritten as:

$$W_2 = \begin{cases} \left(\alpha \frac{\pi - pR}{p} + R \right) (W_1 - C) & \text{if } \alpha (\pi - pR) < 0, \\ \left[\alpha \frac{\pi - pR}{p} (1 - t) + R \right] (W_1 - C) & \text{if } \alpha (\pi - pR) > 0, \end{cases}$$

where the first condition identifies the losses and the second one the gains. Only the capital gains are affected by the tax rate. Before π is revealed and after having observed the private signal, the final wealth is a random variable and its conditional expectation is:

$$E[W_2|s, p] = \begin{cases} \left(\alpha \frac{E[\pi|s, p] - pR}{p} + R \right) (W_1 - C) & \text{if } \alpha (E[\pi|s, p] - pR) < 0 \\ \left[\alpha \frac{E[\pi|s, p] - pR}{p} (1 - t) + R \right] (W_1 - C) & \text{if } \alpha (E[\pi|s, p] - pR) > 0 \end{cases},$$

while the variance is:

$$Var[W_2|s, p] = \begin{cases} \frac{\alpha^2}{p^2} (W_1 - C)^2 Var[\pi|s, p] & \text{if } \alpha (E[\pi|s, p] - pR) < 0 \\ \frac{\alpha^2}{p^2} (1 - t)^2 (W_1 - C)^2 Var[\pi|s, p] & \text{if } \alpha (E[\pi|s, p] - pR) > 0 \end{cases}.$$

After having substituted the expectation and the variance of the final wealth in (4), we can solve the maximization problem. The optimal risky asset demand for agent j is:

$$\alpha_j^* = \frac{p k_j}{\rho (W_1 - C_j)(1 - t)} (E[\pi|s_j, p] - pR),$$

and the following condition holds: $\alpha_j^* (E[\pi|s_j, p] - pR) > 0$. The amount of wealth in risky assets depends on the posterior precision k_j , the risk aversion ρ , the tax rate t and the expected excess return of the risky investment.

Step 4 The equilibrium price clears the market for the risky asset. Aggregating over all traders yields the aggregate demand:

$$\int_{j \in L} \alpha_j^* \frac{W_1 - C_j}{p} dG(j) + \int_{j \in H} \alpha_j^* \frac{W_1 - C_j}{p} dG(j) = \theta.$$

I apply the weak law of large numbers for independent and identically distributed random variables with the same mean, such that $\int_j s_j dG(j) = \pi$. Then, imposing market clearing

condition holds the following equation:

$$\mu_\pi \tau_\pi + \frac{b^2}{d^2} \tau_\theta \xi + \pi [\lambda x_L + (1 - \lambda) x_H] - pR [\lambda k_L + (1 - \lambda) k_H] = \rho(1 - t)\theta,$$

where x_L and k_L are the information choice and posterior precision of the literate agent (type L). Similarly, x_H and k_H for the illiterate agent (type H). Using the definition of aggregate informativeness $I = \lambda x_L + (1 - \lambda) x_H$, I can rewrite the price equation as:

$$pR = (\tau_\pi + I + \frac{b^2}{d^2} \tau_\theta)^{-1} \left[\mu_\pi \tau_\pi + \frac{b^2}{d^2} \tau_\theta \xi + \pi I - \rho(1 - t)\theta \right].$$

Step 5 I impose rationality. ξ involves undetermined coefficients b, d . I substitute the expression for $\xi = \pi - \frac{d(\theta - \mu_\theta)}{b}$ and, rearranging the terms, I have:

$$pR = (\tau_\pi + I + \frac{b^2}{d^2} \tau_\theta)^{-1} \left\{ \mu_\pi \tau_\pi + \frac{b}{d} \tau_\theta \mu_\theta + \pi(I + \frac{b^2}{d^2} \tau_\theta) - \theta \left[\frac{b}{d} \tau_\theta + \rho(1 - t) \right] \right\}.$$

I derive $\frac{b}{d} = \frac{I}{\rho(1-t)}$ and I substitute it back into the price function. I find out the following determined coefficients.

$$a = \frac{\mu_\pi \tau_\pi + \frac{I}{\rho(1-t)} \mu_\theta \tau_\theta}{\tau_\pi + I + \frac{I^2}{\rho^2(1-t)^2} \tau_\theta}, \quad b = \frac{I + \frac{I^2}{\rho^2(1-t)^2} \tau_\theta}{\tau_\pi + I + \frac{I^2}{\rho^2(1-t)^2} \tau_\theta}, \quad d = \frac{\rho + \frac{I}{\rho(1-t)} \tau_\theta}{\tau_\pi + I + \frac{I^2}{\rho^2(1-t)^2} \tau_\theta}. \quad (5)$$

□

To simplify the notation, I rewrite the precision of the public signal ξ :

$$\tau_\xi = \frac{I^2}{\rho^2(1-t)^2} \tau_\theta. \quad (6)$$

Lemma 2. For given aggregate informativeness I , for all agents with $c < \bar{c}(\Theta)$, the optimal information choice x^* solves the following equation:

$$2\rho RC_x(x^*, c) (\tau_\pi + x^* + \tau_\xi) = 1, \quad (7)$$

where $\bar{c}(\Theta)$ is given by (9).

Proof. In order to solve for the information choice x^* , first I need to compute the indirect utility: $v(s_j, p; \Theta) = E[U(W_2(\alpha_j^*)) | s_j, \xi]$, and its expected value. The latter depends on the first two moments of the expected return of trading a unit of risky asset. Once I derive the expected indirect utility function, I apply the concave maximum theorem to find out the optimal

information choice x^* and to identify the threshold $\bar{c}(\Theta)$, according to which is worthwhile to be informed.

Step 1 The outcome of the first step is the indirect utility function. For agent $j \in J$, it is:

$$v(s_j, p; \Theta) = -\frac{1}{\rho} e^{-\frac{1}{2} k_j (E[\pi|s_j, p] - pR)^2 - \rho R [W_1 - C(x_j, c_j)]}.$$

I derive it substituting the optimal asset demand back into the expected utility (4). Thus, for each agent:

$$v(s, p; \Theta) = E\left[-\frac{1}{\rho} e^{-\rho W_2(\alpha^*)} | \mathcal{F}\right] = -\frac{1}{\rho} e^{-\rho\{E[W_2(\alpha^*)|s, \xi] - \frac{\rho}{2} \text{Var}[W_2(\alpha^*)|s, \xi]\}},$$

where

$$\begin{aligned} E[W_2(\alpha^*)|s, \xi] &= E\left[\alpha^* \frac{\pi - pR}{p} (1-t) + R\right] [W_1 - C(x, c)] | s, \xi \\ &= [W_1 - C(x, c)] \left[\alpha^* E\left[\frac{\pi - pR}{p} (1-t) | s, \xi\right] + R\right] \\ &= [W_1 - C(x, c)] \left[\frac{pk}{\rho[W_1 - C(x, c)](1-t)} (E[\pi|s, p] - pR) \frac{E[\pi|s, p] - pR}{p} (1-t) + R\right] \\ &= \frac{k}{\rho} (E[\pi|s, p] - pR)^2 + R[W_1 - C(x, c)], \end{aligned}$$

and

$$\begin{aligned} \text{Var}[W_2(\alpha^*)|s, \xi] &= \frac{[W_1 - C(x, c)]^2 (1-t)^2 (\alpha^*)^2}{p^2} \text{Var}[\pi|s, p] \\ &= \frac{[W_1 - C(x, c)]^2 p^2 k^2 (E[\pi|s, p] - pR)^2}{p^2 [W_1 - C(x, c)]^2 (1-t)^2 \rho^2} \frac{1}{k} \\ &= \frac{k}{\rho^2} (E[\pi|s, p] - pR)^2. \end{aligned}$$

Substituting back the last two terms into the indirect utility of agent j , I get:

$$\begin{aligned} v(s_j, p; \Theta) &= -\frac{1}{\rho} e^{-\rho\left\{\frac{k_j}{\rho} (E[\pi|s_j, \xi] - pR)^2 + R[W_1 - C(x_j, c_j)] - \frac{\rho}{2} \frac{k_j}{\rho^2} (E[\pi|s_j, \xi] - pR)^2\right\}} \\ &= -\frac{1}{\rho} e^{-\frac{1}{2} k_j (E[\pi|s_j, \xi] - pR)^2 - \rho R [W_1 - C(x_j, c_j)]}. \end{aligned}$$

Once I derive the indirect utility, I need to compute the expected value. I call f the expected return: $f = E[\pi|s_j, p] - pR$ that is normally distributed with mean μ_f and variance σ_f^2 .

Step 2 The outcome of the second step is the expected value of the indirect utility function. For agent $j \in J$, it is:

$$E[v(s_j, p; \Theta)] = -\frac{1}{\rho} \left[\left(\frac{1}{k_j} + \sigma_f^2 \right) k_j \right]^{-1/2} e^{\frac{-\frac{1}{2}\mu_f^2}{\left(\frac{1}{k_j} + \sigma_f^2 \right)} - \rho R[W_1 - C(x_j, c_j)]}.$$

The expected return $f = (E[\pi|s, p] - pR)$ is a normal random variable, given that is a linear function of two normally distributed random variables. In order to compute the expected value of the indirect utility function, I need to compute the mean, $\mu_f = E[E[\pi|s, p] - pR]$, and the variance, $\sigma_f^2 = \text{Var}[E[\pi|s, p] - pR]$, of the expected excess return. I start computing the expectation of the posterior belief:

$$E[E[\pi|s, p]] = E \left[\mu_\pi + \frac{1}{k} [x(s - E[s]) + \frac{I^2 \tau_\theta}{\rho^2 (1-t)^2} (\xi - E[\xi])] \right] = \mu_\pi,$$

and the expectation of the market price:

$$\begin{aligned} E[pR] &= E[a + b\pi - d\theta] \\ &= a + b\mu_\pi - d\mu_\theta. \end{aligned}$$

Therefore, the mean of the expected excess return is:

$$\begin{aligned} \mu_f &= \mu_\pi - a - b\mu_\pi + d\mu_\theta \\ &= (1-b)\mu_\pi - a + d\mu_\theta = \frac{\rho\mu_\theta}{K} = \mu_f, \end{aligned}$$

where $K = \tau_\pi + I + \tau_\xi$. To compute the variance, I need the variance of the posterior belief, the market price variance and the covariance of two terms:

$$\sigma_f^2 = \text{Var}[E[\pi|s, p]] + \text{Var}[pR] - 2\text{Cov}[E[\pi|s, p], pR],$$

where the variance of the posterior belief is:

$$\begin{aligned}
Var[E[\pi|s, p]] &= \frac{x^2}{k^2} \left(\frac{1}{\tau_\pi} + \frac{1}{x} \right) + \frac{I^4 \tau_\theta^2}{\rho^4 (1-t)^4 k^2} \left[\frac{1}{\tau_\pi} + \frac{\rho^2 (1-t)^2}{I^2 \tau_\theta} \right] + 2 \frac{x I^2 \tau_\theta}{\rho^2 (1-t)^2 k^2} \frac{1}{\tau_\pi} \\
&= \frac{1}{\tau_\pi k^2} \left[x^2 + x \tau_\pi + \frac{I^4 \tau_\theta^2}{\rho^4 (1-t)^4} + \frac{I^2 \tau_\theta \tau_\pi}{\rho^2 (1-t)^2} + 2x \frac{I^2 \tau_\theta}{\rho^2 (1-t)^2} \right] \\
&= \frac{1}{\tau_\pi k^2} \left[x + \frac{I^2 \tau_\theta}{\rho^2 (1-t)^2} \right] k \\
&= \frac{1}{\tau_\pi} - \frac{1}{k}.
\end{aligned}$$

The market price variance is:

$$Var[pR] = b^2 \frac{1}{\tau_\pi} + d^2 \frac{1}{\tau_\theta},$$

and the covariance between posterior beliefs and market price is:

$$Cov[E[\pi|s, p], pR] = b \frac{1}{\tau_\pi}.$$

Thus, I can rewrite σ_f^2 as:

$$\begin{aligned}
\sigma_f^2 &= \frac{1}{\tau_\pi} - \frac{1}{k} + b^2 \frac{1}{\tau_\pi} + d^2 \frac{1}{\tau_\theta} - 2b \frac{1}{\tau_\pi} \\
&= (1-b)^2 \frac{1}{\tau_\pi} + d^2 \frac{1}{\tau_\theta} - \frac{1}{k} \\
&= \frac{d^2}{\tau_\theta} + \frac{1}{K} \left(\frac{\tau_\pi}{K} - \frac{K}{k} \right).
\end{aligned}$$

Once I have the first two moments of the excess return, I can compute the expected value of the indirect utility:

$$\begin{aligned}
E[v(s, p; \Theta)] &= E \left[-\frac{1}{\rho} e^{-\frac{1}{2} k f^2 - \rho R[W_1 - C(x, c)]} \right] \\
&= -\frac{1}{\rho} e^{-\rho R[W_1 - C(x, c)]} E \left[e^{-\frac{1}{2} k \sigma_f^2 \left(\frac{f}{\sigma_f} \right)^2} \right].
\end{aligned}$$

Recall that $f \sim N(\mu_f, \sigma_f^2)$ and $\left(\frac{f}{\sigma_f} \right)^2 \sim \chi_1^2$. Therefore, I can use the moment generating function of a non central χ_1^2 that it is given by:

$$M(t, h, \lambda) = E[e^{tz}] = \frac{e^{\frac{\lambda t}{1-2t}}}{(1-2t)^{h/2}}.$$

In the model I have $h = 1$ and $t = -\frac{1}{2}k\sigma_f^2$. Moreover, $\lambda = (\frac{\mu_f}{\sigma_f})^2$ and $1 - 2t = k(\frac{1}{k} + \sigma_f^2)$. Thus:

$$\begin{aligned} E \left[e^{-\frac{1}{2}k\sigma_f^2(\frac{f}{\sigma_f})^2} \right] &= \left[k \left(\frac{1}{k} + \sigma_f^2 \right) \right]^{-1/2} e^{\frac{-\frac{1}{2}k\sigma_f^2(\frac{\mu_f}{\sigma_f})^2}{k(\frac{1}{k} + \sigma_f^2)}} \\ &= \left[k \left(\frac{1}{k} + \sigma_f^2 \right) \right]^{-1/2} e^{\frac{-\frac{1}{2}\mu_f^2}{\frac{1}{k} + \sigma_f^2}}, \end{aligned}$$

where the exponential term does not depend on x , given that $(\frac{1}{k} + \sigma_f^2)$ is constant and equal to $(1 - b)^2 \frac{1}{\tau_\pi} + d^2 \frac{1}{\tau_\theta}$. Now, rearranging all the terms, the expected value of the indirect utility for agent j is:

$$E[v(s_j, p; \Theta)] = -\frac{1}{\rho} \left[k_j \left(\frac{1}{k_j} + \sigma_f^2 \right) \right]^{-1/2} e^{\frac{-\frac{1}{2}\mu_f^2}{\left(\frac{1}{k_j} + \sigma_f^2 \right)} - \rho R[W_1 - C(x_j, c_j)]}.$$

Step 3 The outcome of the third step is the optimal information choice x^* and the threshold $\bar{c}(\Theta)$. I called γ the positive expression that is independent from the control variable x :

$$\gamma = \frac{1}{\rho} \left(\frac{1}{k} + \sigma_f^2 \right)^{-1/2} e^{\frac{-\frac{1}{2}\mu_f^2}{\frac{1}{k} + \sigma_f^2} - \rho R W_1},$$

and I rewrite the expected value of the indirect utility function as:

$$E\left[-\frac{1}{\rho}e^{-\rho W_2}\right] = -\gamma k^{-1/2} e^{\rho RC(x, c)} = -\gamma(\tau_\pi + x + \tau_\xi)^{-1/2} e^{\rho RC(x, c)}. \quad (8)$$

The objective function (8) is strictly concave and defined over a compact domain $[0, \bar{x}(c)]$ where $\bar{x}(c)$ solves $W_1 = C(\bar{x}, c)$. The concave maximum theorem guarantees the existence and the uniqueness of the solution. It could be an interior or a corner solution: $x^* = 0$ or $x^* = \bar{x}(c)$.

I specify conditions over parameter space in order to characterize the solution. First, I derive FOC and compute it at $x = 0$:

$$\left. \frac{E[v]}{\partial x} \right|_{x=0} = -\gamma(\tau_\pi + \tau_\xi)^{-1/2} \left[\rho RC_x(0, c) - \frac{1}{2(\tau_\pi + \tau_\xi)} \right].$$

When it is positive, agent has incentive to acquire information, $x^* > 0$. I call $\bar{c}(\Theta)$ the value of c , such that the agent is indifferent between being informed or remain uninformed. Formally,

$\bar{c}(\Theta)$ is implicitly given by:

$$[2\rho R(\tau_\pi + \tau_\xi)]^{-1} = C_x(0; \bar{c}). \quad (9)$$

Therefore, given strictly convexity of the cost function, $\forall c < \bar{c}$, $x^*(I; c, \Theta) > 0$.

For an interior solution, it is enough to show that there exists an $x \in [0, \bar{x}(c)]$ such that FOC is negative. Formally, I check when the following condition holds:

$$\frac{1}{2\rho R(\bar{x} + \tau_\pi + \tau_\xi)} < C_x(\bar{x}, c).$$

I identify a second threshold $\underline{c}(\Theta)$ that it is implicitly given by:

$$\frac{1}{2\rho R(\bar{x}(\underline{c}) + \tau_\pi)} = C_x(\bar{x}(\underline{c}), \underline{c}).$$

For all $\underline{c} < c < \bar{c}$, $x^*(I; c, \Theta)$ is an interior solution belonging to the set $[0, \bar{x}]$ and it is given by:

$$2\rho R C_x(x^*, c) (\tau_\pi + x^* + \tau_\xi) = 1.$$

I derive implicitly the amount of information $x^*(I; c, \Theta)$ that an agent optimally acquires. It depends on the financial literacy cost c and on the aggregate informativeness I .

The agent is indifferent between being informed or uninformed when I goes to infinity (fully revealing market price) or if $c = \bar{c}$. For any $c > \underline{c}$, the optimal information choice is:²¹

$$x^*(I; c, \Theta) = \begin{cases} 0 & \text{if } c > \bar{c} \\ \hat{x} \mid 2\rho R C_x(\hat{x}, c) (\tau_\pi + \hat{x} + \tau_\xi) = 1 & \text{if } c < \bar{c} \end{cases}.$$

□

In order to prove the existence and the uniqueness of a noisy rational expectation equilibrium, I need to check that the aggregation of the optimal information choices of the agents is equal to the amount of aggregate informativeness that agents conjecture in their maximization problems.

Lemma 3. *The optimal information choices of the agents are consistent with the aggregate informativeness.*

Proof. Let agents be distributed over J according a cdf $G(j)$. Let c_j be the financial literacy of agent $j \in J$. Let assume that c_j can take any values greater than $\underline{c}(\Theta)$.²² Let the compact set

²¹I want to avoid the case where agents prefer to spend their whole initial wealth in the information market and nothing in the asset market. This case is possible given the form of the CARA utility function where agents take into account both the mean and the variance of the final wealth.

²²In the paper, I assume that $G(j)$ is a discrete distribution with mass λ on $c = c_L$ and mass $(1 - \lambda)$ on $c = c_H$.

$\mathbf{Y} = [0, \frac{1}{2\rho RC_x(0, \underline{c})}] \subset \mathbb{R}^+$ the domain of the following mapping operator $F : \mathbf{Y} \rightarrow \mathbb{R}^+$:

$$F(y) = \int_j x^*(c_j, y; \Theta) dG(j),$$

where y is an element of $\mathbf{Y}$ and $x^*(c_j, y; \Theta)$ is given by:

$$x^*(c_j, I, \Theta) = \begin{cases} 0 & \text{if } c_j > \bar{c}, \\ \hat{x} \mid 2\rho RC_x(\hat{x}, c_j) \left(\tau_\pi + \hat{x} + \frac{I^2 \tau_\theta}{\rho^2(1-t)^2} \right) = 1 & \text{if } c_j < \bar{c}. \end{cases}$$

Continuity of $F(y)$ is guaranteed by the assumption that C_x is continuous. I need to prove that F maps into itself to apply fixed-point Brouwer's theorem ($F(y) = y$). For all $c_j > \underline{c}$ and $y \in \mathbf{Y}$, $x^*(c_j, y; \Theta) \geq 0$. Moreover, given strictly convexity of the cost function, $C_x(0, \underline{c}) \leq C_x(x^*, c_j)$. This implies that:

$$x^* = \frac{1}{2\rho C_x(x^*, c)} - \left(\tau_\pi + \frac{y^2 \tau_\theta}{\rho^2(1-t)^2} \right) \leq \frac{1}{2\rho C_x(x^*, c)} \leq \frac{1}{2\rho C_x(0, \underline{c})}.$$

For all $c_j > \underline{c}$ and $y \in \mathbf{Y}$,

$$0 \leq x^*(c_j, y; \Theta) = \max\{0, x^*\} \leq \frac{1}{2\rho C_x(0, \underline{c})}.$$

Aggregating over j using the cdf $G(j)$ implies that:

$$0 \leq F(y) = \int_j x^*(c_j, y; \Theta) dG(j) \leq \frac{1}{2\rho C_x(0, \underline{c})}.$$

Thus, $F(y)$ maps into itself and I proved the existence of the equilibrium.

To prove uniqueness of the equilibrium I follow Peress (2004). $N \equiv \{j : c_j \in [\underline{c}, \bar{c}]\}$ and $x^*(c_j, y; \Theta) > 0, \forall j \in N$. Therefore, $\int_{j \in N} x^*(c_j, y; \Theta) dG(j) = \int_{\underline{c}}^{\bar{c}} x^*(c_j, y; \Theta) dG(j)$.

Let $f(y) = y - \int_{\underline{c}}^{\bar{c}} x^*(c_j, y; \Theta) dG(j)$. The equilibrium value y^* is a root of $f(y)$ and, to be uniquely determined, I need monotonicity of $f(y)$. Total differentiation of $f(y)$ yields:

$$f'(y) = 1 - \left(\int_{\underline{c}}^{\bar{c}} \frac{\partial x^*(c_j, y; \Theta)}{\partial y} dG(j) + x^*(\bar{c}, y; \Theta) \frac{\partial \bar{c}}{\partial y} - x^*(\underline{c}, y; \Theta) \frac{\partial \underline{c}}{\partial y} \right).$$

The first term in brackets is the integral of the partial derivative of the optimal information choice with respect to the aggregate informativeness. The second term in brackets is zero given the optimal choice x^* is zero for agents with $c_j = \bar{c}$. The last term is also zero given that

I set $\underline{c}$ independent with respect to y . Differentiating FOC I have:

$$C''_{xx} \frac{\partial x_j^*}{\partial y} \left[x_j^* + \tau_\pi + \frac{y^2}{\rho^2(1-t)^2} \tau_\theta \right] + C'_x \left[\frac{\partial x_j^*}{\partial y} + 2 \frac{y}{\rho^2(1-t)^2} \tau_\theta \right] = 0$$

$$\frac{\partial x_j^*}{\partial y} \left\{ C''_{xx} \left[x_j + \tau_\pi + \frac{y^2}{\rho^2(1-t)^2} \tau_\theta \right] + C'_x \right\} + 2C'_x \frac{y}{\rho^2(1-t)^2} \tau_\theta = 0$$

$$\frac{\partial x_j^*}{\partial y} = - \frac{2C'_x \frac{y}{\rho^2(1-t)^2} \tau_\theta}{C''_{xx} \left[x_j + \tau_\pi + \frac{y^2}{\rho^2(1-t)^2} \tau_\theta \right] + C'_x} \leq 0.$$

As long as the assumptions about the shape of the cost function hold and given $y \in \mathbf{Y} \subset \mathbb{R}^+$, I can conclude that $f'(y)$ is always non negative and $f(y)$ is monotonic. Therefore, there exists a unique value of y such that the information market is in equilibrium. $\square$

Appendix B - Tables and Graphs

If it is not differently specified in the text, the model's parameters are the following: prior mean μ_π and prior precision τ_π are both equal to one. The noisy asset supply has mean zero and variance one. The risk free asset is zero return ($R = 1$) and the risk aversion coefficient (ρ) is one. Initial wealth (W_1) is one. Literate agents are a fourth of the total ($\lambda = 0.25$) and their cost of acquiring information is $c_L = 0.01$, while the information acquisition cost for the illiterates is $c_H = 0.03$. The tax rate is set to zero. In short notation, I have that Θ is given by $\{R = 1, W_1 = 1, \rho = 1, \mu_\pi = 1, \mu_\theta = 0, \tau_\pi = 1, \tau_\theta = 1, \lambda = 0.25, c_L = 0.01, c_H = 0.03, t = 0\}$.

5.1 Information choice

Reducing the information acquisition cost of the illiterates increases the acquired information of the illiterates and decreases that of the literates. Deriving (1) with respect to c_H holds:

$$\left(C_{xx} \frac{dx_H}{dc_H} + C_{xc} \right) k_H + C_x \left[\frac{dx_H}{dc_H} + \frac{2I\tau_\theta}{\rho^2(1-t)^2} \frac{dI}{dc_H} \right] = 0.$$

Then, substituting $\frac{dI}{dc_H} = \lambda \frac{dx_L}{dc_H} + (1-\lambda) \frac{dx_H}{dc_H}$, I get:

$$\frac{dx_H}{dc_H} = - \frac{C_{xc}k_H + \lambda C_x \frac{2I\tau_\theta}{\rho^2(1-t)^2}}{C_{xx}k_H + C_x \left[1 + (1-\lambda) \frac{2I\tau_\theta}{\rho^2(1-t)^2} \right]} \frac{dx_L}{dc_H}.$$

Similar result can be derived also for:

$$\frac{dx_L}{dc_H} = - \frac{(1-\lambda)C_x \frac{2I\tau_\theta}{\rho^2(1-t)^2}}{C_{xx}k_L + C_x \left[1 + \lambda \frac{2I\tau_\theta}{\rho^2(1-t)^2} \right]} \frac{dx_H}{dc_H}.$$

Substituting the last equation in the previous one, I derive the reduced form of $\frac{dx_H}{dc_H}$:

$$\frac{dx_H}{dc_H} = - \frac{\left\{ C_{xx}k_L + C_x \left[1 + \lambda \frac{2I\tau_\theta}{\rho^2(1-t)^2} \right] \right\} C_{xc}k_H}{C_{xx}^2k_Lk_H + C_{xx}C_x \left[k_L + k_H + \frac{2I\tau_\theta}{\rho^2(1-t)^2}K \right] + C_x^2 \left[1 + \frac{2I\tau_\theta}{\rho^2(1-t)^2} \right]^2} < 0$$

and, similarly, the reduced form of $\frac{dx_H}{dc_H}$:

$$\frac{dx_L}{dc_H} = \frac{\left[(1 - \lambda) C_x \frac{2I\tau_\theta}{\rho^2(1-t)^2} \right] C_{xc} k_H}{C_{xx}^2 k_L k_H + C_{xx} C_x \left[k_L + k_H + \frac{2I\tau_\theta}{\rho^2(1-t)^2} K \right] + C_x^2 \left[1 + \frac{2I\tau_\theta}{\rho^2(1-t)^2} \right]^2} > 0.$$

The impact of c_h on the aggregate informativeness is always negative as long as the assumptions on the cost function $C(x, c)$ hold:

$$\frac{dI}{dc_H} < 0 \Leftrightarrow \frac{dx_L/dc_H}{dx_H/dc_H} < -\frac{1 - \lambda}{\lambda}.$$

The previous result holds for all $c_H < \bar{c}(\Theta)$. While for all $c_H > \bar{c}(\Theta)$, $\frac{dx_H}{dc_H} = \frac{dx_L}{dc_H} = \frac{dI}{dc_H} = 0$.

Table 1: Optimal information choices and the endogenous threshold. $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 1$, $\lambda = 0.25$, $c_L = 0.01$, $t = 0$

c_H	x_H	I	x_L	$c_H - \bar{c}$
0.5000	0.0000	0.9599	3.8396	0.2398
0.3000	0.0000	0.9599	3.8396	0.0398
0.1000	0.3868	1.1967	3.6264	-0.1056
0.0500	0.7936	1.4394	3.3768	-0.1128
0.0300	1.1808	1.6664	3.1231	-0.1024
0.0100	2.3385	2.3385	2.3385	-0.0673

5.2 Market Price Variance

Market price function is $pR = a + b\pi - d\theta$ and the market price variance is:

$$\sigma_{pR}^2 = b^2 \frac{1}{\tau_\pi} + d^2 \frac{1}{\tau_\theta}.$$

Deriving with respect to the aggregate informativeness holds:

$$\frac{d\sigma_{pR}^2}{dI} = \frac{2b}{\tau_\pi} \frac{db}{dI} + \frac{2d}{\tau_\theta} \frac{dd}{dI},$$

with

$$\frac{db}{dI} = \frac{dK}{dI} \frac{(1-b)}{K}, \quad \frac{dd}{dI} = \frac{1}{K} \left(\frac{\tau_\theta}{\rho(1-t)} - \frac{dK}{dI} d \right), \quad \frac{dK}{dI} = 1 + 2 \frac{I\tau_\theta}{\rho^2(1-t)^2}.$$

Substituting back into the previous formula, I get:

$$\frac{d\sigma_{pR}^2}{dI} = 2 \left[\left(\frac{b}{\tau_\pi} \frac{dK}{dI} \frac{1}{K} + \frac{d}{\rho(1-t)} \frac{1}{K} \right) - \frac{dK}{dI} \frac{1}{K} \sigma_{pR}^2 \right]. \quad (10)$$

The derivative of market price variance with respect to aggregate informativeness is negative when market price variance is greater than the sum of prior variance multiplied by the sensitivity of market price to asset payoff shocks and the variance of noisy supply multiplied by the sensitivity of market price to noisy supply shocks and the inverse of the marginal impact of aggregate informativeness on the precision of the average agent's variance:

$$\frac{d\sigma_{pR}^2}{dI} < 0 \iff \sigma_{pR}^2 > \frac{b}{\tau_\pi} + \frac{d}{\rho(1-t)} \frac{1}{dK/dI}.$$

Table 2: Market price variance. $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 1$, $\lambda = 0.25$, $c_L = 0.01$, $c_H = 0.03$, $t = 0$

(a)		(b)	
τ_π	σ_{pR}^2	τ_θ	σ_{pR}^2
0.3000	3.2434	0.3000	1.0493
0.5000	1.9089	0.5000	0.9622
1.0000	0.9063	1.0000	0.9063
2.0000	0.4028	2.0000	0.8887

Table 3: Market price variance. $R = 1$, $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $c_L = 0.01$, $c_H = 0.03$, $t = 0$.

ρ	$\tau_\pi = \tau_\theta = .3$	$\tau_\pi = \tau_\theta = .5$	$\tau_\pi = \tau_\theta = 1$	$\tau_\pi = \tau_\theta = 2$
0.1000	3.3180	1.9866	0.9887	0.4904
0.2500	3.2887	1.9599	0.9658	0.4710
0.5000	3.2588	1.9234	0.9284	0.4369
1.0000	3.4164	1.9729	0.9063	0.3900
1.5000	4.0729	2.3062	1.0085	0.3937
2.0000	5.4640	3.0453	1.2767	0.4561

Table 4: Market price variance. $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $c_L = 0.01$, $t = 0$.

c_H	$\tau_\pi = \tau_\theta = .3$	$\tau_\pi = \tau_\theta = .5$	$\tau_\pi = \tau_\theta = 1$	$\tau_\pi = \tau_\theta = 2$
0.0100	3.3308	1.9533	0.9295	0.4268
0.0300	3.4164	1.9729	0.9063	0.3900
0.0500	3.4879	1.9963	0.8982	0.3721
0.1000	3.6221	2.0451	0.8912	0.3485
0.3000	3.9098	2.1569	0.8890	0.3359

Figure 2: The market price variance (continuous line) and the relative threshold (dashed line). The inequality in the financial information costs is computed keeping fixed c_L and letting c_H increase. The four panels refer to different scenarios in the fundamentals' uncertainty: from high uncertainty ($\tau_\pi = \tau_\theta = 0.3$) to low uncertainty ($\tau_\pi = \tau_\theta = 2$). $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $c_L = 0.01$, $t = 0$.

Table 5: Market price variance. $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $c_L = 0.01$, $c_H = 0.03$, $t = 0$.

λ	$\tau_\pi = \tau_\theta = .3$	$\tau_\pi = \tau_\theta = .5$	$\tau_\pi = \tau_\theta = 1$	$\tau_\pi = \tau_\theta = 2$
0.0000	3.4970	2.0000	0.8970	0.3674
0.1000	3.4586	1.9866	0.9007	0.3774
0.2500	3.4164	1.9729	0.9063	0.3900
0.5000	3.3720	1.9605	0.9152	0.4062
0.7500	3.3463	1.9552	0.9229	0.4180
1.0000	3.3308	1.9533	0.9295	0.4268

Appendix C

I run Montecarlo simulations to compute the tax rate that satisfied policy makers' budget constraint: the expected fiscal revenue ($EF R$) is equal to the policy cost. The former is given by:

$$EF R = E[t \int_j \max\{\alpha_j^*(W_1 - C_j^*) \frac{\pi - pR}{p}, 0\} dG(j)].$$

The condition according to which individual capital gain is greater than zero is $\alpha_j^* \frac{(\pi - pR)}{p} > 0$. Thus, substituting α_j^* with (5) and simplifying, I derive the following:

$$EF R = E[\frac{t}{\rho(1-t)}(\pi - pR) \int_j k_j^*(E[\pi|s_j, p] - pR) dG(j)] \quad \text{if} \quad \alpha_j^* \frac{(\pi - pR)}{p} > 0.$$

In order to write the code for the Montecarlo simulation, I distinguish between informed and uniformed agents. To simplify the notation, I drop the subscript about the type.²³

For informed agents, the condition of positive capital gains can be rewritten in terms of private signal: conditional to the realization of the fundamentals, the private signal leads to capital gains or capital losses. I derive a threshold according to which it is possible to distinguish traders who will have positive capital gains. The condition $\alpha_j^* \frac{(\pi - pR)}{p} > 0$ holds when:

$$\begin{aligned} s &> \bar{s} \quad \text{if} \quad \frac{p(\pi - pR)}{p} > 0 \\ s &< \bar{s} \quad \text{if} \quad \frac{p(\pi - pR)}{p} < 0 \end{aligned},$$

where $\bar{s} = (pRk - \mu_\pi \tau_\pi - \xi \tau_\xi)/x$. I can write:

$$\int_j k_j^*(E[\pi|s_j, p] - pR) dG(j) = \begin{cases} k \int_{s>\bar{s}} E[\pi|s_j, p] - pR dF(s) & \text{if} \quad \frac{p(\pi - pR)}{p} > 0 \\ k \int_{s<\bar{s}} E[\pi|s_j, p] - pR dF(s) & \text{if} \quad \frac{p(\pi - pR)}{p} < 0 \end{cases}.$$

Given the properties of truncated normal distribution, I can derive a formula that depends only on fundamentals π and θ :

$$k \int_{s>\bar{s}} E[\pi|s_j, p] - pR dF(s) = x \left[\pi + \sqrt{1/x} \eta((\bar{s} - \pi)\sqrt{x}) \right] + (\mu_\pi \tau_\pi + \xi \tau_\xi - pRk)(1 - F(\bar{s})),$$

where $\eta((\bar{s} - \pi)\sqrt{x}) = \frac{\phi((\bar{s} - \pi)\sqrt{x})}{1 - \Phi((\bar{s} - \pi)\sqrt{x})}$. Similarly, for the other case.

For uninformed agents ($x = 0$), the positive capital gains condition can be rewritten in

²³The Matlab code is available upon request.

terms of public signal. Thus, the condition $\alpha_j^* \frac{(\pi - pR)}{p} > 0$ holds when:

$$\begin{aligned} \xi &> \bar{\xi} & \text{if } \frac{p(\pi - pR)}{p} > 0 \\ \xi &< \bar{\xi} & \text{if } \frac{p(\pi - pR)}{p} < 0 \end{aligned} ,$$

where $\bar{\xi} = (pRk - \mu_\pi \tau_\pi) / \tau_\xi$. I can write:

$$\int_j k_j^* (E[\pi|p] - pR) dG(j) = \begin{cases} k(E[\pi|p] - pR) & \text{if } \frac{p(\pi - pR)}{p} > 0 \quad \vee \quad \xi > \bar{\xi} \\ k(E[\pi|p] - pR) & \text{if } \frac{p(\pi - pR)}{p} < 0 \quad \vee \quad \xi < \bar{\xi} \\ 0 & \text{otherwise} \end{cases} .$$

The budget constraint for the policy makers when both types are informed and $\frac{p(\pi - pR)}{p} > 0$ is the following:

$$\begin{aligned} &\frac{t}{\rho(1-t)} E \left[(\pi - pR) \left\{ \lambda \left[x_L \left(\pi + \sqrt{\frac{1}{x_L}} \eta((\bar{s}_L - \pi) \sqrt{x_L}) \right) + (\mu_\pi \tau_\pi + \tau_\xi \xi - pRk_L)(1 - F_L(\bar{s}_L)) \right] \right. \right. \\ &\left. \left. + (1 - \lambda) \left[x_H \left(\pi + \sqrt{\frac{1}{x_H}} \eta((\bar{s}_H - \pi) \sqrt{x_H}) \right) + (\mu_\pi \tau_\pi + \tau_\xi \xi - pRk_h)(1 - F_H(\bar{s}_H)) \right] \right\} \right] = \lambda. \end{aligned}$$

Similarly for the other cases.

Table 6: Expected utility of the illiterate agents: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 1$, $c_L = 0.01$, $t = 0$.

$c_H - c_L$	$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.25$
0.0000	-0.3387	-0.3387	-0.3387
0.0200	-0.3270	-0.3334	-0.3412
0.0400	-0.3201	-0.3314	-0.3438
0.0900	-0.3081	-0.3286	-0.3476
0.2900	-0.2797	-0.3187	-0.3475

Table 7: Tax rate: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 1$, $\lambda = 0.25$, $c_H = 0.1$.

c_L	$\tau_\pi = \tau_\theta = .3$	$\tau_\pi = \tau_\theta = .5$	$\tau_\pi = \tau_\theta = 1$	$\tau_\pi = \tau_\theta = 2$
0.0100	0.0216	0.0244	0.0295	0.0391
0.0300	0.0131	0.0153	0.0190	0.0246
0.0500	0.0085	0.0101	0.0128	0.0173
0.1000	0.0000	0.0000	0.0000	0.0000

Table 8: Tax rate: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 1$, $c_L = 0.01$, $c_H = 0.1$.

λ	$\tau_\pi = \tau_\theta = .3$	$\tau_\pi = \tau_\theta = .5$	$\tau_\pi = \tau_\theta = 1$	$\tau_\pi = \tau_\theta = 2$
0.0000	0.0000	0.0000	0.0000	0.0000
0.1000	0.0069	0.0080	0.0100	0.0131
0.2000	0.0162	0.0186	0.0223	0.0292
0.2500	0.0216	0.0244	0.0295	0.0391
0.3000	0.0274	0.0311	0.0369	0.0497
0.4000	0.0404	0.0462	0.0545	0.0722
0.5000	0.0551	0.0622	0.0748	0.0910

Figure 3: Expected Utility: The inequality in financial information costs is computed keeping fixed $c_H = 0.1$ and letting c_L decrease. The three panels refer to different policy sizes. It increases from the left ($\lambda = 0.1$) to the right ($\lambda = 0.5$). $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 1$.

Figure 4: Expected Utility: The three panels refer to different degrees of quality of the financial education program: quality decreases from the left ($c_L = 0.01$) to the right ($c_L = 0.05$). $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 1$, $c_H = 0.1$.

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