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normalisation in a
multidimensional analysis of
Social Inclusion**

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Keywords

CES, normalisation, aggregation, weighting, experts, multidimensionality, Social Inclusion

JEL Codes

C43, C83, D63, I32

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The weighting role of normalisation in a multidimensional analysis of Social Inclusion¹

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November 2015

Abstract

In the context of the multidimensional measurement of complex phenomena, the major focus of the recent literature has been on the choice of the dimensions' weights and the shape of the aggregation function, while few studies have concentrated on how normalisation influences the results. With the aim of building a measure of Social Inclusion for European regions between 2004 and 2012, we adopt a CES aggregation framework and compare two alternative normalisation strategies: a *data-driven* min-max function, where the parameters depends solely on the available data, and an *expert-based* function where parameters are elicited through a survey at the University of Venice Ca'Foscari. Regardless of the adopted strategy, we show that normalisation plays a crucial part in defining variables' weighting and trade-offs. The data-driven strategy produces trade-offs that are hard to interpret in economic terms and debatable from a social desirability perspective, thus generating an aggregate measure with a "positive" interpretation. Moreover, it softens the aftermaths of the recent economic crisis on Social Inclusion, by putting a consistent weight on the longevity variable. Conversely, the expert-based normalisation has strikingly different parameters and allows for a normative interpretation of the resulting index. Furthermore, it emphasizes the worsening trends in long-term unemployment and the relevance of early school leaving in the Social Inclusion measure. As a result, numerous rank-reversals occur between regions when switching the normalisation methods.

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1. INTRODUCTION

Although there is quite a consensus on the need for broadening the scope of the analysis of Well-being beyond the monetary dimension (see, e.g., the influential report by [Stiglitz et al. \(2010\)](#)), there is not equal agreement on how such an ambitious task should be operationalized. It is well known that subjectivity and arbitrariness exists with respect to the choice of the dimensions to be included in the composite index, the normalisation of the variables, the choice of the aggregation function and of its parameters (see, e.g., [Ravallion \(2012a\)](#) [Decancq and Lugo \(2013\)](#) [Martinetti and von Jacobi \(2012\)](#)).² The socio-economic literature highlighted that no unanimous method exists to perform such choices, pointing out numerous theoretical issues ([Stiglitz et al. 2010](#); [Ravallion 2011, 2012a](#); [Klugman et al. 2011](#); [Maggino and Nuvolati 2012](#); [Decancq and Lugo 2013](#)), testing empirical robustness ([Kasparian and Rolland 2012](#); [Lefebvre et al. 2010](#); [Saisana et al. 2005](#); [Ravallion 2012b](#)). Yet, although there may be no “absolute cure” for multidimensional evaluations, a good practice could consist in enhancing methodological transparency ([Sen and Anand 1997](#)).

While the major focus of the recent literature has been devoted to the choice of the dimensions’ weights, few studies have concentrated on the role played by normalisation in influencing the final results ([Lefebvre et al. 2010](#); [Pinar et al. 2014](#)). Our contribution highlights that, in fact, normalisation is a crucial stage where an “early” weighting takes place, which can strongly affect the overall results of the multidimensional analysis. Therefore, the unavoidable arbitrariness regarding the choice of the normalisation function should be made as transparent as possible.

In this paper we will build a composite measure of Social Inclusion for 58 European administrative regions (in Belgium, Germany, Italy and Spain) from 2004 to 2012, using data from EUROSTAT. The aggregation framework is a CES function (constant elasticity of substitution), and the selection of variables follows the relevant literature on this topic (stemming from [Atkinson et al. \(2002\)](#)).³

In the first part of the analysis, we adopt a baseline linear aggregation model where the normalised components have equal weights and we look at what happens to the aggregate measure of Social Inclusion when only the normalisation function changes. In particular, we start by applying a widely used normalisation strategy, the *data-driven* min-max function, whose parameters depend only on the available data. These data-driven function generates implicit trade-offs (between the index’ components) and shadow prices with weak economic justification. We therefore propose a novel strategy, an *expert-based* min-max function, whose parameters are grounded on the responses to a survey conducted on a population of 150 professors of Economics or Management at the Ca’ Foscari University of Venice.

Our results indicate that changing the normalisation function substantially affects the relative relevance of each component of the aggregate measure. Therefore, significant differences emerge in the levels and trends of Social Inclusion at the regional level in Europe, leading to different policy implications, which are only due to the normalisation stage.

² The act of synthesizing a composite latent phenomenon encompasses methodological issues that have economic, philosophical (as well as psychological) and political connotations. Indeed, these issues arise from a fundamental mismatch between the kind of multiplicity inherent in the latent concept and the multiplicity characterizing the forged measure (the result of the researcher’s work). In a sense, the latent multidimensional concept (e.g., Well-being or Social Inclusion) is an un-synthesized multiplicity, in that it is composite by nature and perceived as a whole by the human sensibility. Since the phenomenon is unmeasurable per se, the researcher is forced to separate it, operationally, in numerous measurable components, in order to aggregate them back to provide a proxy of the latent phenomenon. In other words, building a synthetic index of Well-being requires that the indeterminate nature of multiplicity is made determinate through a specification of its contents, and of their relationship.

³ Through this analysis, we do not aim at providing efficiency index for the Welfare States, which would require a much more structured set of information. We, rather, limit ourselves at evaluations of performances, as suggested by [Pestieau \(2009\)](#) and [Lefebvre et al. \(2010\)](#).

The second part of the paper shows that, when the linearity assumption is relaxed (e.g., a geometric or a harmonic aggregation is adopted), a change in the normalisation strategy leads to even more striking results in terms of Social Inclusion.

The remaining of the paper is organized as follows. Section 2 briefly describes the concept of Social Inclusion and the data. Section 3 sets a standard framework for multidimensional aggregation and details the baseline model. Section 4 implements the baseline-model with a data-driven normalisation strategy, while Section 5 describes a novel proposal of an expert-based normalisation function. Section 6 discusses the results of applying the aforementioned normalisation functions in the baseline model. Section 7 performs a sensitivity analysis by relaxing the linearity assumption in the CES model. Section 8 concludes.

2. SOCIAL INCLUSION, DEFINITION AND SAMPLE SELECTION

Social Inclusion (as its corresponding opposite, Social Exclusion)⁴ is one of the five priorities selected by the European Commission in the context of the Europe 2020 Strategy. A definition of Exclusion was already drawn in December 1992 by the Commission of the European Communities ([European Communities Commission 1992](#)): “Social Exclusion is a multidimensional phenomenon stemming from inadequacies or weaknesses in the services offered and policies pursued in these various policy areas. Such insufficiencies and weaknesses often combine to affect both people and regions via cumulative and interdependent processes of such a nature that it would be futile to try to combat exclusion by tackling only one of its dimensions. More clearly than the concept of poverty, (...) it states out the multidimensional nature of the mechanisms whereby individuals and groups are excluded from taking part in the social exchanges, from the component practices and rights of social integration and of identity”.

The Laeken European Council in 2001 has developed a set of unanimously agreed indicators that could capture the multifaceted aspects and outcomes of Social Inclusion, thus providing reliable and comparable data to monitor the social and economic conditions of European citizens ([European Council 2001](#)), through the Open Method of Coordination. In particular, four basic dimensions have been identified: the level and distribution of income, the condition of labour market, education and health. For each of them, a set of primary indicators were adopted: income (Poverty rate (after social transfers), Persistent risk-of-poverty rate, Relative median at risk-of-poverty gap, Inequality of income distribution); labour market (Long-term unemployment, Regional cohesion, Persons living in jobless households); education (early school leavers); health (life expectancy at birth, Self-defined health status by income level).⁵

The target of this paper is to build an aggregate index of Social Exclusion at administrative-regional level in Europe. We choose administrative regions as the main territorial unit of this analysis, with the aim of capturing higher variability than it can be inferred from aggregate national data. Data-availability is often mentioned as a serious constraint for analyses which focus on a wide set of countries for a long time-period ([Lefebvre et al. 2010](#); [Martinetti and von Jacobi 2012](#)). In the context of Social Exclusion at administrative regional level, we are able to gather data for four out of the 10 aforementioned indicators, one per dimension: poverty-rate, long-term unemployment, early school-leavers and life expectancy at birth. Our source is the on-line Eurostat Regional Database 2014. We are able to cover a period of nine years, from 2004 to 2012, and 58 administrative regions in four countries (Belgium, Germany, Italy and Spain). Greece, Czech Republic

⁴ The concept of Social Inclusion/exclusion should not be confused with the variable 'at risk of poverty or social exclusion' in the Eurostat database, which defines an individual as at risk of poverty or social exclusion when at least one of the following conditions hold: (a) equivalent household income below 60% of national median; (b) households with at least 4 of the following 9 issues: i) impossibility to bear unexpected expenses, ii) cannot afford a week holiday, iii) issues with the mortgage, rent, bills; iv) cannot afford a proper meal every two days; v) not able to adequately heat the house; vi) not able to afford a washing machine vii) a color TV viii) a phone ix) an automobile; (c) living in families whose members aged 18-59 work less than a fifth of their time.

⁵ We refer to ([Atkinson et al. \(2002\)](#); [Atkinson et al. \(2004\)](#)), as well as to [European Commission \(2009\)](#) [European Commission \(2010\)](#) for further details on the rationale of Social Inclusion indicators and on the issues related to their measurement.

and Norway were not included in the sample since their data are made available for statistical-regions, but not for administrative regions.

As argued in [Lefebvre et al. \(2010\)](#), “these indicators cover the most relevant concerns of a modern welfare state, also reflecting aspects that people who want to enlarge the concept of GDP to better measure social welfare generally take into account”. The latter referenced paper, as do [Atkinson et al. \(2004\)](#), discusses the limitations of these data and the necessary simplifying assumptions that have to be done when translating a complex multidimensional phenomenon like Social Exclusion in empirical terms. Table 2-1 provides a brief definition for our four variables:

Table 2-1, Variable definitions

variable	definition
Poverty rate	Share of individuals living in households with an income below 60% national median equivalised disposable income.
Long-term unemployment rate	Total long-term unemployed population (≥ 12 months; ILO definition) as proportion of total active population.
Early school-leavers	Share of total population of 18-24-year olds having achieved ISCED level 2 or less and not attending education or training.
Life expectancy at birth	Number of years a person may be expected to live, starting at age 0.

The following table and figure report descriptive statistics on the four indicators for Belgium, Germany, Italy and Spain for the period 2004-2012 (regional data are aggregated at the country level with population weighting).

Table 2-2, Descriptive statistics for four indicators of Social Inclusion, years 2004-2012

		BELGIUM	GERMANY	ITALY	SPAIN
Longevity	Min	77.5 Wallonie 2004	77.8 Sachsen-A. 2004	79.5 Campania 2005	78.3 Ceuta 2004
	National mean	79.8	80.2	82	81.4
	Max	81.6 Vlaanderen 2011	82.2 Baden-W. 2012	83.8 Marche 2010	84.2 Madrid 2012
Early school leaving	Min	8.6% Vlaanderen 2008	5.4% Thüringen 2009	10.2% Molise 2012	11.5% P. Vasco 2012
	National mean	12.1%	12.3%	18.4%	29.3%
	Max	20% Brussels 2007	21% Saarland 2006	32.7% Sardegna 2005	54.2% Ceuta 2005
Long-term Unemployment	Min	1.4% Vlaanderen 2008	1.1% Bayern 2012	0.5% TAA 2004	0.9% Aragón 2008
	National mean	3.8%	4.1%	3.6%	4.8%
	Max	9.9% Brussels 2006	13.8 Sachsen-A. 2004	12.2% Campania 2012	18.2% Ceuta 2012
At-risk-of-poverty rate	Min	9.8% Vlaanderen 2011	10% Baden-W. 2007	5.2% VDA 2006	8.1% P. Vasco 2009
	National mean	14.8%	14.5%	16.9%	20.8%
	Max	33.7% Brussels 2011	24.3% Mecklem. 2007	44.3% Sicilia 2011	48.9% Ceuta 2008

Source: Eurostat Regional Database 2014. TAA: Trentino – Alto Adige; VDA: Valle d'Aosta

As for variables' correlations, Table 3 shows a relatively high and positive correlation between unemployment and poverty (0.63). Longevity is negatively correlated with both unemployment and poverty, while the latter is positively correlated with school-dropouts (0.50).

Table 3, correlation between four dimensions of Social Inclusion

	Longevity	Early school leaving	Long-term unemployment	At-risk-of-poverty rate
Longevity	1			
Early school leaving	0.032	1		
Long-term unemployment	-0.298	0.143	1	
At-risk-of-poverty rate	-0.245	0.505	0.631	1

Data for administrative regions of Belgium, Germany, Italy and Spain, years 2004-2012. Eurostat Regional Database 2014.

3. THEORETICAL FRAMEWORK

Let us consider m dimensions (hereinafter also *variables, attributes, components*) of Social Inclusion, observed for n territorial units (in our case, regions). For a generic region i we can therefore build the vector $\mathbf{x}^i = (x_1^i, \dots, x_m^i)$, while $\mathbf{X} \in \mathbb{R}^{n \times m}$ is the distribution matrix of m attributes for n regions. To retrieve an aggregated measure for region i , we consider the function F defined as:

$$(1) \quad F^i(v(\mathbf{x}^i)) = \left[w_1 v_1(x_1^i)^\beta + \dots + w_m v_m(x_m^i)^\beta \right]^{1/\beta}$$

which can be referred to as a CES (constant elasticity of substitution) function, or a generalized mean of order β . Its arguments are the elements $v_1, \dots, v_m$ which are transformations of the original variables $x_1, \dots, x_m$ (defined hereafter). The function F is non-decreasing, separable, weakly scale-invariant and homogenous of degree-one in its arguments v ; we refer to [Blackorby and Donaldson \(1982\)](#) and ([Decancq and Lugo \(2009, 2008\)](#)) for an analytic characterization of these properties.

The parameters $w_1, \dots, w_m$, the weights of the normalised dimensions v , are non-negative and sum to one.

Provided that a choice of the m dimension has been performed, the main methodological task is now the selection of the set of functions $v_1, \dots, v_m$, of parameters $w_1, \dots, w_m$, as well as of β .

3.1 THE MIN-MAX NORMALISATION FUNCTION

Raw variables are usually observed and measured with different measurement units. The component $v_j(x_j)$ is a weakly monotonic and continuous normalisation function that maps the values of the j -th variable x_j on the closed interval $[0, 100]$, i.e., $v_j(x_j) \in [0, 100]$. Moreover, attributes might be, alternatively, positively or negatively related to the latent phenomenon. Hence, in order to ensure comparability and monotonicity of any aggregation function, each individual indicator must be normalised such that better performances in the j -th dimension correspond to non-lower values of $v_j(x_j)$ and therefore of the aggregated value F (see [Giovannini et al. \(2008\)](#) for a comprehensive discussion). The normalisation function thus ensures that F is bounded between 0 and 100 when the weights w sum to one. The vector $\mathbf{v}^i = (v_1^i, \dots, v_m^i)$ represents the normalised values for the m dimensions observed in region i .

Our preferred specification for $v(x)$ is the *min-max normalisation function*, widely used in the literature of multidimensional measures ([Cherchye et al. \(2007\)](#); [Martinetti and von Jacobi \(2012\)](#); [Murias et al. \(2012\)](#); [Lefebvre et al. \(2010\)](#); [Silva and Ferreira-Lopes \(2013\)](#); [Mazziotta and Pareto \(2015\)](#); [Pinar et al. 2014](#)), in the Human Development Index ([Anand and Sen 1994](#)) and in the OECD Better Life ([Boarini and D'Ercole 2013](#)).

For each attribute x observed in region i at a time t (we drop the previously used attribute-specific j index to ease readability), the corresponding normalised value is defined as:

$$(2) \quad \begin{aligned} \nu_+^{i,t}(x_+^{i,t}) &= 100 * \frac{x_+^{i,t} - b_+ \min(x_+)}{b_+ \max(x_+) - b_+ \min(x_+)} & \nu_-^{i,t}(x_-^{i,t}) &= 100 * \frac{b_- \max(x_-) - x_-^{i,t}}{b_- \max(x_-) - b_- \min(x_-)} \\ \nu_+^{i,t}(x_+^{i,t}) &= 0 \text{ if } x_+^{i,t} \leq b_+ \min(x_+) & \nu_-^{i,t}(x_-^{i,t}) &= 0 \text{ if } x_-^{i,t} \geq b_- \max(x_-) \\ \nu_+^{i,t}(x_+^{i,t}) &= 100 \text{ if } x_+^{i,t} \geq b_+ \max(x_+) & \nu_-^{i,t}(x_-^{i,t}) &= 100 \text{ if } x_-^{i,t} \leq b_- \min(x_-) \end{aligned}$$

where ν_+ is used when x is an attribute positively related to Social inclusion (i.e., it is a “good”) and ν_- is used when x is an attribute negatively related to it (i.e., it is a “bad”).

The coefficients $b_{\min}$ and $b_{\max}$ are the highest and lowest values to be used as benchmarks for the x variable for region i . Regardless on how the benchmarks are defined, it is straightforward that, when x is a “good”, $b_{\max}$ corresponds to a more desirable performance than $b_{\min}$, while the opposite is true when x is a “bad”.

The *min-max* strategy rescales indicators into an identical range [0,100].⁶ E.g., for x “good”, 0 is given to values lower or equal to b_{+min} , while 100 is given to those higher or equal to b_{+max} . The values within these benchmarks are proportionally converted into the 0-100 scale. Hence, v is a stepwise continuous function.

Regardless of the definition of such benchmarks, the *min-max* function imposes a link between the original variable x and its normalised $v(x)$, as the partial derivative in (3) illustrates:

$$(3) \quad \frac{\partial v_{\pm}^i(x_{\pm}^i)}{\partial x_{\pm}^i} = \pm \frac{1}{b_{\pm} \max(x_{\pm}) - b_{\pm} \min(x_{\pm})}$$

The effect of a one-unit increment in x on the transformed $v(x)$ is constant, since the transformation function is linear, and depends solely on the benchmarks b_{max} and b_{min} .⁷ Moreover, the higher is the range, the lower is the marginal contribution. Hence, unless all the attributes have very similar distributions and comparable units of measurement (which would make the normalisation itself of secondary importance), the *min-max* is performing a preliminary weighting of the original variables, for any choice of the aggregation function.

3.2 BASELINE LINEAR AGGREGATION MODEL

The parameter β in (1) determines the elasticity of substitution $\epsilon_{k,j}$ between any pair (v_k, v_j) . In the CES function, the elasticity between any pair k, j is constant and equal to $1/1-\beta$. The elasticity of substitution determines the percentage change in v_j/v_k , which would result from a percentage change in the slope along a level-set (the marginal rate of substitution, MRS, along an indifference curve). The parameter β must be lower than one to generate iso-inclusion contours convex to the origin in the two-dimensional region of the space of attributes ([Bourguignon and Chakravarty 2003](#)). The smaller is β , the higher is the increase in dimension v_j needed to keep constant the overall index after a one-unit decrease in dimension v_k .

Since the focus of this paper is on the normalisation choices, let us adopt a standard aggregation framework by setting $\beta=1$ in (1), therefore obtaining a linear weighted average with linear indifference curves, constant MRS and infinite elasticity of substitution between pairs of normalised dimensions. We also let the weights w_j be equal, i.e., $w_1 = \dots = w_m = 1/m = 1/4$ (since $m=4$ in our case study). The resulting model will be, for a generic region i (time subscripts are omitted) an aggregation function L , as in *linear*, such as:

$$(4) \quad L^i(v(\mathbf{x}^i)) = \frac{1}{m}v_1(x_1^i) + \dots + \frac{1}{m}v_m(x_m^i)$$

The arbitrary choice of setting equal weights is a widely adopted strategy in the literature of multidimensional measurement. As [Hoskins and Mascherini \(2009\)](#) and [Decancq and Lugo \(2013\)](#) highlight, this approach is often justified with the argument that all the dimensions are equally important ([Atkinson et al. 2002](#)) or, conversely, that there is insufficient knowledge for setting a more detailed weighting scheme (sometimes referred to as an “agnostic view”). Although being frequently described as a *simple* and relatively *neutral* strategy, “equal weighting” does not mean “no weighting”, because it involves an implicit judgment on the weights being equal, and because it often applies just on the normalised dimensions of the index.⁸

In the following Sections, we will investigate how original attributes contribute to the overall measure, and what characterize the relationship between attributes within the CES framework. In general, such effects can be retrieved via the partial derivative of the aggregate measure F with respect to variable x_j (region-specific indices are dropped for convenience), as follows:

⁶ The choice of multiplying by 100 eases readability of the results in the remaining of the paper, and does not affect any result.

⁷ This hypothesis could be relaxed by imposing a non-linear shape (convex, concave or s-shaped) to the v function, ([Martinetti and von Jacobi \(2012\)](#); [Meyer and Ponthière \(2011\)](#)). Such alternatives were tested and do not alter the implications of this paper.

⁸ In the words of [Martinetti and von Jacobi \(2012\)](#), the implicit assumption for equal weighting is that “in absence of any objective mechanism for determining the relative importance of the considered dimensions, the most neutral method is assigning an equal weight to each of them”. Indeed, both [Chowdhury and Squire \(2006\)](#) and [Nguefack-Tsague et al. \(2011\)](#) provide evidence in favour of equal weighting after collecting expert preferences.

$$\begin{aligned}
(5) \quad \frac{\partial F(v(\mathbf{x}))}{\partial x_j} &= \frac{1}{\beta} \left[w_1 v_1(x_1)^\beta + \dots + w_m v_m(x_m)^\beta \right]^{\frac{1-\beta}{\beta}} \beta w_j v_j(x_j)^{\beta-1} v_j'(x_j) \\
&= w_j v_j'(x_j) \left(\frac{[w_1 v_1(x_1)^\beta + \dots + w_m v_m(x_m)^\beta]^{\frac{1}{\beta}}}{v_j(x_j)} \right)^{1-\beta} = w_j v_j'(x_j) \left(\frac{F(v(\mathbf{x}))}{v_j(x_j)} \right)^{1-\beta}
\end{aligned}$$

From (5) we can identify four main drivers that determine how the aggregate measure F reacts at small changes in the j -th real-valued dimension x_j . The higher is the weight of the normalised j -th dimension, the higher will be the marginal variation in the F . The steeper is the normalisation function, the higher will be the effect of a change in the j -th dimension on the aggregate measure. The worse is the starting condition in the j -th dimension with respect to the aggregate measure, the higher will be its marginal contribution. Finally, the lower is the degree of substitutability β , the more sensitive F will be to bad performances in the j -th attribute.

Within the aggregation function F , the marginal rate of substitution between a pair of observed-indicators x_j and x_k is:

$$(6) \quad MRS_{x_k, x_j} = - \frac{dx_j}{dx_k} = \frac{\frac{\partial F(v(\mathbf{x}))}{\partial x_k}}{\frac{\partial F(v(\mathbf{x}))}{\partial x_j}} = \frac{w_k v_k'(x_k) \left(\frac{F(v(\mathbf{x}))}{v_k(x_k)} \right)^{1-\beta}}{w_j v_j'(x_j) \left(\frac{F(v(\mathbf{x}))}{v_j(x_j)} \right)^{1-\beta}} = \frac{w_k v_k'(x_k) \left(\frac{v_j(x_j)}{v_k(x_k)} \right)^{1-\beta}}{w_j v_j'(x_j) \left(\frac{v_k(x_k)}{v_j(x_j)} \right)^{1-\beta}}$$

When $\beta=1$, both the marginal contribution of the j -th attribute and its MRS will still depend on the shape of the normalisation function v_j . If, however, the transformation function is the identity function ($v_j(x_j) = x_j$), the effect of a change in x_j will be uniquely determined by its weight w_j and the MRS between a pair of dimensions j and k will depend just on the ratio between their weights.⁹

4. AGGREGATION WITH DATA-DRIVEN NORMALISATION

The data-driven min-max normalisation (7) defines the benchmarks *min* and *max* as the best and worst observed performance among selected countries ([Lefebvre et al. 2010](#); [Silva and Ferreira-Lopes 2013](#); [Murias et al. \(2012\)](#)), and across a time-series, in order to take into account the evolution of indicators and offer time-comparability ([Giovannini et al. 2008](#)). In our case study, this corresponds to assigning a value of 0 to the region which reports the worst-observed performance in a given indicator across Belgian German Italian and Spanish regions, in the period from 2004 to 2012, while assigning a value of 100 to the “best-observed” one.

For each region i where an attribute x is observed at a time t , the corresponding normalised value $\tau^{i,t}(x^{i,t})$ is determined as:

$$\begin{aligned}
(7) \quad \tau_+^{i,t}(x_+^{i,t}) &= 100 * \frac{x_+^{i,t} - \min_{i \in I} \min_t(x_+^t)}{\max_{i \in I} \max_t(x_+^t) - \min_{i \in I} \min_t(x_+^t)} \\
\text{or } \tau_-^{i,t}(x_-^{i,t}) &= 100 * \frac{\max_{i \in I} \max_t(x_-^t) - x_-^{i,t}}{\max_{i \in I} \max_t(x_-^t) - \min_{i \in I} \min_t(x_-^t)}
\end{aligned}$$

where x_+ and x_- have the usual meaning of a “good” and a “bad” attribute, respectively.

Table 4 displays the data-driven thresholds for our sample of 58 administrative regions:

⁹ The MRS between two observed dimensions will be equal to their “weights” also if the derivatives of their normalisation functions are equal, i.e., if $v_k'(x_k)/v_j'(x_j) = 1$

Table 4, data-driven benchmarks

	Observed minimum	Observed maximum
Longevity (<i>good</i>)	77.5 years (Wallonie, 2004)	84.2 years (Madrid, 2012)
Early school leaving (<i>bad</i>)	5.4% (Thüringen, 2009)	54.2% (Ciutat de Ceuta, 2005)
Long-term unemployment (<i>bad</i>)	0.5% (Trentino–Alto Adige, 2004)	18.2% (Ciutat de Ceuta, 2012)
At-risk-of-poverty rate (<i>bad</i>)	5.2% (Valle d’Aosta, 2004)	48.9% (Ciutat de Ceuta, 2008)

Note: data from the Eurostat Regional Database 2014, for administrative regions in Belgium, Germany, Italy and Spain.

Three issues on the normalisation in (7) are worth stressing. The first would be sensitivity to variables’ distribution: the presence of outliers in the data would stretch the range over which the normalisation is performed, therefore altering the original variable’s contribution to the overall index. Figure 10 in 0 depicts the distribution of the four observed indicators hereby considered. Possible outlier observations are, e.g., the rates of early school leavers in the autonomous cities of Ceuta and Medilla (Spain), whose values exceed 45% in several years.¹⁰ Moreover, this transformation is not stable when data for new years or new regions become available, which could affect the previously defined thresholds. An example of the latter (new territorial units) could arise when data for more recent EU member states are added to the sample (Lefebvre et al. 2010). Finally, a shift in the territorial dimension of the analysis (e.g., from a regional to a provincial-level analysis) will cause similar changes, since the provincial data are likely to exhibit higher variability than the regional ones.¹¹

Second, the transformations τ_j for each indicator j ensures comparability between the arguments of the aggregation function F , to the extent to which we define two observed attributes x_j and x_k as being *equally satisfied* when their normalised values are equal, i.e., when $\tau_j(x_j) = \tau_k(x_k)$. Consider a pair of indicators j, k (e.g., longevity and unemployment rate), and suppose that we observe (conveniently dropping the time index) $x_j = 81$ years and $x_k = 27\%$ poverty rate. Implementing the data-driven transformation (7), leads us to $\tau_j(81 \text{ years}) = 50$ and $\tau_k(27\% \text{ poverty-rate}) = 50$. Equality in the transformed values τ_j, τ_k implies that the two attributes j (longevity) and k (unemployment-rate) are equally satisfied in our Social Inclusion framework, since both values lie at equal distance between the observed-minimum and the observed-maximum in their respective data-series. Whether it can also be interpreted as an equality in the levels of welfare “revealed” by the two variables, is debatable, as discussed in the next Section.

Finally, computing the first derivative of (7) for each attribute shows (Table 5) that the steepness of the normalisation function varies significantly across dimensions. An increment of 1 unit in longevity-years-at-birth translates in an increase for the τ function much larger than it does a 1-unit increment the remaining indicators. Such coefficients, characterized by limited economic justification, given the choice of data-driven benchmarks, will still inevitably affect the underlying mechanism and trade-offs of the aggregation model (hence, of its implicit preference structure).

¹⁰ Since these territories are substantially different from other Spanish regions (they are both located on the Mediterranean coast of Morocco but belong to Spain since XV century), it could be argued that their exclusion from the sample would be acceptable. This would decrease the upper bound of the distribution to a rate of 43%, reported by the Region of Murcia in 2004 and the Illes Balears in 2007, 2008 (both are Spanish regions), without significant implications on the trade-offs. For long-term unemployment, this exclusion would reduce the maximum bound from 18.2% to 15% (Andalucia, Spain, 2012. For poverty rate, the reduction would be from 48.9% to 44.5% (Sicilia, Italy, 2006).

¹¹ As an example, we tried to enlarge the sample to Greece, Czech Republic and Norway (Eurostat Regional Database 2014, data available for statistical-regions only). The minimum level of early-school-leavers lowers to 1.8% (the Czech area Praha, 2006), as does the minimum threshold of long-term unemployment (to 0.3% in the Norwegian area Vestlandet), and the minimum longevity (to 74.2 years, in the Czech area Severozápad).

Table 5, derivatives of the data-driven normalisation

	1 year increment in longevity	1% increment in early school leavers	1% increment in long-term unemployment	1% increment in poverty-rate
$\frac{\partial \tau_j(x_j)}{\partial x_j}$	14.9	-2.05	-5.65	-2.28

When implementing the data-driven normalisation (7) in the baseline linear model (4) we obtain, for a generic region i , the aggregation function LD as in *linear, data-driven*, hereby defined as:

$$(8) \quad LD^i(\tau(\mathbf{x}^i)) = \frac{1}{m} \tau_1(x_1^i) + \dots + \frac{1}{m} \tau_m(x_m^i) \\ = 100 \left(0.25 * \frac{x_1 - 77.5}{84.2 - 77.5} + 0.25 * \frac{54.2 - x_2}{54.2 - 5.4} + 0.25 * \frac{18.2 - x_3}{18.2 - 0.5} + 0.25 * \frac{48.9 - x_4}{48.9 - 5.2} \right)$$

We can now compute, by the means of partial derivations, the absolute “relevance” of each indicator with respect to the synthetic measure LD, that is, the impact that a unitary change in the original attribute has on the overall measure (Table 6). The row “Relative weight” includes the “relevance” coefficients after they have been normalised, in order for them to sum to one, for comparability purposes.

Table 6, dimensions’ relevance under linear data-driven model

	longevity	early school leavers	long-term unemployment	poverty-rate
$\left \frac{\partial LD(\tau(\mathbf{x}))}{\partial x_j} \right $	3.72	0.512	1.412	0.57
Relative weight	0.59	0.08	0.22	0.09

Note: administrative regional data from the Eurostat Regional Database 2014, for Belgium, Germany, Italy and Spain (2004-2012)

Although the weights were set as equal for each normalised attribute, those related to the actual indicators are highly un-balanced. In particular, the longevity dimension is assigned a predominant role in the aggregation stage, due to the rescaling occurred in the normalisation phase: an increase of one year in life expectancy contributes to increase the synthetic measure by 3.72 points. Much lower effects derive from a decrease of one unit in long-term unemployment, and an even lower one from reductions in school-dropouts and poverty-rate.

Moreover, using (6), the marginal rates of substitution between any pairs of indicators x_j, x_k can be computed. Results are reported in Table 7.

Table 7, MRS in linear data-driven model

MRS for an increase of:	longevity years	% points early school leavers	% points l.t. unemployment	% points poverty rate
one-year in longevity	-	-7.26	-2.63	-6.52
one % point in early school leavers	-0.13	-	0.36	0.89
one % point in l.t. unemployment	-0.37	2.75	-	2.47
one % point in poverty rate	-0.15	1.11	0.4	-

Note: administrative regional data from the Eurostat Regional Database 2014, for Belgium, Germany, Italy and Spain (2004-2012)

A one-year increase in longevity is worth the same, in terms of the Social Inclusion measure, as a reduction in long-term unemployment by 2.63 percentages points, a reduction of 7.26 percentages points in school dropouts, or of 6.52 points of poverty rate. Similarly, e.g., an increase of 1% in the poverty rate lowers the overall Inclusion as an increase of 1.11 points in school-dropouts.

5. EXPERT-BASED NORMALISATION

5.1 COLLECTING EXPERTS' PREFERENCES

In a simple linear model with $\beta=1$ and $w_j = 1/m$ for each j -th indicators, the crucial determinant of a dimension's relevance relies heavily on the normalisation function, as visible from (5). If the normalisation function's parameters are arbitrarily determined through a data-driven strategy, its implications in terms of dimensions' relative weights and MRS have a mathematic interpretation, yet it is harder to determine what do they reflect in economic terms ([Lefebvre et al. 2010](#)). Indeed, a variable with transformed-value equal to "0" just implies it being "the last one", or "the worst one" among the observed, which does not necessarily corresponds to an undesirable condition of Well-being. A similar reasoning, with opposite meaning, can be done for normalised values of "100". In this respect, the data-driven normalisation appears as a tool for a positive- rather than a normative-analysis of observed performances. Yet, as already argued, the absence of economic value judgments does not prevent the rescaling from a potential lack of robustness due to data availability.

An alternative to the data-driven normalisation would require to incorporate some value judgments (e.g., social preferences, policy targets, expert opinions, see [Decancq and Lugo \(2013\)](#), [Mazziotta and Pareto \(2015\)](#), [Kim et al. \(2015\)](#)) in the normalisation. This translates to linking the extreme values "0" and the "100" with, e.g., a certain definition of desirability, thus making the normalisation independent from the data. When an indicator lies above or below such fixed bounds, further variations do not contribute to the latent variable under study (see e.g., the discussion in [Anand and Sen \(1994\)](#), [Klugman et al. \(2011\)](#), [Ravallion \(2012b\)](#), [Lefebvre et al. \(2010\)](#) and [Gidwitz et al. \(2010\)](#)). A major example of fixed threshold is the Human Development Index that, since 1994, adopted "goalposts" as minimum and maximum values in the normalisation function. The interpretation behind these fixed thresholds relies on the belief that objective upper and lower bounds can be identified, defined as "subsistence" minimum or "satiation" points, beyond which additional increments would not contribute to the expansion of capabilities.¹²

Contrary to Human Development, Social Exclusion's concept has been developed with reference to advanced industrialized economies, as are those of the European Union members. Therefore, rather than on "subsistence", its focus is posed on the "unacceptability" and "undesirability" of living conditions, as in an enlarged definition of poverty. Accordingly, a positive threshold for each of our four social-inclusion attributes would refer to a "certainly desirable and favourable condition of Well-being", to which a normalised value of 100 would correspond. Conversely, a negative threshold would refer to a "certainly undesirable and harmful conditions of Well-being", corresponding to a normalised value of 0.

In order to select the actual thresholds, we chose to elicit expert preferences through a survey, rather than pre-determine them in a top-down fashion.

Following [Chowdhury and Squire \(2006\)](#) and [Hoskins and Mascherini \(2009\)](#), we intended to involve informed opinions and therefore selected the population of professors and researchers in the Departments of Economic and Management of the Ca' Foscari University of Venice. Specifically, our population consisted of 149 professors (57 + 38 full or associate professors of Economics and Management, respectively; 29 +

¹² In the worldwide perspective of Human Development, the minimum "subsistence" point for longevity-at-birth was set to 25 years old in 2009, with "satiation" threshold at 85. Literacy rates' boundaries were set at 0% and 100%, as was the gross enrolment ratio, while GDP per capita was limited between 100\$ and 40,000\$. Among the changes made since 2010, the upper values were now set to observed maxima over the time series between 1980 and the most recent year available, while the lower bounds were set equal to subsistence minima (see [UNDP \(2010\)](#) and [Klugman et al. \(2011\)](#)).

25 assistant professors (*ricercatore universitario*) of Economics and Management, respectively)¹³. The survey was worded in Italian and conducted in electronic-form with the QUALTRICS software, a web-based tool that enables users to build custom surveys and distribute them via email.¹⁴ Participants were invited with an email including a link to take part to the on-line questionnaire on an anonymous basis.

The Survey was structured as follows:

- An introductory section discussed the topics, the purpose and the contents of the survey.
- Respondents were asked to select the variables (amongst the four described in Section 2) for which they would be willing to perform an evaluation.
- A randomization led the respondent to a page devoted to one of the selected variables. All pages were homogeneously designed with a consistent phrasing.
- The EUROSTAT definition of the variable at hand was offered, and descriptive statistics were shown through a bar graph, for 25 European countries (years 2000 and 2012).
- The main task of the survey was then detailed. Respondents should identify, according to their own opinion, two main thresholds for the variable at hand: a negative threshold, defined as a “value of the selected variable which conveys a certainly undesirable and problematic condition”, and a positive defined as “a value conveying a certainly desirable and virtuous condition”. The threshold had to be chosen by dragging a slider (using the mouse left-click) on a predetermined discrete interval of values,¹⁵ and releasing it to identify the preferred value (see Figure 1 for a snapshot of the negative-threshold choice for life expectancy).
- An example involving a mock variable “X” explained how to deal with the Qualtrics layout in order to identify the thresholds.
- After choosing the positive and the negative thresholds, a confirmation was required by clicking on “confirm and proceed” button, which would lead the respondent to the next variable-specific page, or to the last section of the survey (if no variables were left).
- The last section of the survey included questions on respondents’ age, gender and affiliation (either Economics or Management).

Appendix B includes further details on the survey pages and wording.

¹³ Although, in principle, it would be of interest to widen the Survey population to professors of other Departments (Asian and North African Studies, Environmental Sciences, Humanities, Linguistic, Molecular Sciences and Philosophy), we were led by time and resources constraints to focus on those Faculty more specifically connected to the issues of Social Inclusion and to the disciplines related to the four indicators over which a judgment was asked.

¹⁴ For further details, please refer to <http://www.qualtrics.com/>

¹⁵ Fixed intervals of values were imposed in order to avoid extreme and implausible choices (like 0 years old of longevity as “harmful” threshold). The predetermined intervals were: [90-60 years] for longevity; [0%, 50%] for early-school-leaving; [0%, 50%] for long-term unemployment; and [0%, 50%] for poverty-rate. No respondents chose one of the non-zero extremes as their preferred threshold.

Figure 1, choice of the negative threshold for life expectancy

5.2 SURVEY'S RESULTS

Out of 149 invitations, we received 88 responses. 59 were faculty members of the Department of Economics, 29 from the Department of Management. The following table provide brief descriptive statistics on our sample.

Table 8, descriptive statistics on the survey' sample

	Economics	Management	Overall
responses	59	28	88
Age:			
less than 40	36.1%	13%	28.6%
between 40 and 49	29.9%	43.5%	34.3%
between 50 and 59	23.4%	26.1%	24.3%
60 or more	10.6%	17.4%	12.6%
Female respondents	40.4%	34.8%	38.6%

Median responses and interquartile range are reported in Table 9¹⁶, while the responses' distribution is described in Figure 11 (Appendix B).

Table 9, survey-elicited benchmarks

	Median elicited minimum (25p – 75p)	Median elicited maximum (25p – 75p)
Longevity	73 years (70 – 75)	83 years (80 – 85)
Early school leaving	10% (5 – 10)	20% (15 – 25)
Long-term unemployment	3% (2 – 4)	9% (5.25 – 10)
At-risk-of-poverty rate	5% (3 – 7)	20% (17 – 21.5)

Note: on-line survey (QUALTRICS software), 88 responses from professors in Economics or Management at the Ca' Foscari University of Venice.

The representative favourable threshold for life expectancy is chosen at 83 years old, while the negative threshold is 73 years old. Early school-leaving's range lies between 10% (which corresponds to the EUROPE 2020's target for members of the European Union). A rate of 9% (or higher) of long-term unemployment denotes a median certainly undesirable condition, while the positive threshold is determined at 3%. As for

¹⁶ We chose the median response as a measure of central tendency to summarize a representative answer, as often done in the literature (e.g., [Hoskins and Mascherini \(2009\)](#)) because of its lower sensitivity to outliers, especially when the sample size is small.

poverty rate, a certainly harmful level has its median value at 20%, while desirability corresponds to 5% (or lower) share of population below the poverty line set by the Eurostat.

The interquartile ranges are always relatively small, except for the negative threshold for early-school-leaving (15%-25%). Nevertheless, we are aware that no “true values” exist, with respect to these thresholds. In the words of [Mascherini and Hoskins \(2008\)](#), “the judgment of one of the outline may be correct, and those who share a consensus view may be wrong”.

A quick comparison of Table 9 and Table 4 suggests that “certain desirability” and “certain undesirability” largely differ from observed minimum or maximum achievement. Indeed, a minimum observed level of longevity (77.5 years in Wallonia) is considered to be “certainly undesirable” by only a small fraction of respondents (Figure 11 in Appendix B). Similarly, any rate of long-term unemployment beyond 9%, or of school dropouts higher than 20%, or of poverty-rate beyond 20%, is regarded as certainly negative, while the actual observed maximums are quite higher. A capping on the positive threshold occurs for those regions which report long-term unemployment lower than 3% or early school leaving rates lower than 10%.¹⁷

5.3 AGGREGATION WITH THE EXPERT-BASED NORMALISATION

The elicited thresholds in Table 9 can be used as benchmarks in the min-max normalisation framework (2), thus obtaining four expert-based (hereafter, *survey-driven*) normalisation functions $\varsigma_j(x_j)$ with $j=1,2,3,4$.

In order to comment and compare such functions with the data-driven ones presented in the previous Section, in Table 10 we compute the first derivatives as in (3), while in Figure 2 we plot both the normalisation strategies applied to our four variables.

Table 10, derivatives of the survey based normalisations

	1 year increment in longevity	1% increment in early school leavers	1% increment in long-term unemployment	1% increment in poverty-rate
$\frac{\partial \varsigma_j(x_j)}{\partial x_j}$	10	-10	-16	-6.6

Note: derivatives refer to unitary increments of the original variable from a starting value within the interval $[\max(x_j)-\min(x_j)]$. For all the values outside the boundaries, a unitary variation would produce zero marginal contribution. Source: administrative regional data from the Eurostat Regional Database 2014, for Belgium, Germany, Italy and Spain (2004-2012).

The steep of the ς normalisation functions are higher because of the shorter min-max range imposed by the experts (except for longevity). Long-term unemployment is now the attribute for which a unitary increase has the highest effect on the normalised variable (a change of one-percentage point leads to a change of 16 points in the normalised 0-100 scale). Conversely, the effect of an increase in life expectancy is lower than in the data-driven framework. To make some examples, a poverty rate of 30% is normalised to zero (with the data-driven normalisation the transformed value was 40), while a life expectancy of 80 years old results in a transformed value of 70 (it was 36 in the data-driven normalisation).

The expert-based normalisation has several technical and economic implications. At first, it is closer in nature to a “social value-functions”, in that the rescale is performed according to how much a value fulfils a “desirability” requirement. Expert-elicited thresholds impose a normative characterization to the normalisation (and therefore to the aggregation) functions, which was lacking from the data-driven model (characterized rather as a tool for positive analysis). Moreover, the normalisation function may become weakly monotonic (instead of being *strongly monotonic*), when the elicited constraints are binding for some observed variable. Indeed, there are regions having attributes with observed performances outside the elicited boundaries. Such dimensions receive a normalised value of 100 or 0, and their marginal contribution to the aggregate measure is zero. In other words, the usual *non-satiation* hypothesis (more of a “good” is always

¹⁷ No territories in our sample reach 5% poverty-rate or 73 years in longevity-at-birth, so no “positive” capping occurs.

preferred to less) is maintained in a weaker form: more of a “good” is *non-ill favoured* with respect to less of it, after a certain performance is reached (and, conversely, more of a “bad” is *non-preferred* to less). As a rough realisation of the diminishing sensitivity hypothesis, the effects of a change in variables’ score on the social utility is zero, after the thresholds are crossed. From a policy-implication point of view, this suggests to focus on those dimensions whose performances lie farther away from the “desirability” level.¹⁸

Figure 2, survey-driven vs data-driven normalisation

When implementing such normalisation function in the linear aggregation framework as in (4), we obtain, for a generic region i (time subscript are omitted) an aggregation function LS as in *linear, survey-driven*:

$$\begin{aligned}
 LS^i(\zeta(\mathbf{x}^i)) &= \frac{1}{m_1} \zeta_1(x_1^i) + \dots + \frac{1}{m} \zeta_m(x_m^i) \\
 &= 100 \left(0.25 * \frac{x_1 - 73}{83 - 73} + 0.25 * \frac{20 - x_2}{20 - 10} + 0.25 * \frac{9 - x_3}{9 - 3} + 0.25 * \frac{20 - x_4}{20 - 5} \right)
 \end{aligned}$$

(9) with

$$\begin{aligned}
 \zeta_1(x_1) &= 0 \text{ if } x_1 < 73 \text{ and } \zeta_1(x_1) = 100 \text{ if } x_1 > 83 \\
 \zeta_2(x_2) &= 0 \text{ if } x_2 > 20 \text{ and } \zeta_2(x_2) = 100 \text{ if } x_2 < 10 \\
 \zeta_3(x_3) &= 0 \text{ if } x_3 > 9 \text{ and } \zeta_3(x_3) = 100 \text{ if } x_3 < 3 \\
 \zeta_4(x_4) &= 0 \text{ if } x_4 > 20 \text{ and } \zeta_4(x_4) = 100 \text{ if } x_4 < 5
 \end{aligned}$$

The marginal contribution of each indicator with respect to the synthetic measure LS, is again computed by applying (5), and reported in Table 11 together with their normalised counterpart.

Table 11, dimensions’ relevance under linear survey-driven model

longevity	early school leavers	long-term unemployment	poverty-rate
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¹⁸ Such a normalisation function can be smoothed, in order to avoid the step-wise shape (see, e.g., the discussion in [Ravallion \(2012b\)](#), [Lefebvre et al. \(2010\)](#), [Martinetti and von Jacobi \(2012\)](#), [Meyer and Ponthière \(2011\)](#) and [Pinar et al. \(2014\)](#))

$\left \frac{\partial LS(\zeta(\mathbf{x}))}{\partial x_j} \right $	2.5	2.5	4	1.65
Relative contribution	23.5%	23.5%	37.6%	15.4%

Note: derivatives refer to unitary increments of the original variable from a starting value within the interval $[\max(x_j) - \min(x_j)]$. For all the values outside the boundaries, a unitary variation would produce zero marginal contribution. Source: administrative regional data from the Eurostat Regional Database 2014, for Belgium, Germany, Italy and Spain (2004-2012).

Moreover, using (6), the marginal rates of substitution between any pairs of indicators x_j, x_k can be computed. Results are reported in Table 12.

Table 12, MRS under linear survey-driven model

MRS for an increase of:	longevity years	% points early school leavers	% points l.t. unemployment	% points poverty rate
one-year in longevity	-	-1	-0.62	-1.51
one % point in early school leavers	-1	-	0.62	1.51
one % point in l.t. unemployment	-1.6	1.6	-	2.42
one % point in poverty rate	-0.66	0.66	0.41	-

Note: these MRS are only valid when the variables take values within the boundaries $[\max(x_j) - \min(x_j)]$. Elseways, the MRS would be either zero, or infinite, or indeterminate, according to whether the numerator, the denominator, or both, in the ratio $v'_k(x_k)/v'_j(x_j)$ in (6) is zero. Source: administrative regional data from the Eurostat Regional Database 2014, for Belgium, Germany, Italy and Spain (2004-2012).

Both Table 11 and Table 12 report trade-offs which are significantly different from those implicit in the previous model (Table 6 and Table 7), only due to the different normalisation strategy. The dimensions' weights are more homogeneous under the expert-based normalisation: longevity and school-dropouts have equal relative relevance (23.5%), while poverty-rate accounts for 15.4% of the weight and the unemployment indicator being the one with the highest marginal effect on the aggregate measure (37.6%).

6. RESULTS

Results for the linear model with data-driven normalisation (LD model) are summarized at country levels in Table 13 (population-weighted averages at country level), together with coefficients of variation within countries. Full results are available in the Appendix (Table 16, Table 17, Table 18).¹⁹

Table 13, Social Inclusion measure and coefficients of variation, baseline model

	2004	2005	2006	2007	2008	2009	2010	2011	2012
Belgium	66.5	65.8	68.0	69.9	70.6	71.9	71.8	73.3	
Germany		65.4	67.5	69.9	71.8	73.1	73.6	75.4	77.1
Spain	59.5	61.8	65.3	65.5	65.3	64.4	62.4	61.6	59.7
Italy	67.2	67.3	70.2	71.6	72.6	73.0	74.1	73.0	71.0
COV regions BE	0.15	0.18	0.18	0.18	0.18	0.16	0.17	0.19	
COV regions DE		0.13	0.12	0.11	0.10	0.09	0.09	0.08	0.08
COV regions ES	0.18	0.22	0.22	0.22	0.21	0.20	0.23	0.23	0.27
COV regions IT	0.19	0.19	0.18	0.17	0.16	0.16	0.15	0.17	0.18
COV all regions	0.19	0.20	0.20	0.19	0.17	0.17	0.18	0.19	0.21

The left graph in Figure 3 helps us to provide an overall description of the trends in the multidimensional phenomenon of Inclusion at country level, according to the data-driven model. Differences between

¹⁹ Throughout this discussions of our results, we will concentrate more on commenting the levels of the composite indices, rather than the rankings of the regions, following the recommendations in [Atkinson et al. \(2004\)](#). The authors argue that the ultimate concern of the policy should lie on performance levels, since rankings might conceal the actual distances between territorial units, thus leading the reader to misleading conclusions.

countries appear rather limited until 2008. Italy reports the highest levels of Inclusion, followed by Germany, Belgium and Spain. There is a general increase in the index for all countries until 2008. Starting from that year, the Spanish performance declines and leads the country back to its 2004 levels of Inclusion. Italy's index is slightly affected by the economic crisis (roughly, from 2010 onward): its aggregate Inclusion ceases to improve and starts to decline, reaching in 2012 the same levels of 2006. Belgium and Germany show a general continuous increase in their levels of Inclusion (the former does not report data for 2012, the latter lacks data for poverty-rate for 2004, thereby their respective aggregate indexes are not available in these years). In particular, since 2011 Italy and Germany switch positions, with the latter becoming the country with the highest index among the four. Overall, the situation in 2012 appear to be more heterogeneous than it was in the early years in our sample: Italy and Spain show a negative trend (increasing exclusion), while Belgium and Germany continue to improve their aggregate performance. The coefficients of variation in Table 13 highlight that the Spanish regional picture is the most heterogeneous one, with Italy and Belgium following closely, and Germany being the country with less territorial variation. Moreover, a look at Table 17 and Table 18 allows us to recognize that, while many Italian regions score very well, some others are consistent bad performers. Most of the top-10 regions between 2004 and 2012 are Italians, but Campania and Sicilia constantly ranks at the very bottom of the tables. With this regard, the Social Inclusion index emphasizes the well-known dichotomous socio-economic picture of Italy as well as the contradictions of Belgium where important differences exist between the Flemish region, the Bruxelles region and Wallonia.

Results for the linear model of Social Inclusion with survey-driven normalisation (LS model) are summarized in Table 14 (aggregated at countries' level), while the right graph in Figure 3 provides a graphical illustration. Full results at regional level are reported in the Appendix D.

Figure 3, baseline models of Social Inclusion with different normalisations

At country levels, trends are confirmed but made more evident, with respect to the results of the data-driven specification. Social Inclusion increases in both Belgium and Germany, while Italy and Spain experience a continuous decline, which starts since the early years of the economic crisis. That being, said, the overall picture is quite different from the one commented before, for at least three reasons. First, Social Inclusion in levels is very different, with Italy being well below both Belgium and Germany for all the time-interval. In particular, Italy (blue circle markers) and Germany (green triangle markers) show similar levels of Inclusion in 2006. After that, the index continues to increase for Germany while it remains constant (and then declines)

in Italy. Thus, there is a clear phenomenon of rank reversal between the two models: in terms of “desirability” (as defined by the expert-panel), the aggregated Italian picture is worse than the German one.

A look at Table 20 and Table 21 highlights how German regions, together with Belgium’s Flanders, achieve more top-10 rankings than they were under the previous specification, especially after 2007. Second, Spain exhibits a decline in Social Inclusion (an increase in Social Exclusion) which is much more dramatic than it appeared before. The negative trend starts after 2006 but the drop in performance is substantial after 2008, leading to levels of Inclusion much lower in 2012 than they were in 2004. Again, we stress the fact that, albeit the negative trend for Spain was already visible from the baseline data-driven model, this picture conveys a much stronger need for intervention. Third, the heterogeneity within each country is much higher, as noticeable from the coefficients of variation in Table 14. Spain, Italy and Belgium still report the highest coefficients, but heterogeneities are rather constant in the latter country while they are increasing in Italy and especially in Spain. An opposite trend appears in Germany, where convergence of Social Inclusion between regions seems to occur. It is important to recall, at this point, that none of these differences are due to changes in the core parameters of the aggregation function (the β , or the weights w_j), but rather to the new re-scaling characterization, which has now an explicit interpretation in terms of desirability.

Table 14, Social Inclusion measure and coefficients of variation, expert-based normalisation

	2004	2005	2006	2007	2008	2009	2010	2011	2012
Belgium	63.5	62.2	64.4	66.1	66.7	69.4	68.5	70.1	
Germany		58.6	60.5	65.4	69.3	71.9	71.9	73.4	76.3
Spain	48.2	50.3	53.6	53.5	52.8	49.0	38.0	34.2	32.0
Italy	51.6	52.2	56.7	59.7	59.0	59.2	59.4	58.3	55.9
COV regions BE	0.37	0.43	0.43	0.42	0.43	0.34	0.39	0.39	
COV regions DE		0.28	0.26	0.24	0.22	0.19	0.17	0.16	0.13
COV regions ES	0.25	0.26	0.27	0.28	0.23	0.31	0.46	0.50	0.50
COV regions IT	0.39	0.34	0.30	0.31	0.31	0.32	0.32	0.38	0.34
COV all regions		0.20	0.22	0.22	0.28	0.21	0.32	0.34	

Changing the normalisation strategy has large implications on the distribution of the Social Inclusion index in our regional sample, as we show by plotting a kernel-density estimation for the distribution of the LD and LS indices, for 2004 and 2012 (Figure 4).²⁰ The left graph (LD model) highlights how the distribution moved to the right (higher inclusion) between the starting and the final period, and became less disperse and more uni-modal. Conversely, the one on the right reports a more heterogeneous starting distribution, which becomes clearly bi-modal in the final period of the analysis, providing another view on the trends already conveyed from Figure 4. Indeed, such differences in trends and levels could lead to very different conclusions in terms of policy implication to assuage and prevent Social Exclusion.

²⁰ The starting year for Germany is 2005, while the final year for Belgium is 2011, due to partial data-unavailability.

Figure 4, distributions of Inclusion Indices

Finally, in order to test whether the two specifications convey similar rankings, we perform a Kendall's tau²¹ tests between the ranking of the data-driven model and the one coming from the survey-driven model, for each year. Results are reported in Table 15. Coefficients' magnitude indicate that rankings' correspondence is far from perfect, yet we can always reject (at 99%) the null-hypothesis of no correlation between the models' rankings. Coefficients are lower than those found in similar analyses, as [Carrino \(2013\)](#). A reason for this is that the latter study was limited to Italian regional data, which presents a rather consolidated pattern of dominance between regions, so that the rankings are less affected by changes in model design.

Table 15, Kendall's τ correlation coefficients

	2005	2006	2007	2008	2009	2010	2011	2012	Average
Kendall's τ	0.78	0.80	0.78	0.75	0.73	0.73	0.78	0.78	0.76

6.1 ON THE RANK-REVERSAL BETWEEN GERMANY AND ITALY

Since 2004, Germany and Italy exhibit similar trends in early school-leaving, life expectancy at birth and poverty rate. Nevertheless, the levels of these variables are quite different: there are much more school-dropouts and poverty rates in Italy, which also presents substantially higher longevity. When it comes to long-term unemployment, the country-trends are crossing: Italy experienced a consistent decline in its labour market performance, while Germany saw a constant improvement (according to many observers, a consequence of the Hartz Reforms). Such trends are summarised in Figure 12 (Appendix D).

Figure 5 and Figure 6 report the values of the four indicators normalised according to, respectively, the data-driven and the expert-based min-max functions. The dashed thick lines in both Figures denote the social exclusion indices obtained by simple averages. For both Germany and Italy, the normalised poverty-rate is quite flat according to both normalisations strategies.

In the data-driven model for Italy (the left graph of Figure 5), the increase in normalised life-expectancy more than counterbalances, due to its substantially higher relative weight, the worsening conditions in the labour market, therefore allowing the overall measure to increase slightly. Almost no role is played by early school-leaving, which has very little variations. In Germany, all the dimensions improve, thus leading to a

²¹ The Kendall- τ test is a non parametric method that allows to measure the degree of correspondence between two rankings. In particular, the Kendall- τ b allows for the possibility of ties in the rankings. Command in STATA: ktau. A resulting test-value of zero would indicate that no correlation exist between the two rankings, while a value of 1 would indicate perfect correlation. Conversely, negative values (down to a minimum of -1) would indicate that rankings are inverted.

regular increase in the composite measure, yet the Inclusion index is lower than for Italy, exactly because of the weight given to life expectancy.

When the elicited benchmarks are implemented (Figure 6), life expectancy becomes the best performing dimension for Italy, while early school-leaving is heavily penalized. In Germany, both these normalised-attributes are much higher than under the data-driven model. Finally, although being rather small in 2004, the spread between the countries' long-term unemployment normalised-rates becomes much more evident. Given the reduced relative weight given to longevity in the expert-based model, and the higher one given to long-term unemployment, the German index is (1) higher than Italy, while it was lower in the previous specification; and (2) increasing at a higher rate.

Figure 5, data-driven normalised time trends in Germany and Italy

Figure 6, survey-driven normalised time trends in Germany and Italy

7. DISCUSSION: SENSITIVITY TO LINEARITY ASSUMPTIONS

When the substitutability parameter β in (1) is set to unity, indifference curves are linear, which implies perfect substitutability between normalised-dimensions.²² If this hypothesis is relaxed (β less than one), the marginal contribution of x_j is not constant (5), but it depends on the observed performance $v(x_j)$ as well as on the other normalised variables which enter the aggregation function. Indeed, the worse is the performance in $v(x_j)$, with respect to the other variables in the index, the higher will be its marginal contribution (hence, its weight) to the synthetic measure. A lower β results in a higher penalization of those territories with heterogeneous dashboards: when some dimensions' performance are relatively bad, they are not fully compensated by good performances in other dimensions.²³ The effect of assuming a $\beta < 1$, thus, will generally lead to lower levels of Social Inclusion with respect to the linear model, yet the reduction will depend on the actual regional performances and, more interestingly for this study, on the shape of the normalisation function.

Indeed, when $\beta < 1$, the normalisation function plays a double role (5): first, through its steepness v' and, second, through $v(x_j)$ at the denominator in the last factor of the last term. The same holds for the marginal rate of substitution (6) between two indicators j and h .²⁴

For convenience, starting from (1), we implement two specifications lying between the extremes of perfect substitutability ($\beta=1$) and perfect complementarity ($\beta=-\text{infinity}$). We first set $\beta=0$, a frequent choice for a non-compensative index (Silva and Ferreira-Lopes 2013; UNDP 2010). Second, we consider a switch to $\beta=-1$, which corresponds to a harmonic mean of the rescaled dimensions. The Geometric (G) and the Harmonic (H) models are defined as follows:

$$(10) \quad \begin{aligned} G^i(v(\mathbf{x}^i)) &= v_1(x_1^i)^w * \dots * v_m(x_m^i)^w \\ H^i(v(\mathbf{x}^i)) &= \left[w \frac{1}{v_1(x_1^i)} + \dots + w \frac{1}{v_m(x_m^i)} \right]^{-1} \end{aligned}$$

where the normalisation function v is a min-max as in (2) and the weights w are all set equal to 0.25, as in the previous Sections. In both models, if any of the rescaled components takes value zero, the whole indices collapses to zero, so that other dimensions become irrelevant.

We compute the Social Inclusion index for both the G and the H models, implementing first the data-driven, then the survey-driven normalisations. Results are shown (aggregated at country level, with regional population weights) in Figure 7, Figure 8 and Appendix E (results at regional level are available upon request), and can be commented under two perspectives: first, the difference between linear and non-linear specifications and, second, the role of different normalisation functions.

As expected, both the geometric and the harmonic Social Inclusion Indices are lower than the linear index. Yet, the extent of this reduction varies with the adopted normalisation. Under the data-driven rescale (Figure 7), the country-trends are relatively consistent to the linear model. Under the survey-driven normalisation (Figure 8), results indicate much higher heterogeneity among countries. While Belgium and Germany keep similar levels and trends with respect to the data-driven geometric and harmonic models, Italy and Spain score much lower levels of Inclusion. Spain, in particular, shows almost no improvements in Inclusion even before the crisis years.

The reason for such a performance drop (for Italy and Spain) relies on the higher steepness of the survey-driven normalisation function, on the consequent higher occurrence of low normalised-values in at least one

²² Moreover, since the min-max normalisation function (2) is linear in its arguments, this implies that also the core indicators x_j , x_k are assumed to be perfect substitutes. This would not be the case if the normalisation function were, e.g., $v(x_j) = \log(x_j)$, as it happens in the Human Development Index with respect to the “income” variable (UNDP (2014)).

²³ Ravallion (2012b) discusses the potentially undesirable (or unjustifiable) trade-offs implicit in the geometric HDI.

²⁴ When an attribute's observed performance lies outside the boundaries of the normalisation function, its corresponding marginal contribution is zero and the MRS between it and any other attribute is either zero or un-defined.

dimension, and in the weights implied by the expert-based rescaling, which penalise Italy and Spain, as already discussed in Section 6.1.

Figure 7, *geometric and harmonic models, data-driven*

Figure 8, *geometric and harmonic models, survey-driven*

Changing the normalisation strategy can sensibly alter the levels of the aggregate measure, as well as its rankings. Figure 9 summarizes the rankings obtained for each administrative region after computing the average indices between 2004 and 2012 under the linear, geometric and harmonic framework, and for both the data-driven and the survey-driven normalisation. Territories are sorted by their ranking in the linear data-driven model. Furthermore, data-driven rankings are characterized by “x” symbols, survey-driven rankings by “circles”. Both the categories appear in markers of different size, depending on the degree of substitutability implied in the models.

The graph highlights rankings’ heterogeneity. Focusing just on the “big X” and “big circle” (the linear specifications), it is visible that several regions are “penalized” by the data-driven normalisation (i.e., the “x” lies below the “circle”), while the opposite occurs when the “x” lies above the “circle”. More importantly, changing the substitutability degree *within* the data-driven framework (i.e., when switching from the big “x” to the smaller ones) produces far less variation in the rankings than it does changing the models *within* the

survey-driven framework (the distance between the big “circles” and the smaller ones).²⁵ Note also that, in many occasions, the switch to complementarity (either geometric or harmonic) causes rankings to change in opposite directions, depending on which normalisation is adopted (e.g., the 6th, the 12th and 13th observations from the left).

Figure 9, rankings' sensitivity

Finally, heterogeneities are not randomly occurring across regions. There are visible groups of “highly ranked” and “badly ranked” territories, which do not move much, regardless of the aggregation and normalisation choices. The former are concentrated in the Northeast and Centre of Italy, the Flemish Belgian region, the Baden-Württemberg and Bayern in Germany, the País Vasco and Comunidad Foral de Navarra in Spain. Among the latter we find most of the Southern Italian and Spanish regions (including the islands), as well as the Belgian Bruxelles region of Belgium.

8. CONCLUSION

The unavoidable subjectivity of composite measures of Well-being are cause of controversies in this field of economic analysis. In this paper, we argued that the lack of transparency on methodological choices can turn out more troublesome than subjectivity per se, and we particularly focus on the choices of the normalisation function. In the context of building a synthetic Index of Social Inclusion for 58 administrative regions in

²⁵

Table 26 reports average Kendall’s τ coefficients between the rankings generated by the data-driven and the survey-driven models computed so far. All Kendall’s τ coefficients allow to significantly reject (at 99%) the null hypothesis of independence in rankings. Yet, the coefficients corresponding to comparisons involving any geometric or harmonic model are lower than those involving linear-models-only: changing both the aggregation model and the normalisation function can indeed lead to substantially different rankings. Furthermore, changing the aggregation model *within* the data-driven framework (reading the table column-wise, e.g. from top to bottom, regardless of the column) produces far less variation in the Kendall’s τ coefficients than it does changing model *within* the survey-driven framework (reading the table row-wise, from left to right, regardless of the row), confirming the results in [Carrino \(2013\)](#) (in Italian).

Europe between 2004 and 2012, we showed the consequences of adopting different normalisation methods while keeping the aggregation model unchanged, with equal weights on the normalised dimensions.

To the extent to which rescaling is a requirement for composite measures, the actual aggregation involves the transformed variables, rather than the observed performances. There is an unavoidable and intrinsic difference between the interpretation of original and normalised performances, and yet social researchers are ultimately interested in the contribution of the *original* variables to the aggregate index. The rescaled unit of measurement (e.g., between zero and one-hundred) can be interpreted as a sort of degree of fulfilment of some criterion. Whether this criterion should be purely statistical (e.g., being far or close to the observed minimum or maximum observed achievements), or whether it should encompass some informed value judgements related to the topic at hand (as in the expert elicitation or in the adoption of policy benchmarks), relies on the researcher's choice.

In the former case, the *agnostic* choice of “letting the data talk”, the standard properties as strong non-satiation and continuity of the normalisation function are guaranteed, yet dimensions' trade-offs might be hard to interpret in economic terms and debatable from a social desirability perspective. Therefore, the resulting aggregate measure would be characterized as a tool for mainly “positive” analysis.

In the latter case, conversely, the normalisation function may become weakly monotonic, when the elicited constraints are binding for some observed variable. Moreover, the elicitation method suffers from the arbitrariness embedded in any survey exercise (choice of the population, bias in the framing of questions). Dimensions trade-offs reflect the preferences of an actual group of experts, are independent from the data-selection, and allow to characterize the final measure with a “normative” connotation.

The main result of our analysis is that neither method is neutral. Data-driven normalisation softens the aftermaths of the recent economic crisis, by putting a consistent weight on the longevity variable. Conversely, the survey-driven normalisation emphasizes the worsening trends in long-term unemployment and the relevance of early school-leaving. As a result, numerous rank-reversals occur between regions when switching the normalisation methods. Finally, the normalisation choice has even stronger consequences when the allowed substitutability between dimensions is lower (e.g., in a geometric or harmonic aggregation).

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Appendix A DISTRIBUTION OF OBSERVED VARIABLES

Figure 10, variables' distribution

Appendix B AN EXAMPLE OF A SURVEY QUESTION

As an example, let us consider the survey-page devoted to the life-expectancy-at-birth indicator. First, a definition of life expectancy was provided. Then, data for 25 European countries (years 2000 and 2012) were shown. At this point, respondents are faced with the summary of what they will be asked to do, i.e., identifying both a favourable and a harmful threshold for life-expectancy-at-birth, according to their own opinion. The harmful threshold is defined as a “level of longevity which represents a certainly negative and undesirable condition”. The favour threshold is defined as a “level of longevity which represents a certainly positive and desirable condition”. Before reaching the actual question, a full example was provided with a generic variable “X”. Respondents had, then, to determine the harmful threshold by dragging a slider on an interval of values (with the left mouse-click), and dropping it at the point that corresponded to their view of a certainly undesirable level of longevity. Figure 1 (in the text) illustrates the choice that respondents were facing for the harmful threshold of longevity. The choice was not entirely free, since we constrained respondents to select a level of life expectancy within a predetermined interval ranging from 60 to 90 years old, in order to avoid extremely implausible choices (like 0 years old). Similar steps characterized the choice of the favourable threshold, where respondents had to select their answer in the same interval between 60 and 90 years old. A cautionary disclaimer was emphasized at this point, stressing the fact that the favourable threshold should, by construction, be higher than- or equal to- the harmful threshold previously selected.²⁶

Figure 11 reports the histograms for the responses' distribution. The blue thick-dashed lines represent the answers for the favourable thresholds.

²⁶ The disclaimer aimed at avoiding inconsistent choices, e.g., a respondent who would choose, say, 81 years old as a harmful threshold, and subsequently choose 80 as a favourable threshold. No such patterns occurred.

Figure 11, distribution of survey responses

Appendix C RESULTS: MODEL WITH DATA-DRIVEN NORMALISATION

The following coefficients are obtained by implementing the LD model (8)

Table 16, Aggregate measure of Social Inclusion, baseline model with data-driven normalisation

nation	region	2004	2005	2006	2007	2008	2009	2010	2011	2012
BE	Bruxelles	51.3	47.7	49.5	50.8	51.8	55.1	52.6	51.3	.
BE	Vlaams Gewest	75.1	75.2	77.6	80.0	81.2	81.7	81.6	83.9	.
BE	Région wallonne	56.1	55.1	56.8	58.2	58.0	59.9	60.4	61.6	.
DE	Baden-Württemberg	.	76.4	79.6	81.2	82.9	83.0	83.0	85.3	86.5
DE	Bayern	.	73.4	74.4	77.4	79.3	80.4	81.2	82.1	84.1
DE	Berlin	.	52.3	53.7	59.4	59.1	62.9	63.4	66.9	67.9
DE	Brandenburg	.	55.0	56.9	60.9	65.5	66.8	68.7	70.2	70.4
DE	Bremen	.	51.3	57.6	59.8	61.4	64.3	65.5	65.0	68.2
DE	Hamburg	.	63.2	67.3	68.1	72.7	74.2	75.2	75.4	77.5
DE	Hessen	.	70.3	72.7	74.0	75.4	77.2	78.9	80.5	81.2
DE	Mecklenburg-Vorpommern	.	45.4	50.6	52.7	58.0	59.5	61.5	63.8	64.0
DE	Niedersachsen	.	63.9	65.7	68.5	69.5	71.0	71.6	72.6	74.9
DE	Nordrhein-Westfalen	.	63.6	65.3	67.2	68.8	69.8	69.6	71.5	72.4
DE	Rheinland-Pfalz	.	66.0	68.9	71.0	73.3	72.8	73.0	75.0	76.7
DE	Saarland	.	57.3	58.5	65.2	65.9	66.8	69.7	68.6	71.9
DE	Sachsen	.	58.1	61.1	62.9	67.1	68.0	66.2	71.2	73.0
DE	Sachsen-Anhalt	.	47.5	51.1	54.1	54.5	57.7	61.0	61.6	.
DE	Schleswig-Holstein	.	66.3	68.7	70.9	71.3	72.8	72.6	74.8	75.8
DE	Thüringen	.	58.7	59.0	61.2	65.8	68.8	71.1	71.9	73.1
ES	Galicia	60.9	65.7	65.8	68.3	68.8	67.9	69.0	68.0	66.0
ES	Principado de Asturias	62.5	65.4	68.5	70.6	72.5	70.5	68.5	69.3	64.7
ES	Cantabria	66.4	72.4	73.6	73.3	77.4	75.5	69.5	70.8	74.6
ES	País Vasco	75.9	78.5	81.4	80.1	82.5	82.1	82.6	81.1	80.4
ES	Comunidad Foral de Navarra	77.7	81.0	85.7	87.2	85.9	86.1	88.4	87.5	85.2
ES	La Rioja	.	64.9	70.2	68.7	65.1	69.1	69.4	64.2	68.4
ES	Aragón	72.7	70.6	74.7	72.5	74.9	76.6	72.0	70.2	69.6
ES	Comunidad de Madrid	74.6	74.6	78.2	77.9	77.3	76.5	77.5	77.7	74.9
ES	Castilla y León	64.8	65.7	70.0	70.9	70.2	71.5	71.0	68.7	71.3
ES	Castilla-la Mancha	54.9	55.3	59.1	58.7	58.7	60.0	57.9	54.1	52.2
ES	Extremadura	42.5	44.2	48.7	47.8	51.4	50.9	46.4	52.3	42.3

nation	region	2004	2005	2006	2007	2008	2009	2010	2011	2012
ES	Cataluña	63.8	66.3	72.5	70.7	71.0	68.0	65.8	65.8	64.1
ES	Comunidad Valenciana	56.5	58.2	63.6	64.8	62.3	62.3	57.2	58.0	53.9
ES	Illes Balears	61.2	61.9	67.8	64.6	64.1	60.6	58.8	62.1	56.8
ES	Andalucía	41.6	47.1	48.8	51.0	49.6	48.5	45.9	43.1	41.8
ES	Región de Murcia	50.4	53.3	55.2	55.3	54.7	52.4	51.1	53.0	50.5
ES	Ciudad Autónoma de Ceuta	.	21.0	23.9	24.9	29.3	33.3	30.0	34.5	26.9
ES	Ciudad Autónoma de Melilla	.	.	42.7	41.1	.	.	48.4	42.2	30.1
ES	Canarias	52.1	52.9	53.5	54.3	53.7	53.1	46.9	42.6	43.0
IT	Piemonte	72.4	74.2	76.0	78.9	77.3	77.5	78.4	78.2	76.6
IT	VDA	74.9	74.0	79.7	74.3	76.3	79.6	80.5	81.1	81.6
IT	Liguria	76.7	74.6	77.8	78.5	80.7	83.0	81.6	81.0	77.3
IT	Lombardia	77.3	77.3	80.4	80.5	81.5	81.3	82.4	84.5	82.5
IT	TAA	79.4	81.7	84.4	87.2	87.9	89.2	89.8	90.7	86.5
IT	Veneto	80.6	79.9	83.3	84.1	83.9	85.5	85.0	84.8	84.8
IT	FVG	80.7	79.0	78.5	84.8	81.6	83.3	83.6	83.9	81.8
IT	ER	80.5	80.7	83.0	84.2	84.8	85.7	86.7	87.4	84.9
IT	Toscana	79.1	81.4	83.2	82.2	83.0	83.7	82.6	82.2	80.6
IT	Umbria	80.6	76.6	80.2	83.0	81.0	84.4	84.5	85.9	81.4
IT	Marche	81.9	79.5	81.6	85.0	84.7	84.7	87.0	85.4	81.5
IT	Lazio	70.1	71.7	74.3	77.1	77.9	77.3	77.6	74.3	73.5
IT	Abruzzo	73.1	73.5	76.4	74.7	76.5	73.5	76.0	75.5	73.4
IT	Molise	64.5	64.7	63.6	67.8	66.2	67.5	73.6	71.8	71.3
IT	Campania	44.0	43.9	47.1	46.2	47.7	50.5	50.2	47.7	45.0
IT	Puglia	50.9	51.0	52.2	56.9	62.4	63.0	64.7	62.6	62.1
IT	Basilicata	59.9	58.0	64.6	66.5	65.0	65.5	68.7	64.6	63.5
IT	Calabria	50.9	50.9	55.5	57.1	58.4	60.3	63.6	61.1	55.9
IT	Sicilia	41.1	40.9	45.6	46.0	49.7	46.9	50.2	46.4	43.8
IT	Sardegna	57.3	54.8	62.4	67.3	64.5	65.1	70.4	64.1	62.2
	BELGIUM	66.5	65.8	68.0	69.9	70.6	71.9	71.8	73.3	.
	GERMANY	.	65.4	67.5	69.9	71.8	73.1	73.6	75.4	77.1
	SPAIN	59.5	61.8	65.3	65.5	65.3	64.4	62.4	61.6	59.7
	ITALY	70.2	70.3	73.2	74.6	75.3	75.8	76.6	75.8	73.9

Table 17, Ranking of Social Inclusion Measure, Baseline model with data-driven normalisation, years 2004-2008

rank	region	2004	region	2005	region	2006	region	2007	region	2008
1	Marche	81.9	TAA	81.7	Navarra	85.7	TAA	87.2	TAA	87.9
2	FVG	80.7	Toscana	81.4	TAA	84.4	Navarra	87.2	Navarra	85.9
3	Veneto	80.6	Navarra	81.0	Veneto	83.3	Marche	85.0	ER	84.8
4	Umbria	80.6	ER	80.7	Toscana	83.2	FVG	84.8	Marche	84.7
5	ER	80.5	Veneto	79.9	ER	83.0	ER	84.2	Veneto	83.9
6	TAA	79.4	Marche	79.5	Marche	81.6	Veneto	84.1	Toscana	83.0
7	Toscana	79.1	FVG	79.0	País Vasco	81.4	Umbria	83.0	Baden-W.	82.9
8	Navarra	77.7	País Vasco	78.5	Lombardia	80.4	Toscana	82.2	País Vasco	82.5
9	Lombardia	77.3	Lombardia	77.3	Umbria	80.2	Baden-W.	81.2	FVG	81.6
10	Liguria	76.7	Umbria	76.6	VDA	79.7	Lombardia	80.5	Lombardia	81.5
11	País Vasco	75.9	Baden-W.	76.4	Baden-W.	79.6	País Vasco	80.1	Vlaams G.	81.2
12	Vlaams G.	75.1	Vlaams G.	75.2	FVG	78.5	Vlaams G.	80.0	Umbria	81.0
13	VDA	74.9	Liguria	74.6	Madrid	78.2	Piemonte	78.9	Liguria	80.7
14	Madrid	74.6	Madrid	74.6	Liguria	77.8	Liguria	78.5	Bayern	79.3
15	Abruzzo	73.1	Piemonte	74.2	Vlaams G.	77.6	Madrid	77.9	Lazio	77.9
16	Aragón	72.7	VDA	74.0	Abruzzo	76.4	Bayern	77.4	Cantabria	77.4
17	Piemonte	72.4	Abruzzo	73.5	Piemonte	76.0	Lazio	77.1	Piemonte	77.3
18	Lazio	70.1	Bayern	73.4	Aragón	74.7	Abruzzo	74.7	Madrid	77.3
19	Cantabria	66.4	Cantabria	72.4	Bayern	74.4	VDA	74.3	Abruzzo	76.5
20	Castilla L.	64.8	Lazio	71.7	Lazio	74.3	Hessen	74.0	VDA	76.3
21	Molise	64.5	Aragón	70.6	Cantabria	73.6	Cantabria	73.3	Hessen	75.4
22	Cataluña	63.8	Hessen	70.3	Hessen	72.7	Aragón	72.5	Aragón	74.9
23	Asturias	62.5	Schleswig	66.3	Cataluña	72.5	Rheinland	71.0	Rheinland	73.3
24	Balears	61.2	Cataluña	66.3	La Rioja	70.2	Castilla L.	70.9	Hamburg	72.7
25	Galicía	60.9	Rheinland	66.0	Castilla L.	70.0	Schleswig	70.9	Asturias	72.5
26	Basilicata	59.9	Galicía	65.7	Rheinland	68.9	Cataluña	70.7	Schleswig	71.3
27	Sardegna	57.3	Castilla L.	65.7	Schleswig	68.7	Asturias	70.6	Cataluña	71.0

rank	region	2004	region	2005	region	2006	region	2007	region	2008
28	Valenciana	56.5	Asturias	65.4	Asturias	68.5	La Rioja	68.7	Castilla L.	70.2
29	Wallonia	56.1	La Rioja	64.9	Balears	67.8	Nieders.	68.5	Nieders.	69.5
30	Castilla M.	54.9	Molise	64.7	Hamburg	67.3	Galicía	68.3	Nordr.-W.	68.8
31	Canarias	52.1	Nieders.	63.9	Galicía	65.8	Hamburg	68.1	Galicía	68.8
32	Bruxelles	51.3	Nordr.-W.	63.6	Nieders.	65.7	Molise	67.8	Sachsen	67.1
33	Puglia	50.9	Hamburg	63.2	Nordr.-W.	65.3	Sardegna	67.3	Molise	66.2
34	Calabria	50.9	Balears	61.9	Basilicata	64.6	Nordr.-W.	67.2	Saarland	65.9
35	Murcia	50.4	Thüringen	58.7	Molise	63.6	Basilicata	66.5	Thüringen	65.8
36	Campania	44.0	Valenciana	58.2	Valenciana	63.6	Saarland	65.2	Br'burg	65.5
37	Extremad.	42.5	Sachsen	58.1	Sardegna	62.4	Valenciana	64.8	La Rioja	65.1
38	Andalucía	41.6	Basilicata	58.0	Sachsen	61.1	Balears	64.6	Basilicata	65.0
39	Sicilia	41.1	Saarland	57.3	Castilla M.	59.1	Sachsen	62.9	Sardegna	64.5
40	Baden-W.	.	Castilla M.	55.3	Thüringen	59.0	Thüringen	61.2	Balears	64.1
41	Bayern	.	Wallonia	55.1	Saarland	58.5	Br'burg	60.9	Puglia	62.4
42	Berlin	.	Br'burg	55.0	Bremen	57.6	Bremen	59.8	Valenciana	62.3
43	Br'burg	.	Sardegna	54.8	Br'burg	56.9	Berlin	59.4	Bremen	61.4
44	Bremen	.	Murcia	53.3	Wallonia	56.8	Castilla M.	58.7	Berlin	59.1
45	Hamburg	.	Canarias	52.9	Calabria	55.5	Wallonia	58.2	Castilla M.	58.7
46	Hessen	.	Berlin	52.3	Murcia	55.2	Calabria	57.1	Calabria	58.4
47	Meck'burg	.	Bremen	51.3	Berlin	53.7	Puglia	56.9	Wallonia	58.0
48	Nieders.	.	Puglia	51.0	Canarias	53.5	Murcia	55.3	Meck'burg	58.0
49	Nordr.-W.	.	Calabria	50.9	Puglia	52.2	Canarias	54.3	Murcia	54.7
50	Rheinland	.	Bruxelles	47.7	Sachsen-A.	51.1	Sachsen-A.	54.1	Sachsen-A.	54.5
51	Saarland	.	Sachsen-A.	47.5	Meck'burg	50.6	Meck'burg	52.7	Canarias	53.7
52	Sachsen	.	Andalucía	47.1	Bruxelles	49.5	Andalucía	51.0	Bruxelles	51.8
53	Sachsen-A.	.	Meck'burg	45.4	Andalucía	48.8	Bruxelles	50.8	Extremad.	51.4
54	Schleswig	.	Extremad.	44.2	Extremad.	48.7	Extremad.	47.8	Sicilia	49.7
55	Thüringen	.	Campania	43.9	Campania	47.1	Campania	46.2	Andalucía	49.6
56	La Rioja	.	Sicilia	40.9	Sicilia	45.6	Sicilia	46.0	Campania	47.7
57	Ceuta	.	Ceuta	21.0	Melilla	42.7	Melilla	41.1	Ceuta	29.3
58	Melilla	.	Melilla	.	Ceuta	23.9	Ceuta	24.9	Melilla	.

Table 18, Ranking of Social Inclusion Measure, Baseline model with data-driven normalisation, years 2009-2012

rank	region	2009	region	2010	region	2011	region	2012
1	TAA	89.2	TAA	89.8	TAA	90.7	TAA	86.5
2	Navarra	86.1	Navarra	88.4	Navarra	87.5	Baden-W.	86.5
3	ER	85.7	Marche	87.0	ER	87.4	Navarra	85.2
4	Veneto	85.5	ER	86.7	Umbria	85.9	ER	84.9
5	Marche	84.7	Veneto	85.0	Marche	85.4	Veneto	84.8
6	Umbria	84.4	Umbria	84.5	Baden-W.	85.3	Bayern	84.1
7	Toscana	83.7	FVG	83.6	Veneto	84.8	Lombardia	82.5
8	FVG	83.3	Baden-W.	83.0	Lombardia	84.5	FVG	81.8
9	Baden-W.	83.0	País Vasco	82.6	FVG	83.9	VDA	81.6
10	Liguria	83.0	Toscana	82.6	Vlaams G.	83.9	Marche	81.5
11	País Vasco	82.1	Lombardia	82.4	Toscana	82.2	Umbria	81.4
12	Vlaams G.	81.7	Vlaams G.	81.6	Bayern	82.1	Hessen	81.2
13	Lombardia	81.3	Liguria	81.6	VDA	81.1	Toscana	80.6
14	Bayern	80.4	Bayern	81.2	País Vasco	81.1	País Vasco	80.4
15	VDA	79.6	VDA	80.5	Liguria	81.0	Hamburg	77.5
16	Piemonte	77.5	Hessen	78.9	Hessen	80.5	Liguria	77.3
17	Lazio	77.3	Piemonte	78.4	Piemonte	78.2	Rheinland	76.7
18	Hessen	77.2	Lazio	77.6	Madrid	77.7	Piemonte	76.6
19	Aragón	76.6	Madrid	77.5	Abruzzo	75.5	Schleswig	75.8
20	Madrid	76.5	Abruzzo	76.0	Hamburg	75.4	Madrid	74.9
21	Cantabria	75.5	Hamburg	75.2	Rheinland	75.0	Nieders.	74.9
22	Hamburg	74.2	Molise	73.6	Schleswig	74.8	Cantabria	74.6
23	Abruzzo	73.5	Rheinland	73.0	Lazio	74.3	Lazio	73.5
24	Schleswig	72.8	Schleswig	72.6	Nieders.	72.6	Abruzzo	73.4
25	Rheinland	72.8	Aragón	72.0	Thüringen	71.9	Thüringen	73.1
26	Castilla L.	71.5	Nieders.	71.6	Molise	71.8	Sachsen	73.0
27	Nieders.	71.0	Thüringen	71.1	Nordr.-W.	71.5	Nordr.-W.	72.4
28	Asturias	70.5	Castilla L.	71.0	Sachsen	71.2	Saarland	71.9
29	Nordr.-W.	69.8	Sardegna	70.4	Cantabria	70.8	Castilla L.	71.3

rank	region	2009	region	2010	region	2011	region	2012
30	La Rioja	69.1	Saarland	69.7	Aragón	70.2	Molise	71.3
31	Thüringen	68.8	Nordr.-W.	69.6	Br'burg	70.2	Br'burg	70.4
32	Sachsen	68.0	Cantabria	69.5	Asturias	69.3	Aragón	69.6
33	Cataluña	68.0	La Rioja	69.4	Castilla L.	68.7	La Rioja	68.4
34	Galicia	67.9	Galicia	69.0	Saarland	68.6	Bremen	68.2
35	Molise	67.5	Basilicata	68.7	Galicia	68.0	Berlin	67.9
36	Saarland	66.8	Br'burg	68.7	Berlin	66.9	Galicia	66.0
37	Br'burg	66.8	Asturias	68.5	Cataluña	65.8	Asturias	64.7
38	Basilicata	65.5	Sachsen	66.2	Bremen	65.0	Cataluña	64.1
39	Sardegna	65.1	Cataluña	65.8	Basilicata	64.6	Meck'burg	64.0
40	Bremen	64.3	Bremen	65.5	La Rioja	64.2	Basilicata	63.5
41	Puglia	63.0	Puglia	64.7	Sardegna	64.1	Sardegna	62.2
42	Berlin	62.9	Calabria	63.6	Meck'burg	63.8	Puglia	62.1
43	Valenciana	62.3	Berlin	63.4	Puglia	62.6	Balears	56.8
44	Balears	60.6	Meck'burg	61.5	Balears	62.1	Calabria	55.9
45	Calabria	60.3	Sachsen-A.	61.0	Wallonia	61.6	Valenciana	53.9
46	Castilla M.	60.0	Wallonia	60.4	Sachsen-A.	61.6	Castilla M.	52.2
47	Wallonia	59.9	Balears	58.8	Calabria	61.1	Murcia	50.5
48	Meck'burg	59.5	Castilla M.	57.9	Valenciana	58.0	Campania	45.0
49	Sachsen-A.	57.7	Valenciana	57.2	Castilla M.	54.1	Sicilia	43.8
50	Bruxelles	55.1	Bruxelles	52.6	Murcia	53.0	Canarias	43.0
51	Canarias	53.1	Murcia	51.1	Extremad.	52.3	Extremad.	42.3
52	Murcia	52.4	Sicilia	50.2	Bruxelles	51.3	Andalucía	41.8
53	Extremad.	50.9	Campania	50.2	Campania	47.7	Melilla	30.1
54	Campania	50.5	Melilla	48.4	Sicilia	46.4	Ceuta	26.9
55	Andalucía	48.5	Canarias	46.9	Andalucía	43.1	Bruxelles	
56	Sicilia	46.9	Extremad.	46.4	Canarias	42.6	Vlaams G.	
57	Ceuta	33.3	Andalucía	45.9	Melilla	42.2	Wallonia	
58	Melilla		Ceuta	30.0	Ceuta	34.5	Sachsen-A.	

Appendix D RESULTS: MODEL WITH SURVEY-DRIVEN NORMALISATION

The following coefficients are obtained by implementing the LS model (9)

Table 19, Social Inclusion Measure, Baseline model with expert-based normalisation

nation	region	2004	2005	2006	2007	2008	2009	2010	2011	2012
BE	Bruxelles	24.9	17.0	17.7	18.3	17.4	31.4	22.5	22.8	.
BE	Vlaams Gewest	81.2	81.8	84.6	86.1	87.3	87.7	88.0	89.8	.
BE	Région wallonne	44.7	42.1	43.7	46.3	46.0	49.2	48.9	50.2	.
DE	Baden-Württemberg	.	79.4	84.5	85.8	88.8	88.3	88.4	89.1	89.5
DE	Bayern	.	79.7	77.6	84.2	85.9	86.0	86.9	86.7	87.4
DE	Berlin	.	28.1	29.1	37.6	34.5	43.1	43.5	51.4	53.6
DE	Brandenburg	.	43.0	44.4	49.2	59.3	61.7	67.6	69.1	68.8
DE	Bremen	.	21.2	32.0	41.0	45.5	55.4	55.5	49.0	60.5
DE	Hamburg	.	44.0	54.3	53.9	67.7	76.0	75.2	73.5	75.9
DE	Hessen	.	66.1	67.4	73.1	77.4	81.0	81.4	83.6	84.9
DE	Mecklenburg-Vorpommern	.	32.0	35.4	38.5	44.8	47.6	52.6	57.2	54.5
DE	Niedersachsen	.	53.8	56.7	63.0	66.8	70.9	68.8	68.1	73.7
DE	Nordrhein-Westfalen	.	52.9	53.8	59.6	63.7	66.9	64.5	67.1	69.6
DE	Rheinland-Pfalz	.	57.6	62.8	67.4	73.8	71.0	66.6	71.2	74.7
DE	Saarland	.	37.9	40.2	62.9	62.8	62.8	69.3	62.4	70.0
DE	Sachsen	.	46.5	48.8	47.5	54.9	57.7	59.8	64.6	69.2
DE	Sachsen-Anhalt	.	35.8	36.4	36.3	35.5	44.8	51.4	48.6	
DE	Schleswig-Holstein	.	58.9	61.7	69.7	70.8	75.0	73.2	74.0	75.0
DE	Thüringen	.	44.1	45.8	47.7	59.8	64.7	69.3	71.9	74.4
ES	Galicia	34.9	48.7	47.1	50.3	48.8	51.8	45.5	36.8	32.9
ES	Principado de Asturias	48.1	48.2	57.4	60.1	58.7	54.7	48.2	46.6	32.2
ES	Cantabria	50.5	57.6	61.1	60.9	63.4	61.5	46.0	42.3	55.3
ES	País Vasco	74.6	76.1	78.3	75.3	78.3	78.3	81.1	74.1	71.5
ES	Comunidad Foral de Navarra	61.6	68.8	82.1	80.6	74.7	71.9	79.3	82.8	74.9
ES	La Rioja	.	46.4	48.7	51.0	50.0	53.5	45.0	34.7	34.8
ES	Aragón	60.5	52.8	60.2	54.9	58.3	63.8	50.5	43.0	35.1
ES	Comunidad de Madrid	65.3	61.3	64.4	63.2	61.1	60.4	51.2	45.1	38.9

nation	region	2004	2005	2006	2007	2008	2009	2010	2011	2012
ES	Castilla y León	43.1	46.8	48.5	48.3	49.0	52.4	43.6	37.8	35.7
ES	Castilla-la Mancha	45.8	45.5	47.8	46.8	47.8	44.3	33.4	25.8	25.8
ES	Extremadura	35.5	36.4	43.1	43.3	43.9	37.2	26.3	23.3	22.5
ES	Cataluña	55.9	57.9	60.1	59.5	60.8	54.8	40.1	32.8	33.6
ES	Comunidad Valenciana	45.3	44.6	49.0	52.0	46.2	46.1	23.7	26.3	23.5
ES	Illes Balears	56.6	52.6	59.5	56.9	58.2	53.9	38.5	35.5	23.5
ES	Andalucía	30.5	38.4	42.5	42.5	41.2	30.5	20.5	21.0	21.0
ES	Región de Murcia	42.2	42.5	44.3	44.3	44.5	40.8	25.7	23.3	23.0
ES	Ciudad Autónoma de Ceuta	.	15.0	19.9	14.8	26.0	16.6	15.8	17.5	20.0
ES	Ciudad Autónoma de Melilla	.	.	30.6	25.8	.	.	24.5	19.0	19.5
ES	Canarias	39.2	41.6	43.5	43.8	42.1	28.8	23.3	22.5	22.8
IT	Piemonte	59.0	61.7	62.1	69.9	65.8	64.8	66.4	68.2	62.2
IT	VDA	62.3	66.0	70.5	64.9	65.2	66.2	67.7	69.5	70.5
IT	Liguria	71.2	63.8	69.2	67.7	78.3	84.8	75.4	75.5	67.4
IT	Lombardia	64.2	65.0	68.4	67.7	67.2	67.1	70.9	75.9	74.3
IT	TAA	65.8	68.8	76.4	79.6	80.7	79.7	78.9	83.4	75.1
IT	Veneto	70.7	69.2	78.1	81.6	76.6	76.0	77.4	74.9	81.2
IT	FVG	81.7	74.8	64.8	84.4	75.8	79.0	83.7	82.3	79.1
IT	ER	68.2	68.7	72.5	74.6	76.6	80.8	83.2	85.8	81.0
IT	Toscana	66.0	74.1	76.6	71.3	75.8	75.2	71.8	69.1	68.4
IT	Umbria	78.8	70.7	71.6	80.4	72.6	81.1	81.6	84.2	75.0
IT	Marche	72.5	64.1	67.3	75.5	78.0	76.7	78.2	79.8	68.3
IT	Lazio	60.2	64.5	70.0	76.5	73.6	73.2	68.8	59.2	61.1
IT	Abruzzo	61.0	61.5	68.3	63.3	65.7	62.2	64.2	63.3	57.4
IT	Molise	46.1	48.7	45.8	53.5	49.1	50.9	60.5	56.2	60.1
IT	Campania	20.6	18.4	25.4	30.4	27.3	26.2	23.3	20.0	20.3
IT	Puglia	21.7	26.1	30.0	35.0	36.5	36.3	33.4	32.7	26.3
IT	Basilicata	36.7	35.6	46.1	53.2	50.4	55.4	43.8	45.1	43.4
IT	Calabria	24.7	26.6	30.4	33.3	37.3	41.6	44.1	34.4	29.8
IT	Sicilia	19.8	19.0	25.2	26.1	26.2	24.3	25.4	25.3	21.8
IT	Sardegna	30.3	29.0	39.2	42.0	35.5	36.9	45.2	32.1	27.4
	BELGIUM	63.5	62.2	64.4	66.1	66.7	69.4	68.5	70.1	.
	GERMANY	.	58.6	60.5	65.4	69.3	71.9	71.9	73.4	76.3
	SPAIN	48.2	50.3	53.6	53.5	52.8	49.0	38.0	34.2	32.0
	ITALY	56.1	56.7	61.1	64.2	63.4	63.8	63.9	62.8	60.7

Table 20, *Ranking of Social Inclusion Measure, Baseline model with expert-based normalisation, years 2004-2008*

rank	region	2004	region	2005	region	2006	region	2007	region	2008
1	FVG	81.7	Vlaams	81.8	Vlaams	84.6	Vlaams	86.1	Baden-W	88.8
2	Vlaams	81.2	Bayern	79.7	Baden-W	84.5	Baden-W	85.8	Vlaams	87.3
3	Umbria	78.8	Baden-W	79.4	Navarra	82.1	FVG	84.4	Bayern	85.9
4	País Vasco	74.6	País Vasco	76.1	País Vasco	78.3	Bayern	84.2	TAA	80.7
5	Marche	72.5	FVG	74.8	Veneto	78.1	Veneto	81.6	País Vasco	78.3
6	Liguria	71.2	Toscana	74.1	Bayern	77.6	Navarra	80.6	Liguria	78.3
7	Veneto	70.7	Umbria	70.7	Toscana	76.6	Umbria	80.4	Marche	78.0
8	ER	68.2	Veneto	69.2	TAA	76.4	TAA	79.6	Hessen	77.4
9	Toscana	66.0	TAA	68.8	ER	72.5	Lazio	76.5	Veneto	76.6
10	TAA	65.8	Navarra	68.8	Umbria	71.6	Marche	75.5	ER	76.6
11	Madrid	65.3	ER	68.7	VDA	70.5	País Vasco	75.3	FVG	75.8
12	Lombardia	64.2	Hessen	66.1	Lazio	70.0	ER	74.6	Toscana	75.8
13	VDA	62.3	VDA	66.0	Liguria	69.2	Hessen	73.1	Navarra	74.7
14	Navarra	61.6	Lombardia	65.0	Lombardia	68.4	Toscana	71.3	Rheinland	73.8
15	Abruzzo	61.0	Lazio	64.5	Abruzzo	68.3	Piemonte	69.9	Lazio	73.6
16	Aragón	60.5	Marche	64.1	Hessen	67.4	Schleswig-H.	69.7	Umbria	72.6
17	Lazio	60.2	Liguria	63.8	Marche	67.3	Lombardia	67.7	Schleswig-H.	70.8
18	Piemonte	59.0	Piemonte	61.7	FVG	64.8	Liguria	67.7	Hamburg	67.7
19	Balears	56.6	Abruzzo	61.5	Madrid	64.4	Rheinland	67.4	Lombardia	67.2
20	Cataluña	55.9	Madrid	61.3	Rheinland	62.8	VDA	64.9	Nieders'n	66.8
21	Cantabria	50.5	Schleswig-H.	58.9	Piemonte	62.1	Abruzzo	63.3	Piemonte	65.8
22	Asturias	48.1	Cataluña	57.9	Schleswig-H.	61.7	Madrid	63.2	Abruzzo	65.7
23	Molise	46.1	Rheinland	57.6	Cantabria	61.1	Nieders'n	63.0	VDA	65.2
24	Castilla M	45.8	Cantabria	57.6	Aragón	60.2	Saarland	62.9	Nordrhein	63.7

rank	region	2004	region	2005	region	2006	region	2007	region	2008
25	Valenciana	45.3	Nieders'n	53.8	Cataluña	60.1	Cantabria	60.9	Cantabria	63.4
26	Wallonne	44.7	Nordrhein	52.9	Balears	59.5	Asturias	60.1	Saarland	62.8
27	Castilla L.	43.1	Aragón	52.8	Asturias	57.4	Nordrhein	59.6	Madrid	61.1
28	Murcia	42.2	Balears	52.6	Nieders'n	56.7	Cataluña	59.5	Cataluña	60.8
29	Canarias	39.2	Molise	48.7	Hamburg	54.3	Balears	56.9	Thüringen	59.8
30	Basilicata	36.7	Galicia	48.7	Nordrhein	53.8	Aragón	54.9	Brand'burg	59.3
31	Extremad.	35.5	Asturias	48.2	Valenciana	49.0	Hamburg	53.9	Asturias	58.7
32	Galicia	34.9	Castilla L.	46.8	Sachsen	48.8	Molise	53.5	Aragón	58.3
33	Andalucía	30.5	Sachsen	46.5	La Rioja	48.7	Basilicata	53.2	Balears	58.2
34	Sardegna	30.3	La Rioja	46.4	Castilla L.	48.5	Valenciana	52.0	Sachsen	54.9
35	Bruxelles	24.9	Castilla M	45.5	Castilla M	47.8	La Rioja	51.0	Basilicata	50.4
36	Calabria	24.7	Valenciana	44.6	Galicia	47.1	Galicia	50.3	La Rioja	50.0
37	Puglia	21.7	Thüringen	44.1	Basilicata	46.1	Brand'burg	49.2	Molise	49.1
38	Campania	20.6	Hamburg	44.0	Thüringen	45.8	Castilla L.	48.3	Castilla L.	49.0
39	Sicilia	19.8	Brand'burg	43.0	Molise	45.8	Thüringen	47.7	Galicia	48.8
40	Baden-W		Murcia	42.5	Brand'burg	44.4	Sachsen	47.5	Castilla M	47.8
41	Bayern		Wallonne	42.1	Murcia	44.3	Castilla M	46.8	Valenciana	46.2
42	Berlin		Canarias	41.6	Wallonne	43.7	Wallonne	46.3	Wallonne	46.0
43	Brand'burg		Andalucía	38.4	Canarias	43.5	Murcia	44.3	Bremen	45.5
44	Bremen		Saarland	37.9	Extremad.	43.1	Canarias	43.8	Meckl'burg	44.8
45	Hamburg		Extremad.	36.4	Andalucía	42.5	Extremad.	43.3	Murcia	44.5
46	Hessen		Sachsen-A.	35.8	Saarland	40.2	Andalucía	42.5	Extremad.	43.9
47	Meckl'burg		Basilicata	35.6	Sardegna	39.2	Sardegna	42.0	Canarias	42.1
48	Nieders'n		Meckl'burg	32.0	Sachsen-A.	36.4	Bremen	41.0	Andalucía	41.2
49	Nordrhein		Sardegna	29.0	Meckl'burg	35.4	Meckl'burg	38.5	Calabria	37.3
50	Rheinland		Berlin	28.1	Bremen	32.0	Berlin	37.6	Puglia	36.5
51	Saarland		Calabria	26.6	Melilla	30.6	Sachsen-A.	36.3	Sardegna	35.5
52	Sachsen		Puglia	26.1	Calabria	30.4	Puglia	35.0	Sachsen-A.	35.5
53	Sachsen-A.		Bremen	21.2	Puglia	30.0	Calabria	33.3	Berlin	34.5
54	Schleswig H.		Sicilia	19.0	Berlin	29.1	Campania	30.4	Campania	27.3
55	Thüringen		Campania	18.4	Campania	25.4	Sicilia	26.1	Sicilia	26.2
56	La Rioja		Bruxelles	17.0	Sicilia	25.2	Melilla	25.8	Ceuta	26.0
57	Ceuta		Ceuta	15.0	Ceuta	19.9	Bruxelles	18.3	Bruxelles	17.4
58	Melilla		Melilla		Bruxelles	17.7	Ceuta	14.8	Melilla	

Table 21, Ranking of Social Inclusion Measure, Baseline model with expert-based normalisation, years 2009-2012

rank	region	2009	region	2010	region	2011	region	2012
1	Baden-W	88.3	Baden-W	88.4	Vlaams	89.8	Baden-W	89.5
2	Vlaams	87.7	Vlaams	88.0	Baden-W	89.1	Bayern	87.4
3	Bayern	86.0	Bayern	86.9	Bayern	86.7	Hessen	84.9
4	Liguria	84.8	FVG	83.7	ER	85.8	Veneto	81.2
5	Umbria	81.1	ER	83.2	Umbria	84.2	ER	81.0
6	Hessen	81.0	Umbria	81.6	Hessen	83.6	FVG	79.1
7	ER	80.8	Hessen	81.4	TAA	83.4	Hamburg	75.9
8	TAA	79.7	País Vasco	81.1	Navarra	82.8	TAA	75.1
9	FVG	79.0	Navarra	79.3	FVG	82.3	Umbria	75.0
10	País Vasco	78.3	TAA	78.9	Marche	79.8	Schleswig-H.	75.0
11	Marche	76.7	Marche	78.2	Lombardia	75.9	Navarra	74.9
12	Hamburg	76.0	Veneto	77.4	Liguria	75.5	Rheinland	74.7
13	Veneto	76.0	Liguria	75.4	Veneto	74.9	Thüringen	74.4
14	Toscana	75.2	Hamburg	75.2	País Vasco	74.1	Lombardia	74.3
15	Schleswig-H.	75.0	Schleswig-H.	73.2	Schleswig-H.	74.0	Nieders'n	73.7
16	Lazio	73.2	Toscana	71.8	Hamburg	73.5	País Vasco	71.5
17	Navarra	71.9	Lombardia	70.9	Thüringen	71.9	VDA	70.5
18	Rheinland	71.0	Thüringen	69.3	Rheinland	71.2	Saarland	70.0
19	Nieders'n	70.9	Saarland	69.3	VDA	69.5	Nordrhein	69.6
20	Lombardia	67.1	Lazio	68.8	Toscana	69.1	Sachsen	69.2
21	Nordrhein	66.9	Nieders'n	68.8	Brand'burg	69.1	Brand'burg	68.8
22	VDA	66.2	VDA	67.7	Piemonte	68.2	Toscana	68.4
23	Piemonte	64.8	Brand'burg	67.6	Nieders'n	68.1	Marche	68.3
24	Thüringen	64.7	Rheinland	66.6	Nordrhein	67.1	Liguria	67.4
25	Aragón	63.8	Piemonte	66.4	Sachsen	64.6	Piemonte	62.2
26	Saarland	62.8	Nordrhein	64.5	Abruzzo	63.3	Lazio	61.1

rank	region	2009	region	2010	region	2011	region	2012
27	Abruzzo	62.2	Abruzzo	64.2	Saarland	62.4	Bremen	60.5
28	Brand'burg	61.7	Molise	60.5	Lazio	59.2	Molise	60.1
29	Cantabria	61.5	Sachsen	59.8	Meckl'burg	57.2	Abruzzo	57.4
30	Madrid	60.4	Bremen	55.5	Molise	56.2	Cantabria	55.3
31	Sachsen	57.7	Meckl'burg	52.6	Berlin	51.4	Meckl'burg	54.5
32	Bremen	55.4	Sachsen-A.	51.4	Wallonne	50.2	Berlin	53.6
33	Basilicata	55.4	Madrid	51.2	Bremen	49.0	Basilicata	43.4
34	Cataluña	54.8	Aragón	50.5	Sachsen-A.	48.6	Madrid	38.9
35	Asturias	54.7	Wallonne	48.9	Asturias	46.6	Castilla L.	35.7
36	Balears	53.9	Asturias	48.2	Madrid	45.1	Aragón	35.1
37	La Rioja	53.5	Cantabria	46.0	Basilicata	45.1	La Rioja	34.8
38	Castilla L.	52.4	Galicia	45.5	Aragón	43.0	Cataluña	33.6
39	Galicia	51.8	Sardegna	45.2	Cantabria	42.3	Galicia	32.9
40	Molise	50.9	La Rioja	45.0	Castilla L.	37.8	Asturias	32.2
41	Wallonne	49.2	Calabria	44.1	Galicia	36.8	Calabria	29.8
42	Meckl'burg	47.6	Basilicata	43.8	Balears	35.5	Sardegna	27.4
43	Valenciana	46.1	Castilla L.	43.6	La Rioja	34.7	Puglia	26.3
44	Sachsen-A.	44.8	Berlin	43.5	Calabria	34.4	Castilla M	25.8
45	Castilla M	44.3	Cataluña	40.1	Cataluña	32.8	Valenciana	23.5
46	Berlin	43.1	Balears	38.5	Puglia	32.7	Balears	23.5
47	Calabria	41.6	Castilla M	33.4	Sardegna	32.1	Murcia	23.0
48	Murcia	40.8	Puglia	33.4	Valenciana	26.3	Canarias	22.8
49	Extremad.	37.2	Extremad.	26.3	Castilla M	25.8	Extremad.	22.5
50	Sardegna	36.9	Murcia	25.7	Sicilia	25.3	Sicilia	21.8
51	Puglia	36.3	Sicilia	25.4	Extremad.	23.3	Andalucía	21.0
52	Bruxelles	31.4	Melilla	24.5	Murcia	23.3	Campania	20.3
53	Andalucía	30.5	Valenciana	23.7	Bruxelles	22.8	Ceuta	20.0
54	Canarias	28.8	Campania	23.3	Canarias	22.5	Melilla	19.5
55	Campania	26.2	Canarias	23.3	Andalucía	21.0	Bruxelles	
56	Sicilia	24.3	Bruxelles	22.5	Campania	20.0	Vlaams	
57	Ceuta	16.6	Andalucía	20.5	Melilla	19.0	Wallonne	
58	Melilla		Ceuta	15.8	Ceuta	17.5	Sachsen-A.	

Figure 12, Time trends in Germany and Italy

Appendix E SUMMARY RESULTS, GEOMETRIC AND HARMONIC MODELS

Table 22, Social Inclusion, geometric aggregation GD (data-driven normalisation) and coefficients of variation

	2004	2005	2006	2007	2008	2009	2010	2011	2012
Belgium	52.8	53.9	60.8	64.5	64.7	67.4	68.2	70.4	
Germany		60.0	63.7	66.6	68.6	70.0	70.9	73.4	75.3
Spain	56.5	58.5	62.8	62.8	62.7	63.0	61.7	60.8	57.9
Italy	65.8	65.8	68.9	70.2	71.5	72.1	73.3	71.8	70.0
COV regions BE	0.34	0.33	0.24	0.21	0.23	0.19	0.19	0.21	
COV regions DE		0.18	0.16	0.13	0.13	0.12	0.11	0.10	0.09
COV regions ES	0.21	0.27	0.26	0.25	0.26	0.22	0.24	0.24	0.32
COV regions IT	0.19	0.19	0.18	0.18	0.17	0.17	0.16	0.19	0.19
COV all regions		0.25	0.23	0.21	0.21	0.19	0.19	0.20	

Table 23, Social Inclusion, harmonic aggregation HD (data-driven normalisation) and coefficients of variation

	2004	2005	2006	2007	2008	2009	2010	2011	2012
Belgium	41.2	43.1	52.2	57.9	57.3	61.9	63.9	66.8	
Germany		53.6	59.2	62.6	64.7	66.3	67.7	71.2	73.3
Spain	53.9	55.6	60.6	60.3	60.3	61.6	61.0	60.0	56.0
Italy	64.3	64.2	67.4	68.7	70.4	71.0	72.5	70.3	68.9
COV regions BE	0.58	0.56	0.37	0.30	0.33	0.25	0.23	0.24	
COV regions DE		0.24	0.20	0.17	0.16	0.15	0.14	0.12	0.10
COV regions ES	0.24	0.31	0.29	0.29	0.31	0.24	0.25	0.25	0.36
COV regions IT	0.20	0.20	0.20	0.20	0.18	0.19	0.17	0.22	0.21
COV all regions		0.32	0.28	0.26	0.25	0.22	0.22	0.22	

Table 24, Social Inclusion, geometric aggregation GS (survey-driven normalisation) and coefficients of variation

	2004	2005	2006	2007	2008	2009	2010	2011	2012
Belgium	61.3	59.9	62.0	63.0	63.3	66.2	65.6	66.5	
Germany		52.4	54.7	60.1	65.1	68.3	68.7	69.3	71.8
Spain	19.9	20.3	21.5	21.6	20.5	20.7	16.5	13.9	11.3
Italy	31.6	33.8	42.3	46.6	43.1	41.1	47.4	47.3	45.6
COV regions BE	0.45	0.52	0.52	0.53	0.54	0.45	0.51	0.51	
COV regions DE		0.48	0.45	0.40	0.35	0.27	0.23	0.25	0.27
COV regions ES	0.79	0.85	0.96	0.87	0.84	0.74	1.26	1.57	1.88
COV regions IT	0.69	0.59	0.52	0.52	0.55	0.59	0.51	0.54	0.54
COV all regions		0.55	0.55	0.55	0.55	0.53	0.56	0.59	

Table 25, Social Inclusion, harmonic aggregation HS (survey-driven normalisation) and coefficients of variation

	2004	2005	2006	2007	2008	2009	2010	2011	2012
Belgium	59.5	58.6	60.6	61.1	61.0	63.6	64.0	64.0	
Germany		48.8	51.2	56.7	61.8	65.0	65.4	65.0	67.1
Spain	6.4	6.8	7.4	7.0	6.8	6.3	7.1	7.1	6.4
Italy	19.6	21.9	32.7	38.3	33.3	30.2	39.4	40.5	40.3
COV regions BE	0.52	0.54	0.54	0.55	0.56	0.53	0.54	0.55	
COV regions DE		0.61	0.58	0.51	0.45	0.35	0.31	0.34	0.38
COV regions ES	2.68	2.61	3.07	2.77	2.48	2.39	2.95	3.20	3.30
COV regions IT	1.19	0.93	0.71	0.67	0.76	0.87	0.67	0.69	0.64
COV all regions		0.76	0.74	0.72	0.73	0.73	0.69	0.71	

Table 26, Average Kendall's τ correlation coefficients between Linear, Geometric and Harmonic specifications: data-driven vs survey- normalisation

	Linear survey-driven	Geometric survey-driven	Harmonic survey-driven
Linear data-driven	0.76	0.56	0.53
Geometric data-driven	0.70	0.51	0.48
Harmonic data-driven	0.63	0.45	0.42