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investigation in
stressed markets, extended
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Abstract

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Keywords

Hedge funds, Tail risk, Diversification, Marginal risk contribution

JEL Codes

G11, G29

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Hedge Fund Tail Risk: An investigation in stressed markets, extended version with appendix *

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November 7, 2016

Abstract

This paper develops several risk measures that captures the tail risk of single hedge fund strategies and the tail risk contribution of these hedge fund strategies to the overall portfolio tail risk, conditional on the level of market distress. We show that, during the recent global financial crisis, all the different hedge fund strategies are contributing to the tail risk of the portfolio of hedge funds, mostly because of the hedge fund strategies' exposure to liquidity and credit risk.

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1 Introduction

The financial industry has witnessed several crises in the past, some of which have had a major global impact. Although hedge funds are expected to have the capability to at least avoid the impact of such crises by hedging against market movements, the LTCM crisis in 1998 and the global meltdown in 2008 showed otherwise. In both cases, most hedge funds suffered from large losses. In fact, almost all Dow Jones Credit Suisse hedge fund indexes experienced huge losses. These systemic losses of hedge funds have been associated with mostly large, simultaneous liquidation by market participants, causing a liquidity freeze that crippled hedge funds and also spilled over to the general economy (see Brown et al. (2009), Buraschi et al. (2014) and for a Lo et al. (2015)). The experiences from these crises motivate investors, academics and also regulators to better understand systematic risk in hedge funds and these funds' contribution to the overall risk of a portfolio (see Billio et al. (2012a)).

In this study, we concentrate on the risk of a portfolio of hedge funds, from the investor's point of view. In this respect, our paper is generally related to Brown et al. (2012), who investigate over-diversification in funds of hedge funds that could generate tail risk and Heuson et al. (2014), who investigate hedge fund performance when returns are skewed. As funds of hedge funds primarily invest in hedge funds, it is important to develop a risk measurement technique that captures the risk exposure of the individual hedge funds and the risk contribution of these hedge funds toward the overall portfolio risk, especially in light of the 1998 and 2008 hedge fund crises. As documented by several studies, hedge funds follow dynamic trading strategies. As a result, the risk exposures and thus the risk contributions of hedge funds are expected to change significantly over time. To capture the dynamic nature of hedge funds, we apply a Markov regime-switching approach, which allows us to estimate time- and state-dependent risk exposures.

There are several ways to measure the risk of a financial asset or portfolio. The common ones include volatility, semi-variance, value-at-Risk and Expected Shortfall. Different studies suggest the use of one or the other of these tools to measure the risk of financial assets and make optimal portfolio

choices. The mean-variance framework of Markowitz (1952) uses volatility (or variance) to understand the risk and select an optimal portfolio. Others, like Roy (1952), suggest a safety first approach, which accounts for the possibility of surprises, that is, returns below a certain, acceptable threshold. These ideas of using downside risk measures are developed further by Bawa (1978), and Fishburn (1977) suggests semi-variance and other partial moments as alternative risk measures. Currently, value-at-risk (VaR) and expected shortfall (ES) are the popular measures of downside/tail risk, VaR being the most widely used. While VaR is used to determine the capital requirements of banks and other financial institutions, it is shown to have some faults, including failing to be coherent. As a result, the academic literature seems to favor ES over VaR. See Artzner et al. (1999) and Inuia and Kijima (2005).

These measures provide different pieces of information about the risk of a portfolio and the contribution of the different hedge fund strategies to the risk of a portfolio. Measuring the risk using these different measures helps one to better diversify the portfolio with respect to “normal” and “extreme events”. It is in fact the case that the contributions of an asset to the portfolio volatility and to the tail risk are not necessarily the same when the asset’s returns are not normally distributed.

The contribution of an individual asset to the portfolio risk can be calculated in two different ways. The first is by taking the difference between the portfolio risk measures computed with and without the asset. A second, and more useful, way is to compute the marginal contribution of individual assets to the portfolio risk, by calculating the first derivative of a risk measure with respect to the portfolio weight of the asset. Examples of literature following this second approach are Scaillet (2004), Fermanian and Scaillet (2005), Hallerbach (2003), Hans (2004), and Litzenberger and Modest (2008). Scaillet (2004) and Fermanian and Scaillet (2005) derive the marginal ES as a linear function of the derivatives and propose a nonparametric estimation based on empirical averages or kernel methods. Litzenberger and Modest (2008) more specifically propose a measure for determining the percentage marginal contribution of individual hedge funds to the ES of a fund of hedge funds. They use a mixture of normal distributions with subjectively chosen parameters, subjectively predefined types of possible crises and fixed (subjective) probabilities of these crises to calculate marginal VaR. The idea of marginal VaR is in line with the systemic risk measure proposed by Adrian and Brunnermeier (2010) in their conditional value-at-risk (CoVaR) measure.

Our work shares this spirit and suggests a methodology for estimating all the parameters based on Markov's regime-switching approach.¹

In summary, our work is perfectly in line with the spirit of Litzenberger and Modest (2008) and, rather than leaving to the risk controller the task of exogenously determining the states of nature and the probabilities of these states, we suggest the use of a Markov regime-switching approach, which allows one to estimate time- and state-dependent risk exposures, as well the state probabilities of the different risk factors. On top of the marginal VaR, we also calculate the marginal variance and the marginal ES. This last measure has been proposed by Acharya et al. (2010) for measuring systemic risk for financial institutions. Our model has the following desired features: (i) it can accurately predict portfolio volatility during normal, non-crisis times, (ii) it captures in a realistic way stress moves during crisis periods, (iii) it is consistent with the empirical observation that returns in many financial markets are characterized by distributions with fat left tails, (iv) it provides a unified framework for comparing trades whose risk may be composed of different proportions of typical day-to-day non-crisis volatility and crisis risk, and (v) it allows one to estimate time- and state-dependent risk exposures as well, as the state probabilities of the different risk factors, in a coherent and objective way.

We demonstrate the performance of our approach by considering an equally weighted portfolio composed of eight hedge fund strategy indexes from the Dow Jones Credit Suisse Hedge Fund Database. We study the hedge fund strategies' marginal contributions to the volatility, VaR and ES of this portfolio through an out-of-sample analysis of a period of 84 months (from January 2005 to December 2011).

The main result of our analysis is that the contribution to the tail risk of a fund of hedge funds is only partially related to the main systematic risk measure identified in the portfolio diversification literature: the market portfolio. Using the S&P500 as a proxy for the market portfolio, we show that, even in a dynamic framework in which the exposure to the market risk factor is changing over time, the contribution of this exposure to the tail risk of a fund of hedge fund portfolios is rather marginal, and mostly appears during crisis periods. Other factors largely affect the tail risk of a fund of

¹For further discussion on regime-switching models characterized by Markov chains, please see Hamilton (1994), Billio and Pelizzon (2000), Billio et al. (2012b), Billio et al. (2014)

hedge fund portfolios, such as the liquidity risk and credit risk. The effects of these other factors are so strong that another strategy, dedicated short bias, which systematically takes a negative position on the equity market, is not able to hedge, or at least not fully, the tail risk of the portfolio of hedge funds during a crisis, but does contribute along with all the other strategies to the tail risk, showing that the Dedicated Short Bias strategy does not work as a hedging instrument when it is most needed.

This study is important in several ways. Previous studies, including Tasche (1999), Gouriéroux et al. (2000), Hans (2004), Scaillet (2004), Fermanian and Scaillet (2005) and Litzenberger and Modest (2008), show that marginal risk contribution can be used in portfolio selection models. Crucially, marginal contribution analysis shows that high risk in individual assets does not necessarily mean a high contribution toward the portfolio risk, but in general the asset dependence structure matters, as several assets could be negatively correlated, leading to a diversification benefit. This is even more relevant if we consider a nonlinear dependence structure, such as that implied by the Markov switching, as the asset correlation could change over time. It is thus very useful to look at the marginal risk contributions of individual hedge funds to a portfolio instead of only the risk measures for individual hedge funds. That is, the marginal volatility, marginal VaR or marginal ES contributions of individual hedge funds tell us more than can be obtained from looking simply at the single hedge fund volatility, VaR or ES respectively.

Even though we concentrate on portfolios of hedge funds when discussing the methodology and results, the study can be extended to the analysis of risk contributions in any portfolio, or even to the general economy. There are several recent studies showing how marginal ES can be used for measuring systemic risk. Acharya et al. (2010) show that marginal ES is a good proxy for an institution's contribution to systemic risk. Following them, Brownlees and Engle (2010) develop a methodology for estimating marginal ES. While the latter employ a method of estimating volatility, correlation and tails separately so as to use them later in the marginal ES estimation, our study estimates the whole distribution for each of the individual strategy indexes and the portfolio. The actual estimation in our study is performed in several steps, which allowed us to minimize the estimation time. Our work shares with all the previous studies the fact that the marginal VaR and marginal ES are calculated conditional on certain risk factors.

The paper is organized as follows. Section 2 presents the different marginal

contribution risk measures and the methodology to determine them. Section 3 reports the empirical analysis applied to a portfolio of hedge funds. Section 4 concludes.

2 Marginal Risk Contribution Measures

We consider three risk measures: (i) Volatility, (ii) VaR, and (iii) ES. Volatility is usually defined as the standard deviation conditional on past information and is a classical measure of dispersion around the mean. Volatility is by definition symmetric and thus it is not capable of capturing any asymmetric aspect of risk. This makes volatility more useful during normal market conditions, while it is less helpful during crises when the market is far below the mean. Common risk measures used to capture downside risk exposure are VaR and ES. The VaR is theoretically defined as the maximum amount of loss that can occur over a given horizon at a certain confidence level. The alternative suggested in the literature is the ES, also known as the Expected Tail Loss (ETL), which overcomes some of the weaknesses of VaR. The ES represents the expected loss when it exceeds the VaR. In other words, it is the expected return conditional on the return being lower than the VaR. For an analytical description of these measures see the Appendix.

We use the regime-switching models detailed in the Appendix to estimate and forecast these risk measures. We propose a general method for computing three marginal risk measures: marginal volatility, marginal VaR and marginal ES. These marginal risk measures are used to estimate the marginal effect of a component of the portfolio P . More specifically, we apply it to estimate the marginal effect of single strategies in a portfolio of hedge funds.

To compute the marginal risk contribution of strategy i , we construct a new portfolio P' , which consists of an amount w_i invested in strategy i , and an amount $(1 - w_i)$ invested in the original portfolio P . Note that w_i is an incremental weight in a strategy that is already a component in the portfolio P . Then, by differentiating with respect to w_i at $w_i = 0$, we can easily account for all the marginal effects of strategy i , either directly through strategy i or through the resulting change in the weight of the remaining strategies in the portfolio. The return on the new portfolio P' is obtained using

$$r_{P',s} = w_i r_{i,s} + (1 - w_i) r_{P,s} \quad (1)$$

Clearly, the state-dependent mean and volatility of the new portfolio are given by:

$$\mu_{P',s} = w_i \mu_{i,s} + (1 - w_i) \mu_{P,s} \quad (2)$$

$$\sigma_{P',s} = \sqrt{w_i^2 \sigma_{i,s}^2 + (1 - w_i)^2 \sigma_{P,s}^2 + 2w_i(1 - w_i) \sigma_{iP,s}} \quad (3)$$

where $\sigma_{iP,s}$ is the state-dependent covariance between strategy i and portfolio P . We know that the covariance $\sigma_{iP,s}$ can be expressed in terms of the correlation $\rho_{iP,s}$ between portfolio P and strategy i and the volatilities of the strategy and portfolio, i.e. $\rho_{iP,s} \sigma_i \sigma_P$. Thus, replacing σ_{iP} with $\rho_{iP,s} \sigma_i \sigma_P$ and differentiating with respect to w_i at $w_i = 0$, we get the marginal volatility (labeled $M\sigma$) as follows:

$$M\sigma_{P,s} = \left[\frac{\partial \sigma_{P',s}}{\partial w_i} \right]_{w_i=0} = \rho_{iP,s} \sigma_i - \sigma_{P,s} \quad (4)$$

Moreover, the state-dependent ES of the portfolio P can be defined as:

$$ES_{P'}(\alpha, h) = \frac{1}{\alpha} \sum_{s \in S} p(s_{t+h}/I_t) \left[\mu_{P',s} \Phi \left(\frac{VaR_{P'}(\alpha, h) - \mu_{P',s}}{\sigma_{P',s}} \right) - \sigma_{P',s} \phi \left(\frac{VaR_{P'}(\alpha, h) - \mu_{P',s}}{\sigma_{P',s}} \right) \right] \quad (5)$$

Again, by differentiating this equation with respect to w_i at $w_i = 0$, we get the marginal ES (labeled MES^2):

$$MES = \left[\frac{\partial ES_{P'}}{\partial w_i} \right]_{w_i=0} = \frac{1}{\alpha} \sum_{s \in S} p(s_{t+h}/I_t) \left[\frac{\partial \mu_{P',s}}{\partial w_i} \Phi(z_s) + \mu_{P',s} \frac{\partial \Phi(z_s)}{\partial w_i} - \frac{\partial \sigma_{P',s}}{\partial w_i} \phi(z_s) - \sigma_{P',s} \frac{\partial \phi(z_s)}{\partial w_i} \right]_{w_i=0} \quad (6)$$

where $z_s = \frac{VaR_{P'} - \mu_{P',s}}{\sigma_{P',s}}$ is the standardized VaR for the portfolio P' .

With the same procedure, the marginal VaR (labeled $MVaR$) can be obtained as follows:

$$MVaR = \left[\frac{\partial VaR_{P'}}{\partial w_i} \right]_{w_i=0} = \frac{\sum_{s \in S} p(s_{t+h}/I_t) \frac{\phi(z_s)}{\sigma_{P',s}} [(\mu_{i,s} - \mu_{P,s}) - z_s(\rho_{iP,s} \sigma_{i,s} - \sigma_{P,s})]}{\sum_{s \in S} p(s_{t+h}/I_t) \frac{\phi(z_s)}{\sigma_{P',s}}} \quad (7)$$

²This derivation is similar to Litzenberger and Modest (2008) derivation of the marginal contribution to the ETL.

On the appendix we describe in details the derivation of the above equations and how to obtain closed-form solutions for all three risk measures ($M\sigma$, $MVaR$ and MES).

3 Empirical Results

3.1 Data

We consider monthly aggregate hedge fund strategy index Returns from the Dow Jones Credit Suisse Hedge Fund Database from January 1994 to December 2011. This database tracks approximately 8000 hedge funds, which fall into ten different strategies according to their investment style. The Dow Jones Credit Suisse hedge fund indexes are asset-weighted indexes of funds with a minimum of \$50 million of assets under management, a minimum one-year track record, and current audited financial statements. Currently, more than 400 funds are used to obtain these indexes. An aggregate index is computed from this universe, and ten sub-indexes based on investment style are also computed using a similar method. Indexes are updated and rebalanced at a monthly frequency and the universe of funds is redefined on a quarterly basis. The performance data used in the indexes are net of all fees. Moreover, the database accounts for survivorship bias in hedge funds (Fung and Hsieh (2000)). New funds are included on a “going-forward” basis only, removing instant history bias. The Dow Jones Credit Suisse indexes also include funds that are closed to new investments, which reduces selection bias. The indexes are computed as to get a maximal representation of their universe, with at least a 85% coverage. Finally, the database tracks hedge funds from all over the world. All of the above makes the Dow Jones Credit Suisse indexes a good proxy to use in studying hedge funds. In this paper we concentrate on the eight equity-related strategy indexes. The detailed definition of the risk factors in the factor model used, selected according to the literature, is reported in Table 1. Table 2 provides summary statistics for these strategies and for the risk factors.

The first part of the Table 2 shows that most of the hedge fund strategy indexes have significant negative skewness and excess kurtosis. This rules out the possibility of using a Gaussian distribution to describe strategy returns. This is in line with the dynamic nature of hedge fund strategies and returns, which are extensively discussed in the literature.

Table 1: Risk Factors Definitions

Risk Factor	Definition
S&P500	Monthly return on the Standard and Poor's Composite Index including dividends
Large-Small	Monthly return differences between Russell 2000 and Russell 1000 indexes
Value-Growth	Monthly return differences between Russell 1000 Value and Growth indexes
USD	Monthly return on Bank of England Value-Weighted US\$ Index
Barclays Government Credit	Monthly return on Barclays US Aggregate Government/Credit Index
Term Spread	The difference between the yields of the 10-year Treasury bond and 3-month Treasury bill
Credit Spread	The difference between Moody's US Corporate Bond indexes for BAA and AAA
MSCI Em. Stock	Monthly return on MSCI Emerging Markets Stock Index
Barclays Em. Bond	Monthly return on Barclays Emerging Markets Bond Index
UMD	Momentum Factor from Prof. Kenneth French's website
Change in VIX	Monthly Change in CBOE's VIX Index

Definitions of risk factors included in the multi-factor regime-switching model. All data, except for UMD, are obtained from Datastream. UMD, the momentum factor, is obtained from Ken French's website.

Table 2: Summary Statistics

Strategy Indexes	Mean	St. Dev	Median	Skewness	Kurtosis	Minimum	Maximum
Convertible Arbitrage	0.37	2.00	0.65	-2.55	15.46	-12.63	5.80
Dedicated Short Seller	-0.46	4.90	-0.84	0.64	1.37	-11.29	22.31
Emerging Markets	0.44	4.33	1.05	-0.81	5.06	-23.43	16.04
Equity Market Neutral	0.21	2.97	0.40	-11.99	164.07	-40.45	3.65
Long Short Equity	0.53	2.86	0.66	-0.06	3.21	-11.83	12.58
Distressed	0.58	1.91	0.84	-2.18	11.35	-12.85	4.14
Event-Driven MS	0.47	1.98	0.61	-1.75	7.80	-11.92	4.77
Risk Arbitrage	0.29	1.18	0.32	-1.09	5.27	-6.55	3.38
Risk Factors							
S&P500	0.56	4.56	0.96	-0.59	0.84	-16.69	11.25
Large-Small	0.05	3.24	-0.08	0.17	4.63	-15.71	16.78
Value-Growth	-0.06	3.19	-0.09	0.13	4.38	-12.51	14.05
USD	-0.36	2.01	-0.30	0.08	0.31	-5.88	5.67
Barclays Gov Credit	0.26	1.28	0.28	-0.10	1.16	-4.27	4.52
Term Spread	1.73	1.18	1.72	-0.06	-1.20	-0.63	3.78
Credit Spread	0.97	0.47	0.85	3.05	11.21	0.53	3.43
Barclays Emerging Bond	0.61	3.80	1.09	-2.38	13.78	-25.96	9.58
MSCI Emerging Stock	0.49	7.12	0.52	-0.69	1.71	-29.31	17.13
UMD	0.45	5.52	0.60	-1.56	9.70	-34.75	18.40
Change in VIX	0.04	4.52	-0.10	0.82	4.84	-15.28	20.50

Summary statistics for the eight hedge fund indexes considered as well as for the risk factors used to explain their systematic risk exposure. A brief definition of the risk factors can be found in Table 1.

The second part of the Table 2 shows that the Gaussian assumption is also rejected for most of the risk factors. This necessitates the use of a nonlinear model, such as the Markov regime-switching model suggested here.

3.2 Hedge Fund Strategies

Our complete sample contains 216 observations for each strategy. Our estimation procedure uses expanding windows to estimate the parameters. That is, we start by using the first 132 observations (January 1994 - December 2004) to estimate the parameters for December 2004 and, when necessary, to compute the out-of-sample forecasts for the subsequent month. We repeat the process, each time including one additional observation, until we reach the end of the sample, thus using all the available data. Through such a process we are able to estimate and forecast for 84 months (January 2005 to December 2011). Later in the paper we refer to this period as the estimation period.

For each strategy index, we estimate two different models. The first one, which we call S&P-RSM (S&P regime-switching model), uses the S&P500 composite index as the only risk factor on the right hand side of the equation. The second model, called MF-RSM (multi-factor regime-switching model), includes several additional risk factors. Details of the estimation procedure are available upon request. Using the S&P-RSM model, we examine the possibility of estimating the marginal risk contributions of individual strategies by considering only their conditional distributions with respect the equity market risk factor. An example of the estimated parameters are presented in Table 3 where we use the whole sample. For this specific case we observe that the S&P500 coefficients are significant for all the strategies except for the Convertible Arbitrage strategy in both regimes and the Equity Market Neutral strategy in the normal market regime³.

The results in Table 3 seem to suggest that the only hedge fund strategy that could contribute to a reduction of risk in the portfolio is the Dedicated Short Bias since it has a negative exposure with respect to the market and therefore potentially a negative correlation with the other strategies. A sim-

³While the second case is in line with the strategy of Equity Market Neutral funds, the case of the Convertible Arbitrage gives the wrong idea that the strategy is independent from the equity market. However, the estimated market exposure for this strategy varies over time and is mostly significant, at least in one of the regimes, specifically the normal market regime in the expanding window estimation.

ilar argument could be applied for the Convertible Arbitrage strategy and the Equity Market Neutral. Therefore, by looking to the exposure to the S&P500 we could identify strategies that are natural hedgers to the risk of the portfolio, at least for its variance risk.

The picture changes when we include additional risk factors in the model, as reported in Table 4. All hedge fund strategies have significant exposure to the S&P500 in both normal and down market regimes, except for the Emerging Market and Equity Market Neutral strategies.⁴

The results reported in Table 4 are extremely important because they highlight that the different hedge fund strategies do present common exposures to the different risk factors. In particular, the Dedicated Short Bias strategy has a negative exposure to the credit risk factor when the S&P500 is in the down market regime. This negative exposure is similar in sign and significance to the one of the Emerging Market strategy and the Distressed strategy. Therefore, the ability of the Dedicated Short Bias strategy to hedge the risk of the portfolio seems to be debatable. In the analysis, detailed in the next section, we show that this common exposure to credit and liquidity risk factors largely reduces the natural ability of some hedge fund strategies to be hedgers to the total portfolio risk, at least during crisis periods.

⁴While the case of the Equity Market Neutral strategy is perfectly in line with our a priori understanding of the strategy, the non-significance of the Emerging Market strategy exposure could seem mind-boggling. However, this can be explained by the fact that we are now including the Emerging Markets Stock Index.

Table 3: Parameter Estimates (S&P-RSM)

Strategy/ Variable	Convertible Arbitrage		=		Emerging Markets		Equity Market Neutral		Long/Short Equity		Distressed		Event-Driven MS		Risk Arbitrage	
	Esti- mate	t-stat	Esti- mate	t-stat	Esti- mate	t-stat	Esti- mate	t-stat	Esti- mate	t-stat	Esti- mate	t-stat	Esti- mate	t-stat	Esti- mate	t-stat
α_0	0.66	6.22	0.41	1.65	0.89	4.12	0.50	8.07	0.18	1.21	0.65	6.41	0.66	6.52	0.25	3.19
α_1	-1.54	-2.49	-0.60	-0.22	-0.66	-1.13	0.24	1.10	0.77	0.37	-3.04	-2.41	-0.94	-1.84	0.01	0.02
β_0	0.07	1.22	-1.37	-7.87	0.49	3.46	0.02	0.57	0.66	6.55	0.36	5.11	0.34	4.43	0.20	4.11
β_1	0.03	1.21	-0.74	-11.40	0.42	7.06	0.07	4.19	0.34	5.79	0.13	4.21	0.15	5.44	0.07	3.20
$(S\&P500)$																
ω_0	0.75	6.77	2.77	14.95	1.86	11.37	0.58	13.04	1.61	11.16	1.13	17.01	0.99	9.80	0.63	8.78
ω_1	3.05	7.53	6.85	2.78	5.18	11.96	1.01	7.38	6.02	3.67	2.88	4.43	2.39	8.34	1.65	7.47
π_{00}^Z	0.93	25.29	0.94	15.23	0.98	57.73	0.98	48.67	0.98	56.07	0.98	69.71	0.97	38.12	0.87	14.14
π_{11}^Z	0.78	8.15	0.00	0.00	0.99	63.28	0.97	19.01	0.77	5.19	0.73	4.48	0.90	12.41	0.72	5.94

Quasi maximum likelihood estimates (QMLE) of the S&P-RSM model, described in the Appendix , with only the S&P500 Composite Index as a risk factor.

$$r_{it} = \alpha_i(Z_t^i) + \beta_i(S_t)M_t + \omega_i(Z_t^i)\epsilon_t$$

where i denotes individual strategies, r_{it} is the return on strategy i , M_t is the return on the market index, and α_i , β_i and ω_i are respectively the state-dependent constant term, market beta and idiosyncratic volatility of strategy i . We consider two Markov chains. The first, S_t , is estimated using the S&P500 Composite Index and it captures the high volatility (down market) and low volatility (up/normal market) regimes of the equity market. The second Markov chain Z_t^i captures two idiosyncratic regimes characterized by low and high volatility. Parameters π_{00}^Z and π_{11}^Z are the transition probabilities for the idiosyncratic Markov chain Z_t^i . The sample period is January 1994 to December 2011.

Table 4: Parameter Estimates (MF-RSM)

Strategy/ Variable	Convertible Arbitrage		Dedicated Short Bias		Emerging Markets		Equity Market Neutral		Long/Short Equity		Distressed		Event- Driven MS		Risk Arbitrage	
	Esti- mate	t-stat	Esti- mate	t-stat	Esti- mate	t-stat	Esti- mate	t-stat	Esti- mate	t-stat	Esti- mate	t-stat	Esti- mate	t-stat	Esti- mate	t-stat
α_0	1.36	6.25	1.23	2.20	0.72	2.26	0.93	5.94	-0.04	-0.11	1.97	6.26	0.67	2.29	0.23	1.09
α_1	0.61	1.35	1.05	1.89	0.64	1.41	0.66	3.08	0.42	0.92	0.74	2.05	-0.74	-1.34	0.49	2.01
β_0 (S&P500)	0.13	2.75	-1.10	-8.74	-0.04	-0.30	0.05	0.92	0.64	8.30	0.28	4.70	0.34	3.99	0.23	4.48
β_1 (S&P500)	-0.06	-2.66	-0.80	-20.71	-0.06	-1.10	0.04	1.97	0.37	7.43	0.14	3.82	0.20	6.14	0.06	1.94
θ_{10} (Large-Small)			-0.80	-6.15			0.03	0.88	0.26	3.85	0.14	2.74	0.09	1.49	0.07	1.71
θ_{11} (Large-Small)			-0.42	-5.47			-0.02	-1.12	0.29	6.29	0.14	3.47	0.16	5.20	0.11	4.09
θ_{20} (Value-Growth)									0.19	2.11	-0.02	-0.26			-0.03	-0.50
θ_{21} (Value-Growth)					-0.02	-0.19	-0.11	-2.00	-0.13	-3.06	0.06	1.27	0.01	0.13	0.07	2.83
θ_{30} (USD)					0.15	2.03	-0.10	-2.40					-0.04	-0.64		
θ_{40} (Barclays Gov. Credit)	-0.16	-1.89									0.04	0.36				
θ_{41} (Barclays Gov. Credit)	0.21	2.78									0.03	0.29				
θ_{60} (Term Spread)	-0.28	-3.21					-0.04	-0.61	-0.21	-1.82						
θ_{61} (Term Spread)	-0.13	-1.20	-0.83	-0.99	-0.72	-1.58	-0.06	-0.69	-0.21	-1.25						
θ_{60} (Credit Spread)	-0.99	-3.05	-1.06	-2.58	-0.54	-2.65	-0.73	-3.11	0.50	1.00	-1.49	-3.45	0.09	0.21	-0.22	-0.72
θ_{61} (Credit Spread)	-0.19	-0.88			0.29	3.03	-0.23	-1.40	0.44	1.11	-0.51	-2.13	-0.06	-0.19	-0.10	-0.61
θ_{70} (Barclays Em. Bond)	0.05	1.36			0.25	4.46										
θ_{71} (Barclays Em. Bond)	0.10	2.64			0.39	7.68										
θ_{80} (MSCI Em. Stock)					0.38	9.39										
θ_{81} (MSCI Em. Stock)	-0.01	-0.30			0.09	1.53			0.01	0.19						
θ_{90} (Momentum Factor)	-0.04	-2.70			0.05	2.02			0.20	7.15						
θ_{91} (Momentum Factor)	0.04	0.81			0.20	2.37			0.16	2.35						
θ_{10} (Change in VIX)	-0.05	-1.47			-0.01	-0.16	-0.03	-1.58	-0.03	-0.70						
ω_0	0.52	9.06	1.21	5.74	0.99	15.90	0.54	13.21	1.01	15.29	0.60	6.69	0.89	14.68	0.63	9.73
π_0^Z	2.84	11.07	2.92	10.77	3.58	12.06	1.26	10.03	2.16	9.79	1.67	13.01	2.25	9.62	1.17	8.97
π_1^Z	0.86	21.36	0.91	11.16	1.00	167.49	0.98	72.74	0.98	82.23	0.90	20.81	0.96	51.95	0.97	43.12
π_{11}^Z	0.76	11.85	0.95	17.10	0.99	115.79	0.98	44.42	0.96	32.49	0.92	23.29	0.90	17.92	0.96	32.35

Quasi maximum likelihood estimates of the MF-RSM model, described in the Appendix, for each of the eight strategies.

$$r_{it} = \alpha_i(Z_t^i) + \sum_{j=1}^K \beta_{ij}(S_t)F_{jt} + \omega_i(Z_t^i)\epsilon_t$$

where i denotes individual strategies, r_{it} is the return on strategy i , F_{jt} are the risk factors defined in table 1, α_i and ω_i are respectively the constant term and idiosyncratic volatility of strategy i , and for each j , β_{ij} is the exposure of strategy i to the j th risk factor. We consider two Markov chains. The first, S_t , is estimated using the S&P500 Composite Index and captures the high volatility (down market) and low volatility (up/normal market) regimes of the equity market. The second Markov chain Z_t^i captures two idiosyncratic regimes characterized by low and high volatility. Parameters π_{00}^Z and π_{11}^Z are the transition probabilities for the idiosyncratic Markov chain Z_t^i . The sample period is January 1994 to December 2011.

3.3 Tail Risk of the Portfolio of Hedge Funds

In this section, we present the estimation of the strategies marginal risk contributions to an equally weighted portfolio, using regime-switching models.⁵ We consider a portfolio P that is composed of eight strategies with equal weights. Combining two states for each strategy and two for the market, the set of all possible states for this portfolio has dimension equal to $2^9 = 512$. The state-dependent means and volatilities of P are computed in the standard way from the state-dependent means and volatilities of the individual strategies. The $M\sigma$, $MVaR$ and MES of the strategies and the portfolio are computed as described in the Appendix.

Before proceeding with presenting the dynamics of the marginal contributions over time, we provide a general picture by presenting the average marginal risk contributions of each strategy over three different periods: the whole estimation period (January 2005 - December 2011), the period before the 2008 crisis (January 2005 - July 2008) and the period during and after the 2008 crisis (August 2008 - December 2011).

The results presented in Panel A of Table 5 show that over the whole estimation period the Emerging Market strategy makes the highest contribution to each of the three risk measures. This is in line with the risk measures in terms of single strategies: the Emerging Market strategy has the highest volatility, the highest VaR, and the highest ES. The strategy with the next highest volatility, VaR and ES, is the Dedicated Short Bias strategy. However, it makes the lowest marginal contribution, in terms of volatility (-20.41), VaR (-51.08) and ES (-53.36). This result can be explained by the fact that the Dedicated Short Bias strategy is designed so as to be negatively related to the other strategies, through its negative exposure to the market. In fact, Table 5 shows that the Dedicated Short Bias strategy makes negative contributions to all three portfolio risk measures. The interpretation of the negative contributions is that those strategies actually decrease the portfolio risk. For instance, a -20% volatility contribution by strategy X can be inter-

⁵We use this portfolio of hedge fund strategies to represent funds of hedge funds. It is obvious that it not straightforward to extend our findings directly to structured portfolios such as those managed by a fund of funds, mostly because funds of funds invest in individual hedge funds instead of indexes and have a different structure, with layers of fees etc. However, to the extent that strategy indexes represent the risk exposure of funds in the specific strategy, we can expect the portfolio to reflect the risk exposure of a fund of funds investing in the eight strategies we consider. Moreover, estimating such a model for each individual hedge fund would be computationally very intensive.

preted as meaning that the volatility of the portfolio is 20% less than it would be if strategy X were not included. That is, strategies with negative contributions generate great hedging benefits. This indicates that, despite having high idiosyncratic risk, such strategies help to reduce the risk when included in a portfolio with other strategies. From Panels B and C of Table 5 we can observe that this feature is shared by the Convertible Arbitrage, Dedicated Short Bias and Equity Market Neutral strategies. However, if we look at the three subsamples (pre-Lehman, Lehman meltdown and post-Lehman), this feature characterizes the Equity Market Neutral strategy only for the pre-Lehman period. In the Lehman meltdown and post-Lehman period, we have that the Equity Market Neutral strategy makes a positive marginal contribution to the variance of the portfolio of funds. The same applies, with a lower intensity, for the Convertible Bond Arbitrage strategy.

On the other hand, there is a striking difference between the $M\sigma$ and both the $MVaR$ and the MES over the whole estimation period: the Convertible Arbitrage and Equity Market Neutral strategies make negative contributions to the portfolio volatility and positive contributions to the portfolio tail risk ($MVaR$ and MES).

The complete results for the three sample periods are reported in Panels B, C and D of Table 5. The general picture of which strategy contributes the most or the least is the same as in the results for the whole period given in Panel A. However, the actual percentage contributions of each strategy differ between the three periods. Comparing the marginal contributions to portfolio volatility, we observe that, while the marginal contributions of the strategies that make positive contributions in the first period decrease in the second one, those of the strategies that make negative contributions in the first period show significant (in terms of percentage change) increases in the second period, with some contributions even switching sign. That is, those strategies that could be used as hedging instruments in the first period lose their hedging benefits and in some cases become risk contributors. This result suggests that, during and after the Lehman meltdown, all strategies systematically become greater risk contributors, with almost no exception: even the Dedicated Short Bias strategy largely reduces its hedging role when considering tail risk (from -49.41 to -14.97) for the MES measure. It would be difficult to capture this effect using the individual strategy risk measures.

In summary, this first analysis shows that there are some strategies, like Dedicated Short Bias, that contribute negatively to the risk of the overall portfolio of hedge funds. Some strategies, like Emerging Market, contribute

the most to the total portfolio risk. As a result, there are some strategies that are natural hedgers and some strategies that tend to amplify portfolio risk. The effect is qualitatively the same for all the three measure of marginal risk contribution but quantitatively largely different.

The second analysis we perform considers the other risk factors that may systematically affect hedge fund strategies. This analysis shows that the results are rather different. As shown in Table 4, other factors than the S&P500 affect hedge fund strategies. The inclusion of these factors in the picture greatly changes the tail risk contributions of the different hedge fund strategies to the portfolio. In particular, we can observe that the exposure to these other risk factors affects the tail risk of the portfolio more than the volatility. In fact, the contribution does not change significantly in terms of marginal standard deviation but does change greatly for the $MVaR$ and the MES . Looking at the Dedicated Short Bias strategy, for example, we can see that the ability to hedge portfolio risk reduces by one half and the Equity Market Neutral and Convertible Arbitrage strategies now positively contribute to the $MVaR$ and MES without providing any hedging benefit.

More specifically, the results presented in Panel A of Table 6 show that, over the whole estimation period, it is again the Emerging Market strategy that makes the highest contribution to all three risk measures. It is interesting to observe that, looking at the individual strategies, the Emerging Market strategy has the highest volatility (3.42) but, in terms of VaR and ES, the Dedicated Short Bias strategy has the higher tail risk (-7.03 and -9.45 respectively). While the Dedicated Short Bias strategy has the second-highest estimated volatility and the highest VaR and ES (-15.02 and -18.27 respectively), this strategy again makes lower marginal risk contributions to the portfolio over all the different samples considered. In fact, Table 6 shows that the Dedicated Short Bias strategy makes negative contributions to all three portfolio risk measures. This means that, for large sample periods, this strategy, even if exposed to various risk factors, is able to reduce the tail risk of the portfolio of hedge funds.

In summary, our analysis evidences that other risk factors and financial crisis times show that all hedge fund strategies tend to be positively exposed to liquidity and credit risk, and as such, all positively contribute to the total tail risk of the portfolio of hedge fund strategies. As a result, the natural ability of some hedge fund strategies to be hedgers to the total portfolio risk disappears during crisis periods. This is important especially during crisis periods, as investors seek diversification and hedging benefits from hedge

funds.

To complete the picture, we present the dynamics of the marginal risk contribution of each strategy over the period January 2005 to December 2011. Figure 1 presents the percentage marginal contribution of each strategy to the portfolio volatility, estimated using the S&P-RSM and MF-RSM models, in Panels A and B respectively. These results show that the Emerging Markets strategy is the highest contributor to the equally weighted portfolio risk throughout the whole period. This is not surprising given that this strategy has the highest estimated volatility among the group of strategies.

Moreover, the Dedicated Short Bias strategy contributes towards reducing the volatility of the portfolio for almost all months in the estimation period, with the exception of a few months in 2009. There are also other strategies that make slightly negative contributions for most of the sample period. Figures show that these negative contributions are minimal for the one-year period from September 2008 to September 2009. This is an important result, indicating that, since most hedge funds (independently of their strategy) were affected by the liquidity freeze, they lost their ability to hedge tail risk. This suggests that, to some extent, the models are able to capture the systemic risk in the portfolio, especially during big crises such as the 2008 liquidity crisis was. We observe that the strategies that help to reduce the volatility of the portfolio in other months lose this characteristic during the Lehman meltdown. In particular, the Dedicated Short Bias strategy, which has the largest effect in terms of reducing portfolio volatility in normal periods, has the smallest capability of reducing volatility, with very small negative contributions and even some positive contributions in some months during the crisis periods.

Although largely different in terms of the magnitudes of individual strategy contributions, we find similar results for the marginal contributions to portfolio tail risk, as measured by VaR and ES (see Figures 2 and 3). The most significant difference between the tail risk and volatility contributions is due to the Equity Long Short and Distressed strategies. While they both make positive contributions to portfolio risk in general, they contribute higher percentages of the portfolio tail risk throughout the estimation period.

Table 5: Average Marginal Risk Contributions (S&P-RSM)

Panel A: The Whole Sample (January 2005 - December 2011)						
Strategy	Strategy Volatility	Strategy MC to Portfolio Volatility	Strategy VaR (95%)	Strategy MC to Portfolio VaR	Strategy ES (95%)	Strategy MC to Portfolio ES
Convertible Arbitrage	1.12	-0.28	-1.27	-0.67	-1.75	0.36
Dedicated Short Seller	2.69	-16.31	-10.53	-64.03	-13.63	-67.62
Emerging Markets	5.29	89.20	-11.89	104.56	-15.14	98.21
Equity Market Neutral	1.02	0.29	-1.51	1.17	-2.01	1.87
Long Short Equity	1.61	14.55	-4.80	34.05	-6.55	36.13
Distressed	1.18	6.66	-2.44	10.27	-3.49	13.05
Event-Driven MS	1.23	4.85	-2.19	9.70	-3.06	11.78
Risk Arbitrage	0.80	1.04	-1.29	4.96	-1.82	6.22
Panel B: The Pre-Lehman Subsample (January 2005 - November 2008)						
Strategy	Strategy Volatility	Strategy MC to Portfolio Volatility	Strategy VaR (95%)	Strategy MC to Portfolio VaR	Strategy ES (95%)	Strategy MC to Portfolio ES
Convertible Arbitrage	1.16	-0.44	-1.63	1.25	-2.20	2.01
Dedicated Short Seller	2.58	-21.33	-9.19	-70.80	-12.16	-74.62
Emerging Markets	5.25	95.69	-11.59	108.89	-14.83	101.54
Equity Market Neutral	0.92	-0.80	-1.26	-0.58	-1.70	0.30
Long Short Equity	1.58	14.41	-4.74	36.01	-6.56	38.42
Distressed	1.19	7.05	-2.48	11.13	-3.62	14.42
Event-Driven MS	1.20	4.67	-2.21	8.81	-3.12	11.16
Risk Arbitrage	0.78	0.75	-1.30	5.28	-1.86	6.78
Panel C: The Lehman Meltdown Subsample (December 2008 - June 2009)						
Strategy	Strategy Volatility	Strategy MC to Portfolio Volatility	Strategy VaR (95%)	Strategy MC to Portfolio VaR	Strategy ES (95%)	Strategy MC to Portfolio ES
Convertible Arbitrage	1.03	-1.16	-0.58	-5.30	-0.85	-4.06
Dedicated Short Seller	2.87	-7.83	-12.14	-51.78	-15.09	-54.53
Emerging Markets	5.38	83.46	-12.69	105.06	-15.91	100.56
Equity Market Neutral	1.08	1.45	-1.73	3.62	-2.29	4.19
Long Short Equity	1.68	14.18	-4.82	29.54	-6.28	30.97
Distressed	1.17	5.24	-2.23	6.80	-3.03	8.66
Event-Driven MS	1.32	3.85	-2.03	8.96	-2.69	10.31
Risk Arbitrage	0.82	0.80	-1.17	3.10	-1.58	3.90
Panel D: The Post-Lehman Subsample (July 2009 - December 2011)						
Strategy	Strategy Volatility	Strategy MC to Portfolio Volatility	Strategy VaR (95%)	Strategy MC to Portfolio VaR	Strategy ES (95%)	Strategy MC to Portfolio ES
Convertible Arbitrage	1.09	2.56	-0.88	-2.51	-1.26	-1.10
Dedicated Short Seller	2.80	-4.70	-12.26	-56.44	-15.60	-59.89
Emerging Markets	5.33	90.84	-12.16	97.38	-15.44	92.17
Equity Market Neutral	1.17	3.76	-1.85	3.32	-2.44	3.79
Long Short Equity	1.63	17.36	-4.88	32.11	-6.60	33.83
Distressed	1.17	7.39	-2.42	9.84	-3.41	12.04
Event-Driven MS	1.26	7.28	-2.19	11.34	-3.08	13.19
Risk Arbitrage	0.82	2.80	-1.30	4.96	-1.81	5.97

Av-

verage marginal contributions of the eight strategy indexes to the volatility, Value-at-Risk and Expected Shortfall of a fund of hedge funds, represented by an equally weighted portfolio of the eight strategies, estimated using the S&P-RSM model. Panel A presents the average marginal risk contributions for the whole test period, January 2005 - December 2011. Panels B, C and D present the results for three sub-periods before, during and after the Lehman meltdown.

Table 6: Average Marginal Risk Contributions (MF-RSM)

Panel A: The Whole Sample (January 2005 - December 2011)						
Strategy	Strategy Volatility	Strategy MC to Portfolio Volatility	Strategy VaR (95%)	Strategy MC to Portfolio VaR	Strategy ES (95%)	Strategy MC to Portfolio ES
Convertible Arbitrage	0.54	-3.08	-4.65	1.23	-6.01	1.20
Dedicated Short Seller	1.72	-12.93	-15.02	-30.30	-18.27	-41.93
Emerging Markets	3.42	92.79	-7.03	52.46	-9.45	59.57
Equity Market Neutral	1.00	-1.33	-1.69	1.76	-2.17	1.21
Long Short Equity	1.31	10.21	-9.62	32.96	-12.67	37.05
Distressed	1.71	15.82	-4.92	22.75	-6.49	22.10
Event-Driven MS	1.09	0.57	-4.59	9.97	-6.33	11.60
Risk Arbitrage	0.77	-2.05	-3.21	9.17	-4.24	9.19
Panel B: The Pre-Lehman Subsample (January 2005 - November 2008)						
Strategy	Strategy Volatility	Strategy MC to Portfolio Volatility	Strategy VaR (95%)	Strategy MC to Portfolio VaR	Strategy ES (95%)	Strategy MC to Portfolio ES
Convertible Arbitrage	0.51	-4.50	-3.48	-0.14	-4.64	0.39
Dedicated Short Seller	1.63	-16.10	-14.65	-35.58	-17.90	-49.41
Emerging Markets	3.51	99.31	-6.07	50.59	-8.37	59.33
Equity Market Neutral	0.91	-2.41	-0.88	0.63	-1.29	0.35
Long Short Equity	1.28	9.81	-8.15	35.26	-11.06	39.58
Distressed	1.69	16.47	-3.98	28.58	-5.69	27.43
Event-Driven MS	1.01	0.56	-4.10	9.31	-5.76	11.22
Risk Arbitrage	0.75	-3.13	-3.05	11.36	-4.09	11.10
Panel C: The Lehman Meltdown Subsample (December 2008 - June 2009)						
Strategy	Strategy Volatility	Strategy MC to Portfolio Volatility	Strategy VaR (95%)	Strategy MC to Portfolio VaR	Strategy ES (95%)	Strategy MC to Portfolio ES
Convertible Arbitrage	0.54	0.49	-7.16	1.66	-8.93	1.30
Dedicated Short Seller	1.82	1.43	-17.10	-7.87	-19.96	-14.97
Emerging Markets	3.32	73.69	-11.58	46.58	-14.65	51.30
Equity Market Neutral	1.10	0.35	-2.63	3.05	-2.88	2.20
Long Short Equity	1.39	9.95	-14.60	26.70	-17.91	29.40
Distressed	1.75	12.88	-7.18	14.04	-8.50	13.86
Event-Driven MS	1.26	0.14	-6.69	9.80	-8.43	10.66
Risk Arbitrage	0.76	1.07	-4.48	6.06	-5.42	6.24
Panel D: The Post-Lehman Subsample (July 2009 - December 2011)						
Strategy	Strategy Volatility	Strategy MC to Portfolio Volatility	Strategy VaR (95%)	Strategy MC to Portfolio VaR	Strategy ES (95%)	Strategy MC to Portfolio ES
Convertible Arbitrage	0.58	-1.74	-5.83	3.33	-7.43	2.48
Dedicated Short Seller	1.83	-11.77	-15.01	-27.95	-18.41	-37.24
Emerging Markets	3.30	87.47	-7.36	57.12	-9.76	62.24
Equity Market Neutral	1.13	-0.04	-2.79	3.24	-3.39	2.34
Long Short Equity	1.33	10.95	-10.71	30.96	-13.82	35.07
Distressed	1.72	15.57	-5.79	15.72	-7.23	15.72
Event-Driven MS	1.19	0.71	-4.85	11.09	-6.69	12.50
Risk Arbitrage	0.83	-1.15	-3.15	6.49	-4.15	6.91

Av-

verage marginal contributions of the eight strategy indexes to the volatility, Value-at-Risk and Expected Shortfall of a fund of hedge funds, represented by an equally weighted portfolio of the eight strategies, estimated using the MF-RSM model. Panel A presents the average marginal risk contributions for the whole test period, January 2005 - December 2011. Panels B, C and D present the results for three sub-periods before, during and after the Lehman meltdown.

Panel A: S&P-RSM

Panel B: MF-RSM

Figure 1: **Dynamics of Marginal Volatility Contribution:** Graphs showing the dynamics of each strategy marginal contribution to the portfolio volatility, estimated using the S&P-RSM model (panel A) and the MF-RSM model (Panel B) over the period of January 2005 to December 2011.

Panel A: S&P-RSM

Panel B: MF-RSM

Figure 2: **Dynamics of Marginal Value-at-Risk Contribution:** Graphs showing the dynamics of each strategy marginal contribution to the portfolio Value-at-Risk, estimated using the S&P-RSM model (Panel A) and the MF-RSM model (Panel B) over the period of January 2005 to December 2011.

Panel A: S&P-RSM

Panel B: MF-RSM

Figure 3: **Dynamics of Marginal Expected Shortfall Contribution:** Graphs showing the dynamics of each strategy marginal contribution to the portfolio Expected Shortfall, estimated using the S&P-RSM model (Panel A) and the MF-RSM model (Panel B) over the period of January 2005 to December 2011.

4 Conclusion

This paper provides a new risk management tool for measuring tail risk for portfolio of hedge funds, based on regime-switching models. Our empirical results show that the contribution to the tail risk of a portfolio of hedge funds is only partially related to the main systematic risk measure identified in portfolio diversification literature: the market portfolio. By using the S&P500 as a proxy for the market portfolio we show that, even in a dynamic framework in which the exposure to the market risk factor is changing over time, the contribution of this exposure to the tail risk of a hedge funds portfolio is largely lower than expected, and mostly occurs during crisis periods. Other factors greatly affect the tail risk of a fund of hedge funds, such as liquidity risk or credit risk. The effects of these other factors are so strong that even a strategy such as Dedicated Short Bias, which is characterized by a negative position on the equity market and therefore a natural hedging strategy, is not able to reduce, even partially, the tail risk during a crisis. In contrast, it contributes like all the other strategies to the tail risk, and does not act as a hedging instrument when the need for it to do so is high.

Therefore, our results show that, mostly during a crisis period, all the different hedge fund strategies contribute to tail risk, largely because of their exposure to the liquidity and credit risks, that affect the entire hedge fund industry, and there is no great distinction between strategies.

The results of the paper have natural extensions to the growing literature on funds of funds. These funds invest in different hedge fund strategies thus providing diversification benefits to investors. However, this benefit was not existent during the recent systemic crisis when all strategies became correlated. Consequently, investors who thought they were hedged lost money due to the fact that all strategies positively contributed to the tail risk of the total portfolio. This may explain the decline in the number of funds of funds and total investments allocated to them that we have observed in the recent years.

A The Regime-Switching Model

A.1 Simple Regime-Switching Model (SRSM)

As in Hamilton (1994), a simple regime-switching model for a time series r_t can be specified as

$$r_t = \mu_t(S_t) + \sigma_t(S_t)\epsilon_t \quad (8)$$

where S_t is a hidden Markov chain with K possible states and ϵ_t is an IID process. This model typically assumes that the series r_t is drawn from a mixture of distributions, where the natures of the distributions depend on the assumption about the distribution of the shock ϵ_t and the latent state variable S_t . To make things simpler, it is also the usual practice to assume that ϵ_t is $IIN(0, 1)$ and we do so in this paper. The other assumption we make is that there are only two possible states (or regimes): normal and crisis.⁶ The normal regime is characterized by low volatility and a high mean, while the crisis regime is characterized by high volatility and a low mean.

When $K = 2$, the Markov chain S_t is defined as

$$S_t = \begin{cases} 0 & \text{if in the normal state,} \\ 1 & \text{if in the crisis state.} \end{cases} \quad (9)$$

with transition probabilities matrix, $P(S_t = i/S_{t-1} = j) = P_{ji}$,

$$\begin{pmatrix} P_{00} & P_{10} \\ P_{01} & P_{11} \end{pmatrix} \quad (10)$$

where P_{00} and P_{11} are the probabilities of staying in regime 0 and regime 1 respectively. $P_{01} = 1 - P_{00}$ and $P_{10} = 1 - P_{11}$ are thus the probabilities of changing from regime 0 to regime 1 and from regime 1 to regime 0 respectively. The unconditional probabilities of regimes 0 and 1 are respectively given by the first and second elements of the following vector:

⁶Note that these two assumptions are not binding in the sense that the probability distribution or the number of states considered can be changed to best fit the problem at hand. See, for example, Billio et al. (2012b), who model the market (S&P500 index) using three regimes, namely up, normal and down. However, considering many regimes may create computational difficulties.

$$\left(\begin{array}{c} \frac{(1-P_{11})}{(1-P_{00}-P_{11})} \\ \frac{(1-P_{00})}{(1-P_{00}-P_{11})} \end{array} \right) \quad (11)$$

Thus, the model in (8) characterizes the time series r_t as

$$r_t = \begin{cases} \mu_0 + \sigma_0 \epsilon_t & \text{if } S_t = 0, \\ \mu_1 + \sigma_1 \epsilon_t & \text{if } S_t = 1. \end{cases} \quad (12)$$

Since we have assumed that $\epsilon_t \sim IIN(0, 1)$, we can write the distribution of r_t as

$$r_t \sim \begin{cases} \mathcal{N}(\mu_0, \sigma_0) & \text{if } S_t = 0, \\ \mathcal{N}(\mu_1, \sigma_1) & \text{if } S_t = 1. \end{cases} \quad (13)$$

A.2 (Multi-)Factor Regime-Switching Models (MF-RSMs)

Based on the SRSM model discussed above, Billio et al. (2012b) proposed an MF-RSM for the hedge fund returns. This model, which also appears in the framework of the asset-based approach discussed in Fung and Hsieh (1997) and Agarwal and Naik (2004), is an extension of linear factor models such as APT and the simple Markov regime-switching models discussed above. In this study, we are particularly interested in the multi-factor model as it enables us to study the risk exposure of hedge fund strategies to several risk factors, during both normal and crisis periods. The MF-RSM used in this study can be written as follows:

$$r_{it} = \alpha_i(Z_t^i) + \sum_{j=1}^J \beta_{ij}(S_t) F_{jt} + \sigma_i(Z_t^i) \epsilon_t \quad (14)$$

$$r_{mt} = \mu_m(S_t) + \sigma_m(S_t) \eta_t \quad (15)$$

Here, r_{mt} is the market return and S_t is a Markov chain characterizing market-wide regimes. As discussed earlier, we assume there are two regimes for the market: normal and down. F_{jt} represents the VaR factor j at time t and $\beta_{ij}(S_t)$ captures the exposure of strategy i to the j^{th} risk factor in state S_t . When $S_t = 0$, that is, when we are in the normal market regime, we get $\beta_{ij}(0)$, which is the sensitivity of r_{it} to the j^{th} risk factor in the normal

market regime. Similarly, $\beta_{ij}(1)$ captures the sensitivity of r_{it} to the j^{th} risk factor in the crisis regime.

The remaining terms in equation (14) capture the series-specific (idiosyncratic) characteristics that cannot be captured by the risk factors. Z_t^i is another Markov chain capturing idiosyncratic regimes and Z_t^i is assumed to be independent of S_t . As with S_t , we assume that Z_t^i has two states: high and low idiosyncratic volatility.⁷ Thus, $\alpha_i(Z_t^i)$ is the idiosyncratic constant term. This is similar to the constant term (alpha) in other multi-factor models except that this one depends on the state variable Z_t^i . Similarly, $\sigma_i(Z_t^i)$ is the idiosyncratic volatility. Finally, ϵ_t is the series-specific shock. This model enables us to study each strategy in four different regimes resulting from the combination of the two Markov chains.

With the assumption that the shocks ϵ_t and η_t are $II\mathcal{N}(0, 1)$, we can write the conditional distribution of r_{it} implied by the above model as follows:

$$r_{it} \sim \mathcal{N}(\mu_i(S_t, Z_t^i), \sigma_i(S_t, Z_t^i)) \quad (16)$$

where

$$\mu_i(S_t, Z_t^i) = \alpha_i(Z_t^i) + \sum_{j=1}^J \beta_{ij}(S_t) \mu_{F_j t} \quad (17)$$

$$\sigma_i(S_t, Z_t^i) = \sum_{j=1}^J \beta_{ij}^2(S_t) \sigma_{F_j t}^2 + \sigma_i^2(Z_t^i) \quad (18)$$

and $(S_t, Z_t^i) \in (0, 0), (0, 1), (1, 0), (1, 1)$, which are the four possible states resulting from the combination of the two Markov chains. The probabilities of these four states are obtained by simply multiplying the probabilities of the states of the two independent Markov chains. Once we have the parameter estimates, which also include the filtered probabilities for the Markov chains, it is easy to compute the expected distribution for each strategy. It

⁷One might argue that there is no significant idiosyncratic risk in hedge fund strategy indexes since the indexes are diversified portfolios of individual hedge funds. While this argument has good grounds and we agree that individual strategies have much higher idiosyncratic components than the strategy indexes, we argue that hedge fund strategy indexes, despite being portfolios of individual hedge funds, are not well diversified. This is because they are composed of hedge funds that use similar trading strategies. We believe that, for each strategy index, there is an idiosyncratic risk associated with the specific trading strategy.

is just a mixture of the four distributions given above, mixed in proportion to their probabilities. This also makes finding a forecasting distribution relatively easier. We can obtain forecasting probabilities for the Markov chains by applying the transition matrices to the corresponding filtered probabilities and these forecasting probabilities along with the estimated distribution parameters are used to obtain the forecasting distribution and compute the various risk measures we consider. As in Hamilton (1994), the parameters can be estimated using quasi maximum likelihood methods and the filtered and forecasting probabilities can be obtained through the Hamilton filter.

B Risk Measures in Financial Markets

B.1 Theoretical Definitions

Before discussing the estimation of risk measures through regime-switching models, we would like to recall the general definition of these risk measures. Volatility, which is usually defined as the standard deviation of the series conditional on past information, is a classical measure of dispersion around the mean. Volatility is by definition symmetric and thus it is not capable of capturing any asymmetric nature of risk. This makes volatility more useful during normal market conditions, while it is not so useful during crises when the market is far below the mean. The common risk measures used to capture downside risk exposure are VaR and ES. The VaR is theoretically defined as the maximum amount of loss that can happen over a given horizon at a certain confidence level. Let $VaR(\alpha, h)$ denote the VaR over the horizon h with confidence level α . This means that the probability of the asset producing a return less than $VaR(\alpha, h)$ over the specified horizon h is α . That is, $Pr(R_{t+h} < VaR(\alpha, h)) = \alpha$. A more detailed discussion of VaR can be found in Jorion (2003). The VaR is thus analytically defined as a specific percentile of the return distribution. If we assume that Φ is the true distribution function of a financial return, we can define VaR as

$$\Phi(VaR(\alpha, h)) = \alpha \tag{19}$$

If ϕ is the density function of the return series, VaR can equivalently be defined as

Figure 4: **Value-at-Risk versus Tail Risk:** Histograms of two different time series with the same VaR but different tail risk

$$\alpha = \int_{-\infty}^{VaR(\alpha, h)} \phi(x) dx \quad (20)$$

Though VaR is a widely used measure of risk, there are some caveats about its use. The first problem with it is that it is not a coherent risk measure. As defined in Artzner et al. (1999), a coherent risk measure should satisfy four axioms, namely translation invariance, sub-additivity, positive homogeneity, and monotonicity. While satisfying three of these axioms, VaR fails to satisfy the sub-additivity axiom. That is, the VaR of the sum of two or more risk factors is not always less than or equal to the sum of the VaRs of the risk factors. In other words, a merger may create extra risk. This goes against the well accepted principle of diversification.

The other caveat when it comes to VaR is that it only tells us the probability of having a certain return at a given level of confidence, and does not measure the possible loss if the return is less than the VaR. This is illustrated in Figure 4. In both cases, the empirical 99% VaR is the same, $VaR_{99\%} = -2.0543$, but the two datasets have different shapes to the left of these VaR values. From the histograms in Figure (4), we can see that, when the 99% VaR threshold is violated, the most probable value for the first case is -2.2023 but it is -6.3 for the second case.

The alternative suggested in the literature is the ES, also known as the expected tail loss (ETL), which overcomes the abovementioned weaknesses of VaR. The ES measures the expected loss when the loss exceeds the VaR. In other words, it is the expected return conditional on the return being less

than the VaR. Analytically, it is defined as

$$ES((\alpha, h) = E[R_{t+h} | R_{t+h} < VaR(\alpha, h)] \quad (21)$$

In terms of the distribution it is equivalently defined as

$$ES(\alpha, h) = \frac{\int_{-\infty}^{VaR(\alpha, h)} x \phi(x) dx}{\int_{-\infty}^{VaR(\alpha, h)} \phi(x) dx} = \frac{1}{\alpha} \int_{-\infty}^{VaR(\alpha, h)} x \phi(x) dx \quad (22)$$

Several works in the literature (for example Artzner et al. (1999), Jorion (2003), Acerbi and Tasche (2002) and Inuia and Kijima (2005)) show the ES to be a coherent risk measure. It also has a better capability of describing what is happening to the distribution in the region to the left of the VaR. It tells us the expected value of the return if the return falls below the computed VaR. This characteristics of the ES makes it a better measure of crisis risk.

B.2 Estimation

We use the regime-switching models described in Appendix A to estimate and forecast these risk measures. The estimation of the conditional volatility in each state is possible since volatilities are among the parameters estimated in the Quasi Maximum Likelihood (QMLE) procedure. The volatility forecast is obtained from the weighted average of the different regime volatilities, weighted with the forecasting probabilities of the corresponding state/regime. The VaR and ES are estimated from the distributions as follows.

The α -level VaR for a forecasting horizon h is given by

$$\alpha = \sum_{s_{t+h} \in S} p(s_{t+h}/I_t) \int_{-\infty}^{VaR(\alpha, h)} \phi(x, \mu_{s_{t+h}}, \sigma_{s_{t+h}}^2) dx \quad (23)$$

where $p(s_{t+h}/I_t)$ are the forecasting probabilities conditional on the available information and S is the set of all possible states of nature, which is obtained as a combination of the possible states of the Markov chains involved. For instance, in the case where we consider two Markov chains with two states each, S has four possible states.

In general, the conditional mean and variance are described as functions of the states of the Markov chains. In this paper, the conditional mean and variance can be written as $\mu_{s_{t+h}} = \mu(S_{t+h}, Z_{t+h}|I_t)$ and $\sigma_{s_{t+h}}^2 = \sigma^2(S_{t+h}, Z_{t+h}|I_t)$ respectively, as functions of the two Markov chains S_t and Z_t , where I_t is

the information available at time t , ϕ denotes the normal probability density function and S is the set of all possible states resulting from the combination of the Markov chains' regimes.

We use a numerical procedure to estimate the VaR using equation (23). The estimation of the ES proceeds in a similar way, as follows. Equation (22) can be rewritten in terms of the state-dependent distributions. If S denotes the set of all possible states, we get

$$ES((\alpha, h)) = \frac{1}{\alpha} \sum_{s_{t+h} \in S} p(s_{t+h}/I_t) \int_{-\infty}^{VaR(\alpha, h)} x \phi(x, \mu_{s_{t+h}}, \sigma_{s_{t+h}}) dx \quad (24)$$

The closed-form solution for the $ES(\alpha, h)$, which can be derived by integrating the right hand side of (24), is given by

$$ES(\alpha, h) = \frac{1}{\alpha} \sum_{s_{t+h} \in S} p(s_{t+h}/I_t) \left[\mu_{s_{t+h}} \Phi \left(\frac{VaR(\alpha, h) - \mu_{s_{t+h}}}{\sigma_{s_{t+h}}} \right) - \sigma_{s_{t+h}} \phi \left(\frac{VaR(\alpha, h) - \mu_{s_{t+h}}}{\sigma_{s_{t+h}}} \right) \right] \quad (25)$$

where Φ is the state-dependent normal cumulative distribution function of the normalized portfolio returns.

C Marginal Risk Contribution

In this section, we present a method for computing three marginal risk measures: marginal volatility, marginal VaR and marginal ES. These marginal risk measures are used to estimate the marginal effect of a component that appears in portfolio risk measures. For simplicity, consider a portfolio P composed of N assets with allocations $(x_1, x_2, \dots, x_N)$, where $\sum_{i=1}^N x_i = 1$. Since the concept is the same for all types of risk measures, we only discuss the idea for the volatility of the portfolio. The marginal volatility of asset i measures the change in portfolio volatility given a marginal (infinitesimal) change in the weight of asset i . While the direct way to compute this marginal volatility measure is to take the partial derivative of the portfolio volatility with respect to x_i , this method presents a difficulty when the weights of the other assets are also functions of x_i . This is because of the constraint $\sum_{i=1}^N x_i = 1$. It is obvious that a marginal change in the weight x_i will result in an opposite change in the weight of the rest of the assets.⁸

⁸For further discussion, see the additional material.

To avoid this difficulty, we use the following method, which is also used in Litzenberger and Modest (2008). To compute the marginal risk contribution of strategy i , we construct a new portfolio P' , which consists of an amount w_i invested in strategy i , and an amount $(1 - w_i)$ invested in the portfolio P . Note that w_i is an incremental weight in a strategy that is already a component in the portfolio P . Then, by differentiating with respect to w_i at $w_i = 0$, we can easily account for all the marginal effects of strategy i , either directly through strategy i or through the resulting change in the weight of the rest of the strategies in the portfolio. The return on the new portfolio P' is obtained using

$$r_{P',s} = w_i r_{i,s} + (1 - w_i) r_{P,s} \quad (26)$$

Obviously, the state-dependent mean and volatility of the new portfolio are given by

$$\mu_{P',s} = w_i \mu_{i,s} + (1 - w_i) \mu_{P,s} \quad (27)$$

$$\sigma_{P',s} = \sqrt{w_i^2 \sigma_{i,s}^2 + (1 - w_i)^2 \sigma_{P,s}^2 + 2w_i(1 - w_i) \sigma_{iP,s}} \quad (28)$$

where $\sigma_{iP,s}$ is the state-dependent covariance between strategy i and portfolio P . We know that the covariance $\sigma_{iP,s}$ can be expressed in terms of the correlation $\rho_{iP,s}$ between portfolio P and strategy i and the volatilities of the strategy and portfolio, as $\rho_{iP} \sigma_i \sigma_P$. Thus, replacing σ_{iP} with $\rho_{iP} \sigma_i \sigma_P$ and differentiating with respect to w_i at $w_i = 0$, we get the marginal volatility (labeled $M\sigma$) as follows:

$$M\sigma_{P,s} = \left[\frac{\partial \sigma_{P',s}}{\partial w_i} \right]_{w_i=0} = \rho_{iP,s} \sigma_i - \sigma_{P,s} \quad (29)$$

Moreover, the state-dependent ES of a portfolio can be defined as

$$ES_{P'}(\alpha, h) = \frac{1}{\alpha} \sum_{s \in S} p(s_{t+h}/I_t) \left[\mu_{P',s} \Phi \left(\frac{VaR_{P'}(\alpha, h) - \mu_{P',s}}{\sigma_{P',s}} \right) - \sigma_{P',s} \phi \left(\frac{VaR_{P'}(\alpha, h) - \mu_{P',s}}{\sigma_{P',s}} \right) \right] \quad (30)$$

Differentiating this equation with respect to w_i we get the marginal ES (labeled *MES*):⁹

$$MES = \left[\frac{\partial ES_{P'}}{\partial w_i} \right]_{w_i=0} = \frac{1}{\alpha} \sum_{s \in S} p(s_{t+h}/I_t) \left[\frac{\partial \mu_{P',s}}{\partial w_i} \Phi(z_s) + \mu_{P',s} \frac{\partial \Phi(z_s)}{\partial w_i} - \frac{\partial \sigma_{P',s}}{\partial w_i} \phi(z_s) - \sigma_{P',s} \frac{\partial \phi(z_s)}{\partial w_i} \right]_{w_i=0} \quad (31)$$

where $z_s = \frac{VaR_{P'} - \mu_{P',s}}{\sigma_{P',s}}$ is the standardized VaR for the portfolio P' . Note that we have the constraint

$$\sum_{s \in S} p(s_{t+h}/I_t) \Phi(z_s) = \alpha \quad (32)$$

which can also be stated as

$$\left[\sum_{s \in S} p(s_{t+h}/I_t) \frac{\partial \Phi(z_s)}{\partial w_i} \right]_{w_i=0} = 0 \quad (33)$$

We can solve for $\left[\frac{\partial ES_{P'}}{\partial w_i} \right]_{w_i=0}$ and $\left[\frac{\partial VaR_{P'}}{\partial w_i} \right]_{w_i=0}$ from equations (31) and (33). All we need do is compute the partial derivatives in equation (31) and then apply the constraint in equation (33) when necessary. This is done below. We have already obtained the partial derivative of the volatility in equation (29). Similarly, we can easily get the partial derivative for the mean from equation (28):

$$\left[\frac{\partial \mu_{P',s}}{\partial w_i} \right]_{w_i=0} = \mu_{i,s} - \mu_{P,s} \quad (34)$$

Now we have the solution for two of the four terms in equation (31). We need to solve for the other two terms to obtain a closed-form solution for the partial derivative of the ES. Applying the chain rule gives

$$\frac{\partial \Phi(z_s)}{\partial w_i} = \frac{\partial z_s}{\partial w_i} \frac{\partial \Phi(z_s)}{\partial z_s} \quad (35)$$

$$\frac{\partial \phi(z_s)}{\partial w_i} = \frac{\partial z_s}{\partial w_i} \frac{\partial \phi(z_s)}{\partial z_s} \quad (36)$$

⁹This derivation is similar to Litzenberger and Modest (2008)'s derivation of the marginal contribution to the ETL.

Since Φ and ϕ are respectively the standard normal cumulative distribution and probability density functions we can immediately obtain their derivatives:

$$\frac{\partial \Phi(z_s)}{\partial z_s} = \phi(z_s) \quad (37)$$

$$\frac{\partial \phi(z_s)}{\partial z_s} = -z_s \phi(z_s) \quad (38)$$

The marginal VaR can be obtained from its definition:

$$\begin{aligned} \frac{\partial z_s}{\partial w_i} &= \frac{\partial}{\partial w_i} \left[\frac{VaR_{P'} - \mu_{P',s}}{\sigma_{P',s}} \right] \\ &= \frac{1}{\sigma_{P',s}^2} \left[\sigma_{P',s} \frac{\partial (VaR_{P'} - \mu_{P',s})}{\partial w_i} - (VaR_{P'} - \mu_{P',s}) \frac{\partial \sigma_{P',s}}{\partial w_i} \right] \end{aligned} \quad (39)$$

After a little bit of rearrangement we obtain the expression

$$\left[\frac{\partial z_s}{\partial w_i} \right]_{w_i=0} = \frac{1}{\sigma_{P',s}} \left\{ \left[\frac{\partial VaR_{P'}}{\partial w_i} \right]_{w_i=0} - (\mu_{i,s} - \mu_{P,s}) - z_s (\rho_{iP,s} \sigma_{i,s} - \sigma_{P,s}) \right\} \quad (40)$$

The only part left to solve is the partial derivative of $VaR_{P'}$. It can be obtained by substituting equation (40) into the constraint given by equation (33). This gives us the marginal VaR (labeled $MVaR$):

$$MVaR = \left[\frac{\partial VaR_{P'}}{\partial w_i} \right]_{w_i=0} = \frac{\sum_{s \in S} p(s_{t+h}/I_t) \frac{\phi(z_s)}{\sigma_{P,s}} [(\mu_{i,s} - \mu_{P,s}) - z_s (\rho_{iP,s} \sigma_{i,s} - \sigma_{P,s})]}{\sum_{s \in S} p(s_{t+h}/I_t) \frac{\phi(z_s)}{\sigma_{P,s}}} \quad (41)$$

Now we have all the necessary pieces for the solution of equation (31). By substituting these results into (31) we obtain a closed-form solution for the partial derivative portfolio expected shortfall (MES) with respect to the strategy weights, conditional on the VaR of the portfolio. Moreover, with equation (41) we have the partial derivative for the portfolio VaR. Thus, we now have closed-form solutions for all three risk measures, $M\sigma$, $MVaR$ and MES .

References

Carlo Acerbi and Dirk Tasche. On the Coherence of Expected Shortfall. *Journal of Banking and Finance*, 26(7):1487–1503, 2002.

- Viral V. Acharya, Lasse H. Pedersen, Thomas Philippon, and Matthew Richardson. Measuring Systemic Risk. Stern School of Business, New York University, 2010.
- Tobias Adrian and Markus K. Brunnermeier. CoVaR. Federal Reserve Bank of New York Staff Reports 348, 2010.
- Vikas Agarwal and Narayan Y. Naik. Risk and Portfolio Decision Involving Hedge Funds. *The Review of Financial Studies*, 17(1):62–98, 2004.
- Philippe Artzner, Freddy Delbaen, Jean-Marc Eber, and David Heath. Coherent measures of risk. *Mathematical Finance*, 9(3):203–228, 1999.
- Vijay S. Bawa. Safety First, Stochastic Dominance and Optimal Portfolio Choice. *Journal of Financial and Quantitative Analysis*, 13(2):255 – 271, 1978.
- Monica Billio and Lorian Pelizzon. Value-at-Risk: A Multivariate Switching Regime Approach. *Journal of Empirical Finance*, 7(5):513–554, 2000.
- Monica Billio, Mila Getmansky, Andrew W Lo, and Lorian Pelizzon. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics*, 104(3):535–559, 2012a.
- Monica Billio, Mila Getmansky, and Lorian Pelizzon. Dynamic risk exposures in hedge funds. *Computational Statistics & Data Analysis*, 56(11):3517–3532, 2012b.
- Monica Billio, Lorenzo Frattarolo, and Lorian Pelizzon. A Time-Varying Performance Evaluation of Hedge Fund Strategies through Aggregation. *Bankers, Markets & Investors*, (129):40–58, 2014.
- Stephen J. Brown, Marcin Kacperczyk, Alexander Ljungqvist, Anthony Lynch, Lasse H. Pedersen, and Matthew Richardson. Hedge Funds in the Aftermath of the Financial Crises. In Viral V. Acharya and Matthew Richardson, editors, *Restoring Financial Stability*, pages 157–177. Wiley Finance, 2009.
- Stephen J. Brown, Greg N. Gregoriou, and Razvan Pascualu. Diversification in Funds of Hedge Funds: Is It Possible to Overdiversify? *Review of Asset Pricing Studies*, 2(1):89–110, 2012.

- Christian T. Brownlees and Robert Engle. Volatility Correlations and Tails for Systemic Risk Management. Stern School of Business, New York University, 2010.
- Andrea Buraschi, Robert Kosowski, and Fabio Trojani. When There Is No Place to Hide: Correlation Risk and the Cross-Section of Hedge Fund Returns. *Review of Financial Studies*, 27(2):581–616, 2014.
- Jean-David Fermanian and Olivier Scaillet. Sensitivity analysis of VaR and expected shortfall for portfolios under netting agreements. *Journal of Banking & Finance*, 29(4):927–958, 2005.
- Peter C. Fishburn. Mean-Risk Analysis with Risk Associated with Below Target Returns. *The American Economic Review*, 67(2):116 – 126, 1977.
- William K.H. Fung and David A. Hsieh. Empirical Characteristics Of Dynamic Trading Strategies: The Case Of Hedge Funds. *The Review of Financial Studies*, 10(2):275–303, 1997.
- William K.H. Fung and David A. Hsieh. Performance Characteristics Of Hedge Funds And Commodity Funds: Natural Versus Spurious Biases. *The Journal of Financial and Quantitative Analysis*, 35(2):291–307, 2000.
- Christian Gouriéroux, Jean Paul Laurent, and Olivier Scaillet. Sensitivity analysis of values at risk. *Journal of empirical finance*, 7(3):225–245, 2000.
- Winfried G. Hallerbach. Decomposition of Portfolio Value-at-risk: a General Analysis. *Journal of Risk*, 5(2), 2003.
- James D. Hamilton. *Time Series Analysis*. Princeton University Press, New Jersey, 1994.
- Rau-Bredow Hans. Value at Risk, Expected Shortfall, and Marginal Risk Contribution. In Giorgio Szego, editor, *Risk Measures for the 21st Century*, pages 61–68. Wiley, 2004.
- Andrea J Heuson, Mark C Hutchinson, and Alok Kumar. Assessing Hedge Fund Performance When Fund Returns Are Skewed. *Available at SSRN 2448486*, 2014.

- Koji Inuia and Masaaki Kijima. On the significance of expected shortfall as a coherent risk measure. *Journal Banking and Finance*, 29(4):853–864, 2005.
- Philippe Jorion. *Financial Risk Manager Handbook*. Wiley, 2 edition, 2003.
- Robert H. Litzenberger and David M. Modest. Crisis and Non-Crisis Risk in Financial Markets: A Unified Approach to Risk Management. University of Penn and University of California, 2008.
- Andrew W. Lo, Mila Getmansky, and Peter A. Lee. Hedge Funds: A Dynamic Industry In Transition. *Annual Review of Financial Economics*, 7(1), 2015.
- Harry Markowitz. Portfolio Selection. *Journal of Finance*, 7:77–91, 1952.
- Andrew Donald Roy. Safety First and the Holding of Assets. *Econometrica*, 20(3):431–449, 1952.
- Olivier Scaillet. Nonparametric estimation and sensitivity analysis of expected shortfall. *Mathematical Finance*, 14(1):115–129, 2004.
- Dirk Tasche. Risk contributions and performance measurement. Zentrum Mathematik (SCA), TU Munchen, 1999.