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Keywords
Business Cycle, Financial Cycle, Granger causality, Regime-switching models, Dynamic Factor Models, Dynamical interaction

JEL Codes
C32, C34, C38, E32

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Dynamical Interaction Between Financial and Business Cycles*

October 15, 2017

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Abstract

We adopt the Dynamical Influence model from computer science and transform it to study the interaction between business and financial cycles. For this purpose, we merge it with Markov-Switching Dynamic Factor Model (MS-DFM) which is frequently used in economic cycle analysis. The model suggested in this paper, the Dynamical Influence Markov-Switching Dynamic Factor Model (DI-MS-FM), allows to reveal the pattern of interaction between business and financial cycles in addition to their individual characteristics. More specifically, this model allows to describe quantitatively the existing regimes of interaction in a given economy and to identify their timing, as well as to evaluate the effect of the government policy on the duration of each of the regimes. We are also able to determine the direction of causality between the two cycles for each of the regimes. The model estimated on the US data demonstrates reasonable results, identifying the periods of higher interaction between the cycles in the beginning of 1980s and during the Great Recession, while in-between the cycles evolve almost independently. The output of the model can be useful for policymakers since it provides a timely estimate of the current interaction regime, which allows to adjust the timing and the composition of the policy mix.

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1 Introduction

Throughout the history, the financial sector has been given an increasing role with respect to the business cycle: from neutral intermediary in the theory of Modigliani-Miller to the early-warning indicator revealing the expectations of the economic agents about the business cycle in the framework of the efficient market hypothesis, then further to financial accelerator exacerbating the shocks in the real economy in models with financial frictions, and finally, to the independent source of shocks, on a par with technology and preference shocks in the New Keynesian DSGE models. Given the fast development and the increasing importance of the financial sector, the understanding of the interaction between the financial sector and the business cycle has become crucial for coordination of fiscal, monetary and macroprudential policies. For this purpose, the quantitative estimates of the role of the financial sector are essential.

The study of the financial sector and financial crises in particular gave rise to the notion of the financial cycle. For the moment, there is no single definition of the financial cycle. Instead, in most applied papers researchers refer to the fluctuations of credit, equity and house prices. In spite of the fact that these represent different parts of the financial sector, they possess similar cyclical features, which are therefore considered as the features of the financial cycle. Hubrich et al. (2013), Borio (2014), Stremmel (2015) find that the financial cycles are longer than the real business cycles and last about 12-15 years in US, France and Italy. Drehmann et al. (2012), Ciccarelli et al. (2016), Canova and Ciccarelli (2009), Canova and Ciccarelli (2012) find that the amplitude and duration of the financial cycle evolve. Borio (2006) states that the financial cycle depends on financial regime (liberalized market, controlled market), monetary policy (high and variable inflation causes financial instability) and the state of the business cycle (recession or expansion). In the same time, most of the studies agree that the business cycle, in turn, depends on the financial cycle, with the real shocks being more significant during the episodes of financial instability (see, for example, Bernanke and Gertler (1999), Kiyotaki and Moore (1997), Borio (2014), Hubrich et al. (2013), Claessens et al. (2012)).

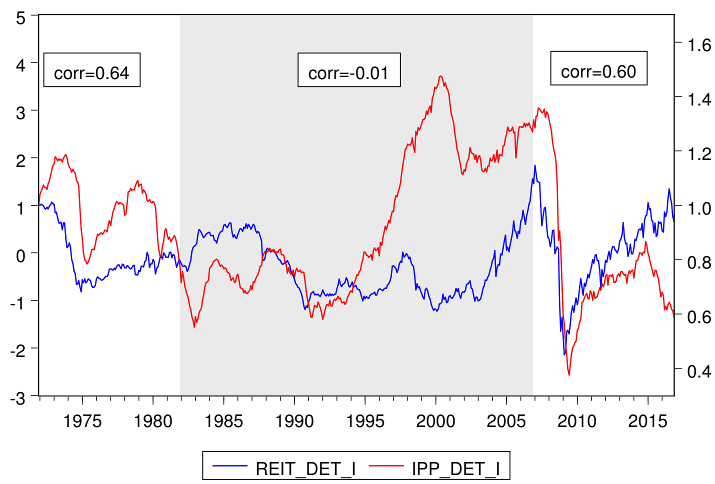
One particularly interesting feature of interdependence - the causality direction between the cycles - has been studied in many papers. In the same time, there is no consensus on whether the financial cycle leads the real cycle (Borio (2014), Adrian et al. (2010), Bandholz and Funke (2003), Chauvet (1999), Chauvet and Senyuz (2012)) or lags behind it (Runstler and Vlekke (2015)). This, however, is consistent with the fact that the financial cycles evolve over time and are longer than business cycles.

Given the changing character of the cycles, it is natural to expect that the interaction between them is also evolving. Indeed, a brief look on the dynamics of the business and financial cycle in the US (approximated by the index of industrial production and the index of house prices[2], respectively) shows that the degree of synchronization is different in different periods of time (Figure 1). The cycles are much more correlated in 1970s-beginning of 1980s and after the Global Financial Crisis (with correlation about 0.60), and much less in-between (the correlation is zero). The absolute value of the cross-correlations is even higher during these periods (see the dynamics of the absolute value of correlation and cross-correlation estimated on a moving window with width $w = 141$ on Figure 2).[3]

[2]see ECB (2009) for discussion of the indicator characterizing the financial cycle

[3]The results are similar when the financial cycle is approximated with a time series of credit, as suggested by Drehmann

Figure 1: US industrial production index and US index of house prices



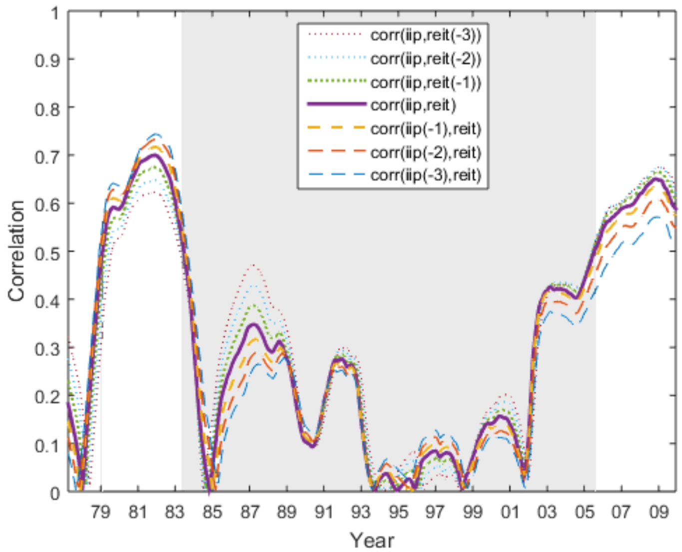
Note: US index of industrial production (red line, right axis, source: Federal Reserve Bank of St. Louis), US index of house prices (blue line, left axis, source: FTSE NAREIT US Real Estate Composite Index). Both series are detrended and seasonally-adjusted.

Taking into consideration the stylized facts mentioned above, an econometric framework that is used to study the joint dynamics of business and financial cycles should allow for the dynamical feedback between them. This idea was implemented in several different approaches. Among them are the time-varying VAR (in Hubrich et al. (2013)), Markov-Switching VAR model with time-varying transition probabilities (as, for example, in Billio et al. (2007)), versions of multivariate structural time series models (STSMs) (see Runstler and Vlekke (2015)), and time-varying Panel Bayesian VAR (see Ciccarelli et al. (2016)) for the analysis of the macro-financial linkages between countries.

In this paper we suggest an alternative model, the Dynamical Influence Markov-Switching Dynamic Factor Model (DI-MS-FM) that provides rich statistical inference due to its three components: Dynamical Influence model by Pan et al. (2012), Markov-Switching model by Hamilton (1989) and Dynamic Factor model by Geweke (1977). Importantly, in contrast to the models mentioned above, the DI-MS-FM does not require an exogenous variable to drive the interaction but allows it to evolve intrinsically. More precisely, we assume that each of the cycles can be in several states (expansion and recession in case of the business cycle, boom and downturn in case of the financial cycle), and that there are several regimes of interaction which differ in degree of interdependence and leading/lagging relation. This assumption is formalized with the help of an hierarchical structure, where an exogenous unobservable Markov chain governs the mutual impact of the two other discrete processes characterizing the cycles. Besides average duration, qualitative characteristics, and filtered and smoothed probabilities of each state for each of the cycles, we get the same inference for the existing influence regimes. Additionally, for each of the influence regimes, we are able to identify the direction of causality between cycles and evaluate the relative importance of the past of each cycle on their present states. These estimates allow to perform a retrospective analysis of the cycles and their interaction as well

et al. (2012).

Figure 2: Cross-correlations (in absolute value) between industrial production index and US index of house prices



Note: Cross-correlations between US index of industrial production and US index of house prices (FTSE NAREIT US Real Estate Composite Index) estimated on a moving window with width $w = 141$, i.e. a estimate for a date t is obtained using observations from $t - 70$ to $t + 70$.

as to make probabilistic inference on the current situation. Finally, they allow to provide forecasts of future states of each cycle given the current influence regime. Moreover, the estimate of the filtered probability of the influence regime corresponding to high interaction (as influence regime 2 in our empirical exercise below) can serve as an early-warning indicator of systemic risk (if one considers the notion of systemic risk in a broader sense, i.e. as a risk of a joint recession both in the financial and the business cycle simultaneously). These estimates can be useful for policymakers to design and adjust the policy mix.

The paper is organized as follows. In Section 2 we introduce the model, describe the underlying interaction mechanism and define Granger causality and suggest a possible extension of the model allowing to evaluate the effect of government policies. In Section 3 we discuss the estimation procedure, derive h-step ahead forecasts, examine in-sample and out-of-sample performance of the model. Section 4 contains the results of the application of the model to the US data. Section 5 concludes.

2 The DI-MS-FM

2.1 The general presentation

We adopt the Dynamical Influence model from computer science by Pan et al. (2012) and transform it to study the interaction between business and financial cycles. For this purpose, we merge it with Markov-Switching Dynamic Factor Model (MS-DFM) which is frequently used in economic cycle analysis. The resulting model, the Dynamical Influence Markov-Switching Dynamic Factor Model

(DI-MS-FM), is presented below.

At date t, $t = 1, ..., T$, economic agents observe (or infer) the business cycle RF_t and the financial cycle FF_t[4] which have the following dynamics

$$RF_t = \mu(S_t^1) + \varphi(L)RF_t + \sigma(S_t^1)\varepsilon_t, \tag{1}$$

$$FF_t = \beta(S_t^2) + \psi(L)FF_t + \theta(S_t^2)\xi_t, \tag{2}$$

where S_t^1 and S_t^2 are unobservable discrete processes which are associated with a finite number of states and which govern the dynamics of the business cycle and the financial cycle, correspondingly, $\varphi(L) = \varphi_1 L + ... + \varphi_{p_1} L^{p_1}$ and $\psi(L) = \psi_1 L + ... + \psi_{p_2} L^{p_2}$ are lag polynomials of finite order p_1 and p_2 correspondingly, $\{\varepsilon_t\}$ and $\{\xi_t\}$ are independent standard Gaussian white noises. The functions $\mu(\cdot)$,$\sigma(\cdot)$, $\beta(\cdot)$, $\theta(\cdot)$ are known functions of the specified arguments with unknown parameters.

We assume that the interaction between the cycles happens at the level of unobservable processes S_t^1 and S_t^2, but not observations, which means that (1)-(2) is a restricted VAR.[5]

The current values of S_t^1 and S_t^2 are each dependent on the past of both processes and a variable r_t governing the interaction between S_t^1 and S_t^2, which is the crucial feature of the model:

$$P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t) = A(S_{t-1}^1, S_{t-1}^2, r_t), \tag{3}$$

$$P(S_t^2 | S_{t-1}^1, S_{t-1}^2, r_t) = B(S_{t-1}^1, S_{t-1}^2, r_t), \tag{4}$$

$$P(r_t | r_{t-1}) = Q, \tag{5}$$

where $A(\cdot)$, $B(\cdot)$ are known functions with unknown parameters. We assume that the initial r_0, S_0^1, S_0^2, RF_0, FF_0 are not random. The process r_t, which we call the interaction regime process, is a Markov chain of first order[6] with a finite number of regimes and a transition probability matrix Q.

For the sake of simplicity, we suppose here that the variables r_t, S_t^1 and S_t^2 can take only two values (states) each ($S_t^1 = 1$ in case of expansion and $S_t^1 = 2$ in case of recession; $S_t^2 = 1$ in case of financial boom and $S_t^2 = 2$ in case of financial downturn; the interpretation of the states of $r_t \in \{1, 2\}$ is determined by the degree of mutual influence between the two chains in each regime estimated within the model[7].). Nevertheless, the analysis can be easily extended to incorporate chains of a higher (and different) order and with more states. Similarly, it is also feasible to allow the past of RF_t, FF_t or some observable covariate cause S_t^1 and S_t^2.

Unlike classic Markov-switching models used in business cycle analysis, strictly speaking, the processes S_t^1 and S_t^2 are not Markov chains since the current state of each of them depends on the past of the other chain, too. Moreover, the process (S_t^1, S_t^2) is not Markov either as it depends on all its lags.

---

[4]The construction of RF_t and FF_t will be described later on in section 2.2.

[5]The lags of FF_t do not enter the equation for RF_t (equation (1)) and vice versa. When the interaction on the level of observation is also allowed for, the identification of each channel can be an issue.

[6]This assumption is not restrictive.

[7]The model can be easily extended for the case when the number of states of S_t^1 and S_t^2 is not equal

Nevertheless, for the ease of exposition, we address to S_t^1 and S_t^2 as "chains".

To understand the dynamics of the model, we present the conditional distributions of $\underline{RF_t}$, $\underline{FF_t}$, $\underline{S_t^1}$, $\underline{S_t^2}$, $\underline{r_t}$ using a generic notation $\underline{x_t} = (x_t, x_{t-1}, ..., x_0)$:

$$\mathfrak{L}(r_t|\underline{RF_{t-1}}, \underline{FF_{t-1}}, \underline{S_{t-1}^1}, \underline{S_{t-1}^2}, \underline{r_{t-1}}) = \mathfrak{L}(r_t|r_{t-1}), \tag{6}$$

$$\mathfrak{L}(S_t^1|\underline{RF_{t-1}}, \underline{FF_{t-1}}, \underline{S_{t-1}^1}, \underline{S_{t-1}^2}, \underline{r_t}) = \mathfrak{L}(S_t^1|S_{t-1}^1, S_{t-1}^2, r_t), \tag{7}$$

$$\mathfrak{L}(S_t^2|\underline{RF_{t-1}}, \underline{FF_{t-1}}, \underline{S_t^1}, \underline{S_{t-1}^2}, \underline{r_t}) = \mathfrak{L}(S_t^2|S_{t-1}^1, S_{t-1}^2, r_t), \tag{8}$$

$$\mathfrak{L}(RF_t|\underline{RF_{t-1}}, \underline{FF_{t-1}}, \underline{S_t^1}, \underline{S_t^2}, \underline{r_t}) = N(\mu(S_t^1) + \varphi(L)RF_t, \sigma^2(S_t^1)), \tag{9}$$

$$\mathfrak{L}(FF_t|\underline{FF_{t-1}}, \underline{RF_t}, \underline{S_t^1}, \underline{S_t^2}, \underline{r_t}) = N(\beta(S_t^2) + \psi(L)FF_t, \theta^2(S_t^2)). \tag{10}$$

The fundamental assumptions of the model are:

1. r_t is autonomous, i.e. S_t^1, S_t^2, RF_t and FF_t do not cause r_t in the Granger sense since $\underline{S_{t-1}^1}, \underline{S_{t-1}^2}, \underline{RF_t}, \underline{FF_t}$ do not appear in its conditional distribution.
2. RF_t and FF_t do not Granger cause S_t^1, S_t^2 and r_t.
3. S_t^1 and S_t^2 are conditionally independent given $\underline{RF_{t-1}}, \underline{FF_{t-1}}, \underline{S_{t-1}^1}, \underline{S_{t-1}^2}, \underline{r_t}$.
4. The process (S_t^1, S_t^2, r_t) is an autonomous Markov chain.
5. RF_t and FF_t are conditionally independent given $\underline{r_t}$, $\underline{S_t^1}$ and $\underline{S_t^2}$.

To summarize, the dynamics of the model can be represented in the following way (with $\omega_t = (r_t, S_t^1, S_t^2, RF_t, FF_t)$):

$$r_t|\omega_{t-1} = r_t|r_{t-1} \tag{11}$$

$$S_t^1|r_t, \omega_{t-1} = S_t^1|r_t, S_{t-1}^1, S_{t-1}^2, \tag{12}$$

$$S_t^2|r_t, S_t^1, \omega_{t-1} = S_t^2|r_t, S_{t-1}^1, S_{t-1}^2, \tag{13}$$

$$RF_t|r_t, S_t^1, S_t^2, \omega_{t-1} = RF_t|S_t^1, \tag{14}$$

$$FF_t|RF_t, r_t, S_t^1, S_t^2, \omega_{t-1} = FF_t|S_t^2. \tag{15}$$

2.2 Construction of RF_t and FF_t

To construct the proxies for business and financial cycles RF_t and FF_t, we adopt the Dynamic Factor Model approach by Stock and Watson (1989). Following the concept of the business cycle by Burns and Mitchell (1946) as comovement of economic series, they assume that each of the indicators of the real sector of an economy (industrial production, consumption, stock, consumer and business surveys, etc.) can be decomposed into two parts. The first one refers to the comovement of series of the real sector (the business cycle) while the second part corresponds to the idiosyncratic dynamics:

$$x_t = \lambda RF_t + y_t, \tag{16}$$

where x_t is a $N \times 1$ vector of stationarized and deseasonalized economic indicators, RF_t is a $r \times 1$ vector of common factors of x_t, y_t is a $N \times 1$ vector of idiosyncratic components uncorrelated with RF_t at all leads and lags, λ is a $N \times r$ vector of factor loadings.

Bai (2003), Stock and Watson (2002) showed that $\hat{RF_t}$ can be consistently estimated with PCA when N and T are large. The use of PCA for factor extraction in the two-step procedures is very convenient since it is robust to some types of misspecifications, as was shown by Stock and Watson (2002). For example, under the number of series and observations sufficiently large, PCA provides consistent estimates of factors when the series of the database are weakly cross-sectionally correlated or autocorelated. Also, PCA does not require normality of the series. In the business cycle analysis, the first principal component usually explains most of the variance of x_t, so RF_t is actually one-dimensional. Therefore, the first principal component of a rich database of macroeconomic variables is commonly accepted as a proxy to the business cycle. The proxy of the financial cycle $\hat{FF_t}$ is obtained similarly from the database of financial indicators.[8] These two proxies are then used to estimate (1)-(5).

To keep the notations simple, in what follows RF_t and FF_t (but not $\hat{RF_t}$ and $\hat{FF_t}$) refer to the proxies of business and financial cycles estimated with PCA.

2.3 The interaction mechanism

In order to describe the interaction between the chains, let us consider their joint dynamics. As we have mentioned above, the process (S_t^1, S_t^2, r_t) is a Markov chain. Each its component taking two values, the joint Markov chain has 8 states and thus a 8×8 transition matrix with 56 free parameters. By imposing a particular interaction mechanism, we parametrize this transition probability matrix with only 14 parameters, thus rendering the model more parsimonious. The interaction mechanism is organized as follows.

Consider two auxiliary variables, E_t^1 and E_t^2 (E standing for "effect"). Each of these variables is a binary variable and determines the current driving force for each of the corresponding chains, i.e.:

$$E_t^1 = \begin{cases} d, & \text{if } S_t^1 \text{ is impacted by } S_{t-1}^1 \text{, direct effect} \\ c, & \text{if } S_t^1 \text{ is impacted by } S_{t-1}^2 \text{, cross effect} \end{cases}, \tag{17}$$

$$E_t^2 = \begin{cases} d, & \text{if } S_t^2 \text{ is impacted by } S_{t-1}^2 \text{, direct effect} \\ c, & \text{if } S_t^2 \text{ is impacted by } S_{t-1}^1 \text{, cross effect} \end{cases}. \tag{18}$$

The chances of being under cross or direct effect for each of the chains depend on the interaction regime variable r_t. The exogenous process r_t is an ergodic first-order Markov Chain with 2 states, i.e.

$$P(r_t = j | r_{t-1} = i, r_{t-2} = k, ...) = P(r_t = j | r_{t-1} = i) = q_{ij}, \quad j, i, k \in \{1, 2\},$$

[8] One can use single series to approximate business and financial cycles (industrial production index and housing prices index, for example). However, in practice factors are commonly used as they reflect a larger information set on each of the sectors.

so r_t switches states according to the transition probabilities matrix

$$Q = \begin{bmatrix} q_{11} & 1-q_{11} \\ 1-q_{22} & q_{22} \end{bmatrix}. \quad (19)$$

The dynamic causality structure is the following:

1. the values of r_t are generated from a two-state Markov chain with the transition probability matrix Q;

2. for each value of r_t, E^1_t is drawn in $\{d, c\}$ from the Bernoulli distribution $\mathcal{B}(R^{r_t}_{11})$, where $R^{r_t}_{11}$ is the probability of drawing d and $1 - R^{r_t}_{11} = R^{r_t}_{21}$ is the probability of drawing c;

3. for each value of r_t, E^2_t is drawn in $\{d, c\}$ from the Bernoulli distribution $\mathcal{B}(R^{r_t}_{22})$, where $R^{r_t}_{22}$ is the probability of drawing d and $1 - R^{r_t}_{22} = R^{r_t}_{12}$ is the probability of drawing c;

4. for $E^1_t = d$, $S^1_{t-1} = i$, $S^2_{t-1} = j$ $(i, j \in \{1, 2\})$, S^1_t is drawn in $\{1, 2\}$ from the Bernoulli distribution $\mathcal{B}(D^1_{i1})$, where D^1_{i1} is the probability of drawing 1 and $1-D^1_{i1} = D^1_{i2}$ is the probability of drawing 2;
   for $E^1_t = c$, $S^1_{t-1} = i$, $S^2_{t-1} = j$ $(i, j \in \{1, 2\})$, S^1_t is drawn in $\{1, 2\}$ from the Bernoulli distribution $\mathcal{B}(C^{21}_{j1})$, where C^{21}_{j1} is the probability of drawing 1 and $1-C^{21}_{j1} = C^{21}_{j2}$ is the probability of drawing 2;

5. for $E^2_t = d$, $S^1_{t-1} = i$, $S^2_{t-1} = j$ $(i, j \in \{1, 2\})$, S^2_t is drawn in $\{1, 2\}$ from the Bernoulli distribution $\mathcal{B}(D^2_{j1})$, where D^2_{j1} is the probability of drawing 1 and $1-D^2_{j1} = D^2_{j2}$ is the probability of drawing 2;
   for $E^2_t = c$, $S^1_{t-1} = i$, $S^2_{t-1} = j$ $(i, j \in \{1, 2\})$, S^2_t is drawn in $\{1, 2\}$ from the Bernoulli distribution $\mathcal{B}(C^{12}_{i1})$, where C^{12}_{i1} is the probability of drawing 1 and $1-C^{12}_{i1} = C^{12}_{i2}$ is the probability of drawing 2.

Therefore, the interaction between the chains is fully described by a set of 14 parameters (q_{11}, q_{22}, R^1_{11}, R^1_{22}, R^2_{11}, R^2_{22}, D^1_{11}, D^1_{22}, C^{12}_{11}, C^{12}_{22}, D^2_{11}, D^2_{22}, C^{21}_{11}, C^{21}_{22}), which we organize in matrices Q defined above,

$$R^1 = \begin{bmatrix} R^1_{11} & 1-R^1_{22} \\ 1-R^1_{11} & R^1_{22} \end{bmatrix}, R^2 = \begin{bmatrix} R^2_{11} & 1-R^2_{22} \\ 1-R^2_{11} & R^2_{22} \end{bmatrix},$$

$$D^1 = \begin{bmatrix} D^1_{11} & 1-D^1_{11} \\ 1-D^1_{22} & D^1_{22} \end{bmatrix}, D^2 = \begin{bmatrix} D^2_{11} & 1-D^2_{11} \\ 1-D^2_{22} & D^2_{22} \end{bmatrix},$$

$$C^{12} = \begin{bmatrix} C^{12}_{11} & 1-C^{12}_{11} \\ 1-C^{12}_{22} & C^{12}_{22} \end{bmatrix}, C^{21} = \begin{bmatrix} C^1_{11} & 1-C^1_{11} \\ 1-C^1_{22} & C^{21}_{22} \end{bmatrix}.$$

At period t the probability of being in a particular state of the business cycle S^1_t depends on its own past S^1_{t-1} and also on the previous state of the financial cycle S^2_{t-1}. The relative importance of each chain is determined by the matrix R^{r_t} with $r_t \in \{1, 2\}$, which assigns weights to S^1_{t-1} and S^2_{t-1}, thus determining their self effect and the effect of the other chain given the current interaction regime r_t. Therefore, the probability that the business cycle is in state S_t^1, given the states S_{t-1}^1, S_{t-1}^2 and r_t, is a weighted average of probabilities to switch from $S_{t-1}^1 = i$ to $S_t^1 = k$ and from $S_{t-1}^2 = j$ to $S_t^1 = k$, where $i, j, k \in \{1, 2\}$, with weights determined by r_t. Formally, the probability that a chain S_t^1 is in state k given the past of both chains and the current values of r_t is:

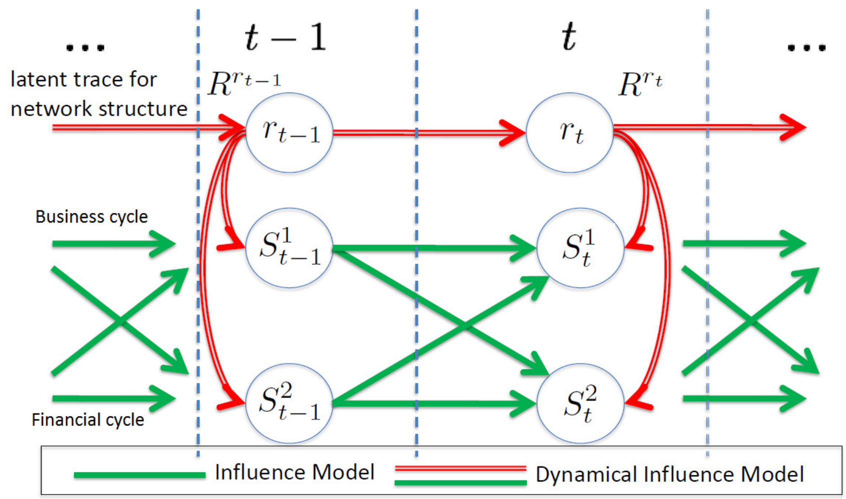
$$
\begin{aligned}
P(S_t^1 = k | S_{t-1}^1 = i, S_{t-1}^2 = j, r_t) &= \sum_{l=d,c} P(S_t^1 = k, E_t^1 = l | S_{t-1}^1 = i, S_{t-1}^2 = j, r_t) \\
&= P(S_t^1 = k | E_t^1 = d, S_{t-1}^1 = i, S_{t-1}^2 = j, r_t) P(E_t^1 = d | r_t) \\
&+ P(S_t^1 = k | E_t^1 = c, S_{t-1}^1 = i, S_{t-1}^2 = j, r_t) P(E_t^1 = c | r_t) \\
&= D_{ik}^1 R_{11}^{r_t} + C_{jk}^{21} (1 - R_{11}^{r_t}) \\
&= D_{ik}^1 R_{11}^{r_t} + C_{jk}^{21} R_{21}^{r_t}.
\end{aligned} \tag{20}
$$

with $i, j, k \in \{1, 2\}$ and where X_{ij} denotes the element of the i-th row and j-th column of the matrix X. Similar logic applies to the financial cycle giving

$$
P(S_t^2 = k | S_{t-1}^1 = i, S_{t-1}^2 = j, r_t) = D_{jk}^2 R_{22}^{r_t} + C_{ik}^{12} R_{12}^{r_t}. \tag{21}
$$

Here D^i, $i \in \{1, 2\}$, is a matrix of parameters capturing the transition due to the direct effect, so that, for example, the element D_{11}^1 shows the probability that the first chain stays in the regime 1 "expansion". Similarly, the matrix C^{ki}, $i, k \in \{1, 2\}$, $i \neq k$, is a matrix of parameters that capture cross effect transitions, so that, for instance, the element C_{11}^{12} shows the probability that an expansion in the business cycle induces a boom in the financial cycle. Importantly, direct effect transitions and cross effect transitions do not depend on r_t. The value $R_{ki}^{r_t}$ shows the relative importance (the weight) of the past of chain k on the present of the chain i given the current interaction regime $r_t \in \{1, 2\}$. Therefore, the larger are the diagonal elements of this matrix, the higher is the self-impact, and more independent are the chains. The most important feature of this framework arises from the fact that the weights vary over time with r_t, thus rendering the interaction between the two chains dynamical. We illustrate schematically the Dynamical Influence model in Figure 1.

Figure 3: A graphical representation of the Dynamical Influence Model



Note: This is a modified version of Figure 2 from the paper by Pan et al. (2012)

This type of interaction is new in the economic literature. The existing methods based on the modeling of the joint process (S_t^1, S_t^2) allow either for a fixed relation between the chains (in case of static transition probability matrix) or exogenously driven relation (in case of transition probability matrix depending on some covariates). On the contrary, in this model the interaction is designed to be intrinsically dynamical, whether dependent on the covariates or not.

As we show in the next section, after introduction of a new state variable the model boils down to the classic Hamilton (1989) Markov-switching model. Therefore, once the estimates of the coefficients of (1) and (2), D^1, D^2, C^{12}, C^{21}, R^1, R^2 are obtained, the standard filtered and smoothed probabilities of each state of each chain can be calculated, including the smoothed probability $P(r_t = j|I_T)$, $j \in \{1, 2\}$ of being in a particular interaction regime j, where $I_\tau = (\underline{RF_\tau}, \underline{FF_\tau})$ is the information available up to time τ. On top of that, it would be possible to calculate the joint filtered and smoothed probabilities $P(S_t^1 = i, S_t^2 = j|I_t)$ and $P(S_t^1 = i, S_t^2 = j|I_T)$, $i, j \in \{1, 2\}$, which is useful for the purpose of analysis of joint crises in real and financial sectors.

2.4 Granger causality

As we said above, in this framework the two cycles RF_t and FF_t interact on the level of chains. Importantly, the estimated matrices of coefficients R^1, and R^2 can give us an idea about the causality relation between the two chains S_t^1 and S_t^2.

Consider a process $\tilde{S}_t = (S_t^1, S_t^2, r_t)$ which is a Markov process with 8 states. We can decompose the transition probabilities as follows:

$$P(S_t^1, S_t^2, r_t | S_{t-1}^1, S_{t-1}^2, r_{t-1}) = P(S_t^1 | S_t^2, r_t, S_{t-1}^1, S_{t-1}^2, r_{t-1}) \times P(S_t^2 | r_t, S_{t-1}^1, S_{t-1}^2, r_{t-1}) P(r_t | S_{t-1}^1, S_{t-1}^2, r_{t-1}). \tag{22}$$

Using the assumptions (2) and (3) and equation (6), this expression can be simplified:

$$P(S_t^1, S_t^2, r_t | S_{t-1}^1, S_{t-1}^2, r_{t-1}) = P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t) P(S_t^2 | S_{t-1}^1, S_{t-1}^2, r_t) P(r_t | r_{t-1}). \tag{23}$$

Now, like Billio and Sanzo (2015), we can define Granger non-causality between S_t^1 and S_t^2 in *strong* sense, since it is specified by imposing restrictions on the parameters characterizing conditional distributions:

1. S_{t-1}^2 does not strongly cause S_t^1 one-step ahead given S_{t-1}^1 and r_t if

$$P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t) = P(S_t^1 | S_{t-1}^1, r_t) \quad \forall t. \tag{24}$$

2. S_{t-1}^1 does not strongly cause S_t^2 one-step ahead given S_{t-1}^2 and r_t if

$$P(S_t^2 | S_{t-1}^2, S_{t-1}^1, r_t) = P(S_t^2 | S_{t-1}^2, r_t) \quad \forall t. \tag{25}$$

We can also define the independence of two chains as follows:

3. S_t^1 and S_t^2 are independent given r_t if

$$P(S_t^1, S_t^2, r_t | S_{t-1}^1, S_{t-1}^2, r_{t-1}) = P(S_t^1 | r_t, S_{t-1}^1) P(S_t^2 | r_t, S_{t-1}^2) P(r_t | r_{t-1}). \tag{26}$$

Following the approach of Billio and Sanzo (2015), for a given parametrization (20), the conditions of the strong one-step ahead non-causality and independence can be derived as restrictions on the parameter space.

The restriction $H_{1 \not\Rightarrow 2}$ of the strong non-causality from S_t^1 to S_t^2 given r_t implies that the parameter related to S_{t-1}^1 is equal to zero. So, since

$$P(S_t^2 = k | S_{t-1}^1 = i, S_{t-1}^2 = j, r_t) = R_{22}^{r_t} \times D_{jk}^2 + R_{12}^{r_t} \times C_{ik}^{12}, \tag{27}$$

the strong non-causality is implied by

$$H_{1 \not\Rightarrow 2}: \quad R_{12}^{r_t} = 0 \tag{28}$$

Under $H_{1 \not\Rightarrow 2}$ S_{t-1}^1 does not cause one-step ahead S_t^2 given S_{t-1}^2 and r_t. Since the terms related to S_{t-1}^1 are excluded from (27), therefore $P(S_t^2 | S_{t-1}^2, S_{t-1}^1, r_t) = P(S_t^2 | S_{t-1}^2, r_t)$.

On the other hand, the strong one-step ahead non-causality from S_t^2 to S_t^1 given S_{t-1}^1 and r_t, given the parametrization

$$P(S_t^1 = k|S_{t-1}^1 = i, S_{t-1}^2 = j, r_t) = R_{11}^{r_t} \times D_{ik}^1 + R_{21}^{r_t} \times C_{jk}^{21}, \tag{29}$$

is implied by

$$H_{2\not\Rightarrow 1}: \quad R_{21}^{r_t} = 0 \tag{30}$$

The term related to S_{t-1}^2 is excluded from (29), so $P(S_t^1|S_{t-1}^1, S_{t-1}^2, r_t) = P(S_t^1|S_{t-1}^1, r_t)$.

Finally, the restriction of the independence of S_t^1 and S_t^2 given r_t is implied by both restrictions (28) and (30) simultaneously:

$$H_{2\perp 1}: \quad R_{21}^{r_t} = R_{12}^{r_t} = 0 \tag{31}$$

Therefore, the value and significance of the off-diagonal coefficients of the matrices R^1 and R^2 allow to make inference on the causality between the two chains within each regime j. Moreover, since the elements in R^1 and R^2 are not necessarily 0 and 1, we can quantify the relative importance of each of the affecting chains.

Note that the values of the elements C^{ij}, $i, j \in \{1, 2\}$, $i \neq j$ give the idea of the global character of Granger causality between the two cycles, defining the channels of interaction irrespective of the current interaction regime. At the same time, the conditions on $R_{ij}^{r_t}$ refer to local changes in Granger causality, and can modify the intensity of the channel if it exists (the relevant element of C^{ij} is non-zero).

For the parametrization of the interaction described above, it is also possible to test for global non-causality, i.e. irrespective of the current interaction regime. However, in contrast to the local Granger non-causality, the null for the global non-causality can not be formulated in terms of restrictions on the elements of matrices R^{r_t}, for example, $H_0 : R_{12}^1 = R_{12}^2 = 0$ in case of testing for global non-causality of business cycle with respect to the financial cycle. Indeed, in this case, the parameters of the matrix C^{12} are not identified, and tests based on this null are not standard.[9] Instead, one can reformulate the null in terms of restrictions on the elements of C^{12} and C^{21}, thus avoiding the non-identification problem. Thus, S_t^1 does not strongly cause one-step ahead S_t^2 globally given S_{t-1}^2 and r_t if:

$$H_{1\not\Rightarrow^g 2}: C_{11}^{12} = C_{21}^{12}, \tag{32}$$

i.e. the impact of the recession is the same as the one of expansion, so the state of the business cycle is irrelevant for the future state of the financial cycle. Note that the null also implies $C_{22}^{12} = C_{12}^{12}$ since

[9] A possible solution for this task would be to simulate the distribution of the test statistics under the null. Hansen (1996) also suggests a transformation of the test statistics based on a conditional probability measure which yields an asymptotic distribution free of the unidentifiable parameters.

$C_{11}^{12} = 1 - C_{12}^{12}$, so under the null the matrix C^{12} has the form $\begin{bmatrix} C_{11}^{12} & 1 - C_{11}^{12} \\ C_{11}^{12} & 1 - C_{11}^{12} \end{bmatrix}$.

The null hypotheses for the strong global one-step ahead non-causality of S_t^2 with respect to S_t^1 given S_{t-1}^1 and r_t and strong global one-step ahead independence can be formulated in a similar way.

2.5 Extension: policy analysis

It is natural to assume that government policies may affect the cycles themselves as well as their interaction. One of possible ways to take this impact into account is through imposing dependence of the parameters describing state transitions on the policy variable vector z_t. Possible candidates for z_t series are the Federal Funds rate, the term premium as well as the series of tax shocks (see, for example, Mertens and Ravn (2013) and Romer and Romer (2010)).

Depending on assumptions on the impact of a particular policy measure, the dependence on policy variables can be introduced in different ways. While the rest of the framework stays unmodified, the changes may concern the matrices Q (impact on the duration of each of the interacting regimes), D^1 (impact on the business cycle), D^2 (impact on the financial cycle), C^{12} and C^{21} (impact on the mechanisms of transmission of states between the cycles). In the first case, for example, the transition probability matrix Q for the interaction regime variable r_t becomes dynamic, i.e. Q_t:

$$Q_t = \begin{bmatrix} q_{11}(z_{t-1}) & 1 - q_{11}(z_{t-1}) \\ 1 - q_{22}(z_{t-1}) & q_{22}(z_{t-1}) \end{bmatrix}. \tag{33}$$

Different functional forms of the transition probabilities mapping z_t into the unit interval can be considered (for example, the logistic function, probit function, Cauchy integral and other). The logistic function is a common case, therefore:

$$q_{ii}(z_{t-1}) = \frac{exp(\delta_{i0} + \sum_{j=1}^{J} \delta_{ij} z_{t-j})}{1 + exp(\delta_{i0} + \sum_{j=1}^{J} \delta_{ij} z_{t-j})}, \tag{34}$$

where $\delta_{i0}, ..., \delta_{iJ}$, $i \in \{1, 2\}$ are parameters to estimate. The matrices D^1, D^2, C^{12} and C^{21} can be modified in a similar way.

3 Estimation and Forecasting

3.1 Maximum Likelihood Estimation

On the basis of observable data, we need to infer the distributions of the underlying latent variables and system parameters for the DI-MS-FM. If the interaction regime were constant, a standard approach to estimate (1)-(5) would be to construct an auxiliary state variable (S_t^1, S_t^2) with 2^2 states:

$$P(S_t^1 = k, S_t^2 = l | S_{t-1}^1 = i, S_{t-1}^2 = j) = (R_{11} \times D_{ik}^1 + R_{21} \times C_{jk}^{21})(R_{22} \times D_{jl}^2 + R_{12} \times C_{il}^{12}), \tag{35}$$

where $i, j, k, l \in \{1, 2\}$. However, when different interaction regimes come into play, the coefficients of matrices R_1 and R_2 are dependent on r_t, and the transition probability matrix of (S_t^1, S_t^2) becomes

Markov-switching itself:

$$P(S_t^1 = k, S_t^2 = l | S_{t-1}^1 = i, S_{t-1}^2 = j, r_t) = (R_{11}^{r_t} \times D_{ik}^1 + R_{21}^{r_t} \times C_{jk}^{21})(R_{22}^{r_t} \times D_{jl}^2 + R_{12}^{r_t} \times C_{il}^{12}), \tag{36}$$

so the standard estimation procedures can not be applied. This problem is easily overcome by using the joint state variable $\tilde{S}_t = (S_t^1, S_t^2, r_t)$ with $2^3 = 8$ states instead of (S_t^1, S_t^2). In this case, the transition probability matrix Π is constant and is computed as follows:

$$\begin{aligned} \Pi = P(\tilde{S}_t | \tilde{S}_{t-1}) &= P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t = j) \times P(S_t^2 | S_{t-1}^1, S_{t-1}^2, r_t = j) \times P(r_t = j | r_{t-1} = k) \\ &= P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t = j) \times P(S_t^2 | S_{t-1}^1, S_{t-1}^2, r_t = j) \times Q_{kj}, \end{aligned} \tag{37}$$

$j, k \in \{1, 2\}$. Note that, as we have mentioned above, due to the hierarchical structure that we impose on the chains (S_t^1, S_t^2, r_t), the matrix Π has a more parsimonious representation than a transition matrix of a Markov chain with 8 states would usually have. Indeed, matrix Π contains only 14 parameters instead of 56, which certainly facilitates the numerical optimization of the likelihood. For notational use, we arrange the eight states of $\tilde{S}_t$ in the following order: $(S_t^1, S_t^2, r_t) = \{(0,0,0)\,(1,0,0)\,(0,1,0)\,(1,1,0)\,(0,0,1)\,(1,0,1)\,(0,1,1)\,(1,1,1)\}$.

The classical Hamilton (1989) filter can then be applied. At each step, it updates the filtered probability $P(\tilde{S}_{t-1} = j | I_{t-1})$ to the next period $P(\tilde{S}_t = j | I_t)$, giving the likelihood $f(y_t | I_{t-1})$ as a by-product. Once the starting filtered probability $P(\tilde{S}_0 = j | I_0)$ is initiated (we suppose that the probability of starting in any of eight states of $\tilde{S}_0$ is equal, $P(\tilde{S}_0 = j | I_0) = 1/8$, $\forall j = 1, ..., 8$), the filtered probability for steps $t = 1, ..., T$ are calculated by iterating the following:

$$P(\tilde{S}_t = j, \tilde{S}_{t-1} = i | I_{t-1}, \gamma) = P(\tilde{S}_t = j | \tilde{S}_{t-1} = i, \gamma) P(\tilde{S}_{t-1} = i | I_{t-1}, \gamma), \tag{38}$$

$$f(y_t, \tilde{S}_t = j, \tilde{S}_{t-1} = i | I_{t-1}, \gamma) = f(y_t | \tilde{S}_t = j, \tilde{S}_{t-1} = i, I_{t-1}, \gamma) P(\tilde{S}_t = j, \tilde{S}_{t-1} = i | I_{t-1}, \gamma) \tag{39}$$

$$f(y_t | I_{t-1}, \gamma) = \sum_{j=1}^{8} \sum_{i=1}^{8} f(y_t, \tilde{S}_t = j, \tilde{S}_{t-1} = i | I_{t-1}, \gamma). \tag{40}$$

$$\begin{aligned} P(\tilde{S}_t = j, \tilde{S} = i | I_t, \gamma) &= \frac{f(y_t, \tilde{S}_t = j, \tilde{S}_{t-1} = i | I_{t-1}, \gamma)}{f(y_t | I_{t-1}, \gamma)} \\ &= \frac{f(y_t | \tilde{S}_t = j, \tilde{S}_{t-1} = i, I_{t-1}, \gamma) \times P(\tilde{S}_t = j, \tilde{S}_{t-1} = i | I_{t-1}, \gamma)}{f(y_t | I_{t-1}, \gamma)}, \end{aligned} \tag{41}$$

$$f(y_t | \tilde{S}_t = j, \tilde{S}_{t-1} = i, I_{t-1}, \gamma) = (2\pi)^{-1} (\sigma_{S_t^1}^2 \theta_{S_t^2}^2)^{-1/2} exp\{-\frac{1}{2} \frac{(\tilde{RF}_t)^2}{\sigma_{S_t^1}^2} - \frac{1}{2} \frac{(\tilde{FF}_t)^2}{\theta_{S_t^2}^2}\}, \tag{42}$$

$$P(\tilde{S}_t = j|I_t) = \sum_{i=1}^{2} P(\tilde{S}_t = j, \tilde{S}_t = i|I_t, \gamma), \tag{43}$$

where

$y_t = (RF_t, FF_t),$

$\gamma = (D^1, D^2, C^{12}, C^{21}, R^1, R^2, \mu_1, \mu_2, \beta_1, \beta_2, \sigma_1^2, \sigma_2^2, \theta_1^2, \theta_2^2, \varphi_1...\varphi_{p_1}, \psi_1...\psi_{p_2}),$

$\mu_{S_t^1} = \mu_2(S_t^1 - 1) - \mu_1(S_t^1 - 2),$

$\sigma^2_{S_t^1} = \sigma_2^2(S_t^1 - 1) - \sigma_1^2(S_t^1 - 2),$

$\beta_{S_t^2} = \beta_2(S_t^2 - 1) - \beta_1(S_t^2 - 2),$

$\theta_{S_t^2} = \theta_2(S_t^2 - 1) - \theta_1(S_t^2 - 2),$

$\tilde{RF}_t = RF_t - \mu_{S_t^1} - \varphi(L)RF_t,$

$\tilde{FF}_t = FF_t - \beta_{S_t^2} - \psi(L)FF_t.$

As a by-product of the Hamilton filter above, we obtain the log-likelihood function for the whole sample for any given value of γ:

$$\mathscr{L}(y, \gamma) = ln(f(y_T, y_{T-1}, ..., y_0|I_T, \gamma) = \sum_{t=1}^{T} ln(f(y_t|I_{t-1}, \gamma)), \tag{44}$$

where $f(y_t|I_{t-1}, \gamma)$ can be computed using formulas (38) to (43).

Once the filtered probability $P(\tilde{S}_t = j|I_t)$ is obtained for all $t = 1, ..., T$, it is possible to compute the smoothed probability $P(\tilde{S}_t = j|I_T)$ (we refer the reader to Hamilton (1989) for details). The filtered and smoothed probabilities for each chain can be obtained by integrating out the other chains in $\tilde{S}_t$, i.e.:

$$P(S_t^i = k|I_t) = \Sigma_{k=1}^2 \Sigma_{j=1}^2 P(S_t^i = i, S_t^{3-i} = k, r_t = j|I_t), \tag{45}$$

$$P(S_t^i = k|I_T) = \Sigma_{k=1}^2 \Sigma_{j=1}^2 P(S_t^i = i, S_t^{3-i} = k, r_t = j|I_T), \tag{46}$$

$$P(r_t = j|I_t) = \Sigma_{i=1}^2 \Sigma_{k=1}^2 P(S_t^i = i, S_t^{3-i} = k, r_t = j|I_t), \tag{47}$$

$$P(r_t = j|I_T) = \Sigma_{i=1}^2 \Sigma_{k=1}^2 P(S_t^i = i, S_t^{3-i} = k, r_t = j|I_T), \tag{48}$$

where $i \in \{1, 2\}$. Since the maximum likelihood is obtained with numerical algorithms, this estimation method can be applied only when the number of parameters is not too big. When more interacting chains with more states are involved, or when more interaction regimes are allowed for, the optimization algorithms may have difficulties to converge. In this case, the Forward-Backward algorithm and variational EM suggested by Pan et al. (2012) can be used. Pan et al. (2012) have successfully applied this approach to model the interaction between 50 states with 6 latent states each and 3 regimes of influence in order to evaluate flu epidemics.

The extended version of the model (see Section 2.5) can also be estimated with Maximum Likelihood after corresponding modifications in the transition probability matrix of $\tilde{S}_t$. Once any (or all) of the matrices Q, D^1, D^2, C^{12} and C^{21} becomes time-dependent, the matrix Π becomes dynamic as well, i.e. $\Pi_t = P(\tilde{S}_t|\tilde{S}_{t-1}, z_{t-1})$. Note that, since z_t is observable, the general form of the Hamilton filter (38)-(43) does not change and it can still be applied for the calculation of the likelihood function.

3.2 Forecasting

The in-sample analysis tools, such as filtered and smoothed probabilities discussed above, give a posteriori insight into the dating of both financial and business cycles and the types and timing of different interaction regimes. The out-of-sample analysis is a valuable complement, providing a probabilistic draft of future periods.

H-step ahead forecast of ergodic probability of the future state. Since the chain $\tilde{S}_t$ is the Markov chain of order one, it is straightforward that

$$P(\tilde{S}_{t+h}|\tilde{S}_t) = \Pi^h. \tag{49}$$

Then, the h-step ahead forecast for each individual chain can be computed by integrating the other two chains entering $\tilde{S}_t$ out. For example, the h-step ahead forecast for S^1_{t+h} is:

$$P(S^1_{t+h} = k|\tilde{S}_t) = \Sigma^2_{i=1}\Sigma^2_{j=1}P(S^1_{t+h} = k, S_{t+h} = i, r_{t+h} = j|\tilde{S}_t) = \Pi^h v, \tag{50}$$

where $i, j, k \in \{1, 2\}$, the vector v selects the columns of Π^h to be summed. For example, for $P(S^1_{t+h} = 1|\tilde{S}_t)$ the vector v is $v = (1\,0\,1\,0\,1\,0\,1\,0)'$.

H-step ahead forecast of the future state. It is also possible to compute an h-step ahead forecast of the state variable $\tilde{S}_t$

$$P(\tilde{S}_{t+h}|I_t) = \Sigma^8_{i=1}P(\tilde{S}_{t+h}|\tilde{S}_t = i)P(\tilde{S}_t = i|I_t) = P(\tilde{S}_t|I_t)'\Pi^h, \tag{51}$$

where $P(\tilde{S}_t|I_t)$ is the vector of filtered probabilities of being in state $\tilde{S}_t = i$, $i = \{1, ..., 8\}$.
As in the previous case, the h-step ahead forecast for each chain separately can be calculated by integrating the other chains out. For example, for S^1_{t+h} we obtain:

$$P(S^1_{t+h} = k|I_t) = \Sigma^2_{i=1}\Sigma^2_{j=1}P(S^1_{t+h} = k, S^2_{t+h} = i, r_{t+h} = j|I_t) = P(\tilde{S}_t|I_t)'\Pi^h v, \tag{52}$$

where, as before, the vector v selects the columns to sum over. For example, for $P(S^1_{t+h} = 2|\tilde{S}_t)$ the vector v is $v = (0\,1\,0\,1\,0\,1\,0\,1)'$.

H**-step ahead forecast of factors**. Given equations (1)-(5), the h-step ahead forecasts of the factors are obtained recursively as for regular $AR(p)$ forecasts. For example, if

$$RF_{t+h} = \mu(S^1_{t+h}) + \varphi(L)RF_{t+h-1} + \sigma(S^1_{t+h})\varepsilon_{t+h}, \tag{53}$$

then

$$\hat{RF}_{t+h} = E(RF_{t+h}|I_t) = E(\mu(S^1_{t+h})|I_t) + \varphi(L)E(RF_{t+h}|I_t), \tag{54}$$

where the $E(\mu(S^1_{t+h})|I_t)$ is a known function of $P(S^1_{t+h}|I_t)$ defined in (52), and $\varphi(L)E(RF_{t+h}|I_t)$ can be calculated using the forecasts obtained in the previous iterations, i.e. for $h-1$, $h-2$, etc. The h-step ahead forecast of the financial factor $\hat{FF}_{t+h|t}$ can be obtained in a similar way.

It is important to notice, however, that the DI-MS-FM is designed for the identification of the latent interaction regime and performs poorly in the forecasts of factors. For this reason in the following sections we focus solely on the in-sample and out-of-sample performance for the forecasts of states.

3.3 In-sample and out-of-sample performance

In this section we evaluate and compare the quality of in-sample and out-of-sample forecasts. We also verify whether the dynamical influence feature, which is obviously a complication to a regular two-factor Markov-switching Dynamic Factor model, actually helps to obtain better forecasts, both in-sample and out-of-sample.

In order to evaluate the performance of the model in terms of identification of the current state of each of the chains, it is difficult to use empirical data since we have no reference dating for the financial cycle and the interaction regimes. For this reason, we run a Monte Carlo experiment on the simulated data. We use the data generating process described in equations (1)-(5) with the parameters set to their estimated values that we obtained using data described in the following section (see Table **4.1**).[10] For simplicity we assume that there is no external intervention into the system, so z_t is omitted. The generated sample has $T = 500$ observations and is simulated 1000 times.

For the analysis of the accuracy of identification of states, we use the following indicators (we use a generic notation X_t for any of the chains S^1_t, S^2_t or r_t and X^*_t for the corresponding sequence of true states; T is the total number of observations, T_1 is the out-of-sample period, indices *is* and *oos* correspond to in-sample and out-of-sample cases, respectively):

1. QPS, the quadratic probability score. This indicator is conceptually similar to the mean squared error and is calculated in the following way:

$$QPS_{is}(X) = \frac{\Sigma^T_{t=1}(P(X_t = 2|I_T, \hat{\gamma}) - (X^*_t - 1))^2}{T}, \tag{55}$$

[10] The simulations show that model is very sensitive to the difference between the interaction regimes. For this reason, to generate our data, we use the estimates with a large difference between $\hat{R}_1$ and $\hat{R}_2$. However, according to our observations, in case the regimes are close, this does not deteriorate the accuracy of the identification of the states of the financial and business cycles.

$$QPS_{oos}(X) = \frac{\Sigma_{t=0}^{T_1-1}(P(X_{T+t+1} = 2|I_{T+t}, \hat{\gamma}) - (X^*_{T+t+1} - 1))^2}{T_1}, \tag{56}$$

where $P(X_t = 2|I_T, \hat{\gamma})$ is the smoothed probability of state 2 given in equation (46), $P(X_{T+t+1} = 2|I_{T+t}, \hat{\gamma})$ is the one-step-ahead forecast of probability of state 2 given in equation (52).

2. FPS, the false positive score. This indicator gives the proportion of misidentified states and is calculated as

$$FPS_{is}(X) = \frac{\Sigma_{t=1}^{T}(I_{P(X_{T+t+1}=2|I_{T+t},\hat{\gamma})>\alpha} - (X^*_t - 1))^2}{T}, \tag{57}$$

$$FPS_{oos}(X) = \frac{\Sigma_{t=0}^{T_1-1}(I_{P(X_{T+t+1}=2|I_{T+t},\hat{\gamma})>\alpha} - (X^*_{T+t+1} - 1))^2}{T_1}, \tag{58}$$

where $I_{\hat{P}>\alpha}$ is the indicator function taking value one when $\hat{P}$ is higher than a threshold α, conventionally set at 0.5.

3. $AUROC$, the area under the Receiver Operating Characteristic (ROC) curve. ROC curve gives the information on the accuracy of identification of each state as the threshold varies. In other words, it provides pairs of ratios - a fraction of correctly identified recession (financial downturn) periods and a fraction of missed expansion (financial boom) periods - for each arbitrary chosen level of α.[11]

   A better identification performance would imply a higher ratio of correct guesses and a lower percentage of mistakes for a given α. Then, the Area Under the ROC curve calculated as an integral over α measures discrimination, i.e. the general ability of the model to distinguish the states of a process (independently of α). $AUROC$ takes the value in $[0; 1]$, $AUROC = 1$ meaning that the state identification performance is perfect.

4. J, Youden's J statistic. J shows the identification performance at each given level of α and is calculated as a sum of fractions of correct guesses for each of the states, i.e.:[12]

$$J = \frac{TP}{TP + FN} + \frac{TN}{TN + FP} - 1.$$

   J takes values in $[-1; 1]$, with $J = 1$ meaning that the states are identified correctly in all periods, so the state identification performance is perfect. $J = 0$ means that the discrimination ability of the model is the same as of a regular coin, so the model is useless.

[11] According to its definition, ROC curve is a graphical plot which juxtaposes the false positive rate (FPR, on horizontal axis) and the true positive rate (TPR, on vertical axis) as the threshold of the classifier (in this case, the cut-off smoothed probability for a state to be identified as recession state) varies. TPR and FPR are defined as

$$TPR = \frac{TP}{TP+FN}, \; FPR = \frac{FP}{TN+FP},$$

where where TP is the number of true positives (correctly identified recessions or financial downturns), FN is the number of false negatives (incorrectly identified expansions or financial booms), TN is the number of true negatives (correctly identified expansions or financial booms) and FP is the number of false positives (incorrectly identified recessions or financial downturns).

[12] See the previous footnote.

For comparability, we set the threshold level α equal to 0.5 for all chains.

QPS can be considered as a mean squared error computed for the forecasts (nowcasts) of states, and is informative only in comparison of several models. The other three measures can be used independently, as the absolute values of FPS, $AUROC$ and J-statistic are informative b themselves.

3.3.1 In-sample performance

We present below the indicators of the in-sample performance of the DI-MS-FM. These results are opposed to the ones estimated with the help of a one-regime Markov-Switching VAR (see Billio and Sanzo (2015) for more details) with factors used as observable variables, i.e. discarding r_t from the framework, so that the interaction between S_t^1 and S_t^2 is described by an unrestricted transition probability matrix with four states (see Table 3.1). In this way, we intend to measure the potential losses of quality due to time-invariance of the interaction between the cycles.

We observe that the DI-MS-FM performs very well in the identification of the individual cycles - the error rate measured with QPS_{is} and FPS_{is} is low, whereas the classification quality measured with $AUROC_{is}$ and J_{is} is high. The interaction regime is more difficult to identify (both QPS_{is} and FPS_{is} are higher, whereas $AUROC_{is}$ and J_{is} are lower), however values of QPS_{is} and FPS_{is} do not exceed the ones usually obtained in the empirical papers for the business cycle.

When comparing the performance of DI-MS-FM to one-regime MS-VAR-DFM (see Table 3.1), one may notice that, all four measures of quality pointing in the same direction, neglecting the dynamics of the interaction deteriorates the accuracy of the identification of states of the individual cycles.

Table 3.1: In-sample performance: smoothed probabilities of the second state

	DI-MS-FM			
	QPS_{is}	FPS_{is}	$AUROC_{is}$	J_{is}
S^1- business cycle	0.0182	0.0238	0.9754	0.9222
S^2- financial cycle	0.0444	0.0567	0.9527	0.6819
r - interaction regimes	0.2034	0.2537	0.7343	0.3951

	One interaction regime MS-DFM			
	QPS_{is}	FPS_{is}	$AUROC_{is}$	J_{is}
S^1- business cycle	0.0765	0.0888	0.9340	0.8163
S^2- financial cycle	0.2454	0.2618	0.8353	0.6259
r - interaction regimes	-	-	-	-

Note: The table describes the ability of the models to identify state two of each of the chains: "recession" for S_t^1, "high volatility" for S_t^2 and "Interdependent chains" for r_t.

3.3.2 Out-of-sample performance

One-step ahead forecasts of states. For the out-of-sample analysis on simulated data, the sample is split into in-sample period with $T_1 = 1, ..., T - 60$ observations and out-of-sample period with $T_2 = 60$ observations, so that the number of observations in the in-sample period corresponds to the one we used in the real sample (395 observations)). The out-of-sample forecasts $P(\tilde{S}_{T_1+t+1}|I_{T_1+t})$, for $t = 1, ..., T - T_1 - 1$ are then constructed using the equations (51). As in case of in-sample analysis, we fit both DI-MS-FM and MS-VAR-DFM ('One interaction regime MS-DFM') to the generated data.

The results of the simulation are given in Table 3.2.

As expected, the out-of-sample behavior is inferior compared to in-sample performance. However, the quality is still satisfactory, the values of QPS_{oos} and FPS_{oos} for the business cycle corresponding to the ones usually obtained in the empirical exercises (see, for example, Matas-Mir et al. (2008)). Similarly to the in-sample performance, the introduction of switches in the interaction regime improves the quality of the out-of-sample identification of the individual cycles.

Table 3.2: Out-of-sample performance: one-step ahead forecast of the future state

	DI-MS-FM			
	QPS_{oos}	FPS_{oos}	$AUROC_{oos}$	J_{oos}
S^1- business cycle	0.0615	0.0712	0.8952	0.6574
S^2- financial cycle	0.1309	0.1279	0.6683	0.2435
r - interaction regimes	0.2857	0.3639	0.6195	0.1628

	One interaction regime MS-DFM			
	QPS_{oos}	FPS_{oos}	$AUROC_{oos}$	J_{oos}
S^1- business cycle	0.1128	0.1583	0.8721	0.6576
S^2- financial cycle	0.3412	0.4076	0.8004	0.4367
r - interaction regimes	-	-	-	-

4 Interaction between financial and business cycles in the US

In this empirical exercise we apply the DI-MS-FM to the US data in order to identify the existing interaction regimes between the financial cycle and the business cycle and to determine when each of them was activated. We leave the analysis of the impact of particular government policies on this interaction for further research.

We set $\varphi_{p_1} = \psi_{p_2} = 0$, $\forall p_1, p_2$.[13] We also impose several technical constraints in order to increase the convergence to the correct local maximum. More specifically, we set $diag(Q) > 0.5e$ (where e is a vector of ones) to avoid the situations when the influence regimes are not persistent. The initial values of β_0, β_1 and σ_0^2, σ_1^2 are set to the mean and the variance for the business cycle observations above and below 0 (the initial values of μ_0, μ_1 and θ_0^2, θ_1^2 are set similarly for the financial factor). The initial values of the matrices R^1 and R^2 are set to their potential values, for example, $diag(R^1) = [0.9, 0.9]'$, $diag(R^2) = [0.9, 0]'$.[14]

[13]The number of lags has been chosen according to the information criteria. Inclusion of lags of the dependent variables in equations (1) and (2) does not change the estimates significantly.

[14]Setting the initial values of these parameters to random leads to instability in the results. To solve this problem, we try different plausible values: 1) $diag(R^1) = [0, 0]'$, $diag(R^2) = [0.9, 0.9]'$, 2) $diag(R^1) = [0, 0.9]'$, $diag(R^2) = [0.9, 0.9]'$, 3) $diag(R^1) = [0.9, 0]'$, $diag(R^2) = [0.9, 0.9]'$, 4) $diag(R^1) = [0.9, 0.9]'$, $diag(R^2) = [0.9, 0.9]'$, 5) $diag(R^1) = [0, 0]'$, $diag(R^2) = [0, 0]'$, 6) $diag(R^1) = [0.5, 0]'$, $diag(R^2) = [0.5, 0.5]'$, 7) $diag(R^1) = [0, 0.5]'$, $diag(R^2) = [0.5, 0.5]'$. The output obtained with different these sets of initial values are equivalent qualitatively and very similar quantitatively.

4.1 Data description

We perform our analysis for the business and the financial cycle of the United States. To construct the business cycle indicator we use the Stock-Watson database of indicators from CITIBASE and available in the databank of the Federal Reserve Bank of Saint Louis.[15] The first principal component explains just 18% of the total variance, however it is highly correlated with the GDP growth, contrary to the other components. In practice, the first component is usually enough to describe the business cycle, the other inclusion of the other components giving only marginal improvement (see Doz and Petronevich (2015), for example). The full list of variables and the corresponding factor loadings can be found in Table A.2 and Figure A.1 in the Appendix.

We approximate the financial cycle with the first principal component extracted from the database containing 31 indicators of different segments of the financial sector most used in the empirical papers on financial cycles. In particular, we extended the list of indicators used by Guidolin et al. (2013) with the information on deposits, monetary aggregates, loans, reserve balances and other. The complete list is given in Table A.1 in the Appendix, while the factor loadings can be found in Figure A.2.[16]

All data are seasonally adjusted, stationarized (by taking first differences of logarithms) and standardized. The time-span covers the period 1976m06-2014m12. The dynamics of the factors and the correlation between them is presented in Appendix B.

4.2 Characteristics of cycles and identified interaction regimes

The estimation results are given in Table 4.1. According to the estimates, switches in the regime of the business cycle happen mostly in mean, whereas the variance stays relatively stable. On the contrary, the financial factor switches primarily in variance. We also find that expansions in both cycles, as well as recessions of the business cycle, are very persistent ($\hat{D}^1_{11}$, $\hat{D}^2_{11}$, $\hat{D}^1_{22}$, are close to one). Recessions in the financial cycle are less persistent ($\hat{D}^2_{22}$ is below 0.9). These estimates match the findings in the previous literature.

Now consider the parameters characterizing the influence. The business cycle is capable of transmitting both expansion and recession to the financial cycle (the coefficients $\hat{C}^{12}_{11}$ and $\hat{C}^{12}_{22}$ are above 0.9). The transmitting ability is reciprocal, although the financial cycle less likely to transmit expansion to the business cycle ($\hat{C}^{21}_{11}$ is only 0.74). A similar asymmetry of influence between the business cycle and the financial cycle (measured as industrial production growth rate and excess returns, correspondingly) was also detected by Billio and Sanzo (2015).

[15] see Stock and Watson (2005)

[16] Other datasets were also tested, see section 4.4.

Table 4.1: Estimation results

Business cycle			Financial cycle		
	$\hat{\gamma}$	$\hat{\sigma}_{\hat{\gamma}}$		$\hat{\gamma}$	$\hat{\sigma}_{\hat{\gamma}}$
$\hat{\mu}_1$	0.5718	(0.0374)	$\hat{\beta}_1$	0.1584	(0.0343)
$\hat{\mu}_2$	-0.8512	(0.0712)	$\hat{\beta}_2$	-0.8587	(0.2806)
$\hat{\sigma}_1^2$	0.3103	(0.0276)	$\hat{\theta}_1^2$	0.2742	(0.0311)
$\hat{\sigma}_2^2$	0.8107	(0.0851)	$\hat{\theta}_2^2$	4.0480	(0.9117)
$\hat{D}_{11}^1$	0.9897	(0.0156)	$\hat{D}_{11}^2$	0.9888	(0.0192)
$\hat{D}_{22}^1$	0.9809	(0.0121)	$\hat{D}_{22}^2$	0.8799	(0.2985)
$\hat{C}_{11}^{12}$	0.9059	(0.1311)	$\hat{C}_{11}^{21}$	0.7481	(0.0121)
$\hat{C}_{22}^{12}$	0.9899	(0.7864)	$\hat{C}_{22}^{21}$	0.9889	(0.0029)

Influence regimes			
	"Independent chains"		"Interdependent chains"
$\hat{R}_{11}^1$	0.9815	$\hat{R}_{11}^2$	0.8562
$\hat{R}_{22}^1$	0.9426	$\hat{R}_{22}^2$	0.1853
$\hat{q}_{11}$	0.9900	$\hat{q}_{22}$	0.9677

Note: The estimated specification is $RF_t = \mu_{s_t^1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma^2_{S_t^1})$, $FF_t = \beta_{s_t^2} + \xi_t$, $\xi_t \sim N(0, \theta^2_{S_t^2})$.

The detected abilities of state transmission are clearly necessary for understanding of the relation between the chains. In our framework these should be considered together with the parameters responsible for the influence regimes. The model identified two distinct and very persistent influence regimes ($\hat{q}_{11}$ and $\hat{q}_{22}$ are above 0.96). The values $\hat{R}_{11}^1$, $\hat{R}_{22}^1$, $\hat{R}_{11}^2$, $\hat{R}_{22}^2$ suggest that the first and the second regimes can be interpreted as "Independent cycles" and "Interdependent cycles", correspondingly. According to information criteria, the two regimes are not redundant: in case of a single influence regime the values of the information criteria are $AIC = 2047.6$, $BIC = 2135.3$, $HQ = 2080.2$, which is above $AIC = 2285.8$, $BIC = 2368.5$, $HQ = 2318.3$ for the DI-MS-FM.

We perform a Likelihood-ratio test in order to find out the direction of causality in each of the regimes. More precisely, we test if the high values of $\hat{R}_{11}^1$, $\hat{R}_{22}^1$ can be interpreted as the absence of causality in the first regime, and if the high value of $\hat{R}_{11}^2$ and the low value of $\hat{R}_{22}^2$ actually implies that in the second regime the business cycle leads the financial cycle. We test a joint hypothesis H_0 versus the alternative H_1, where

$$H_0: \begin{cases} \hat{R}_{11}^1 = 1 \\ \hat{R}_{22}^1 = 1 \\ \hat{R}_{11}^2 = 1 \\ \hat{R}_{22}^2 = 0 \end{cases}, \qquad H_1: \begin{cases} \hat{R}_{11}^1 \neq 1 \\ \hat{R}_{22}^1 \neq 1 \\ \hat{R}_{11}^2 \neq 1 \\ \hat{R}_{22}^2 \neq 0 \end{cases}. \tag{59}$$

The value of the test statistics is $LR = 81.91$ and largely overcomes the critical value at 5% of confidence probability ($\chi^2_{0.95,4} = 9.49$), so H_0 is not rejected.

4.3 Identifying the periods of recession, financial downturn and high interdependence between the cycles

The estimated smoothed probabilities of recession $P(S_t^1 = 2|I_T)$, financial downturn $P(S_t^2 = 2|I_T)$ and second influence regime $P(r_t = 2|I_T)$ are presented in Figures 4-6. Shaded areas correspond to NBER business cycle recessions and are given to verify the validity of the obtained estimates. On Figure 4 one can see that the model captures all business cycle recessions well. The smoothed probability of recession spikes exactly with the beginning of the NBER recession, without either false signals or missed recessions. Whereas the double-dip crisis of 1980 and 1981-1982 is identified very accurately, the duration of the other three recessions observed in the time-span - the early 1990s recession, the dot-com bubble and the Great Recession - appears to be overestimated by the model. This imprecision might be due to the fact that the US business cycle is reported to have at least three states (recession, expansion and slow growth), one of which we have omitted in this simple specification of the model.[17]

The adequacy of the estimated smoothed probabilities of financial downturns is difficult to evaluate since there is no benchmark dating of financial cycles. To provide at least some reference, we use the dates of the beginning of banking crises as identified by Laeven and Valencia (2013)) and Reinhart (2009) to pinpoint the gravest events in the US banking sector (September 1988 and July 2007) which certainly correspond to financial crises, even though it is possible that they do not cover all financial crises but only those in the banking sector. Comparing the graphs of the smoothed probability with these reference dates on Figure 5, we can see that the model captures the banking crisis of 2008 with much precision, but foreruns the crisis of 1988 by about 10 months. In general, smoothed probability of financial downturn detects all the major events in the last 40 years: the savings and loans crisis and bank crisis during the double-dip recession of 1980 and 1981-1982, Black Monday of 1987, early 1990s recession, the Russian crisis of 1998, bursting of dot-com bubble in 2001, the global financial crisis of 2008.

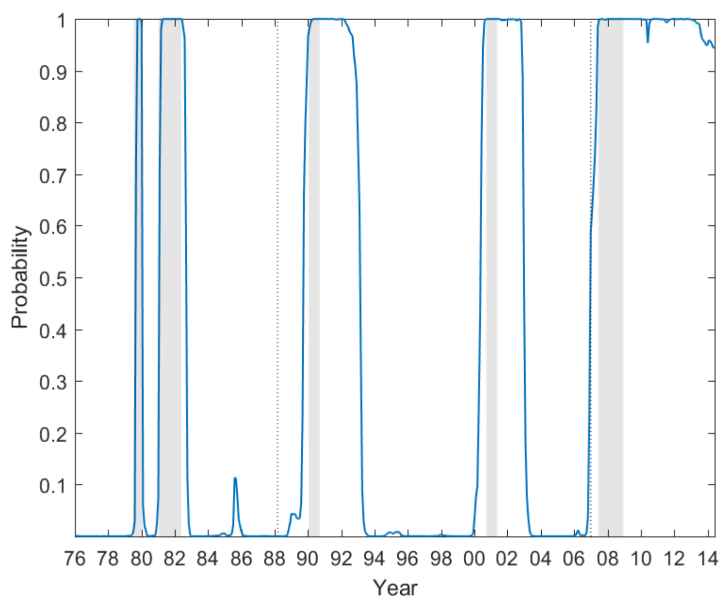
The ongoing influence regime at each point of time is clearly visible from Figure 6. The "Interdependent cycles" regime was active during the double-dip recession and the Great Recession. Both cases (and not during the other two observed recessions during the period under consideration) were marked with increased panic on the stock exchange, which can probably be an explanation of the higher interaction between the financial and the business cycles during these periods. This idea is consistent with the theory of sunspot equilibria: the exogeneous random Markov-Switching process r_t can be viewed as an extrinsic variable, influencing the economy through expectations but not affecting the fundamentals. In other words, if the agents' beliefs are such that the current shock (either financial or economic) is likely to be devastating, they act accordingly on the stock exchange, launching a self-reinforcing mechanism of transition of the shock from the financial sector to the real and back - the economy enters the "Interdependent cycles regime". Otherwise, if the agents are sure that the

[17] Indeed, the GDP growth rate was recovering much slower during the last three recessions comparing to the preceding ones.

shock is temporary (as the Black Monday of 1987, for example), the interaction is just not activated ("Independent cycles" influence regime is on), and the shock does not propagate.

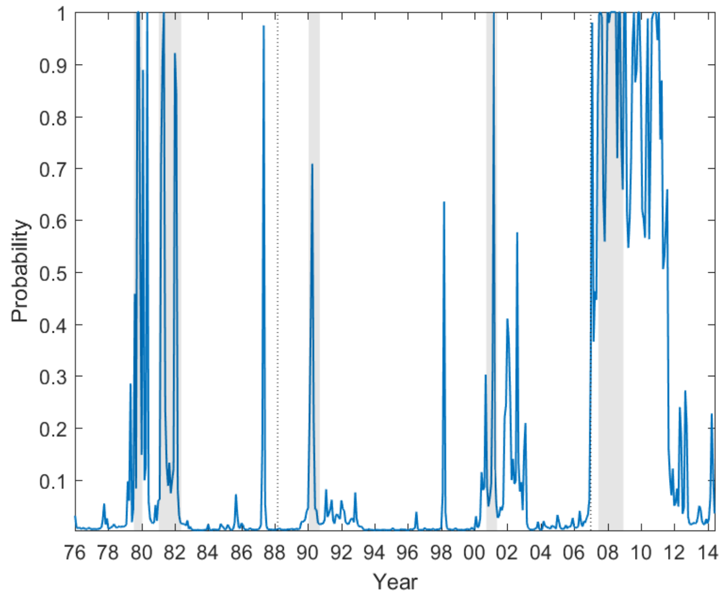
The estimates of the periods of high interaction seem reasonable. However, one may argue that the direction of causality between the two cycles (identified as business cycle leading the financial cycle) might not be the same in 1980-1982 and 2008. This misidentification of causality in the second case might arise from the fact that in this empirical exercise we allow for just two influence regimes. Given the relatively long period of low correlation between the two cycles in the middle of the sample, the model identified the regime of independent cycles and attributed any sort of other relation to the other regime. Therefore, once more influence regimes are allowed for, the model might be able to distinguish different types of interdependence. A certain evidence for this hypothesis is shown in the robustness check exercise below, where the causality direction in the second regime is shown to be different in different subsamples.

Figure 4: Smoothed probability of recession in the business cycle



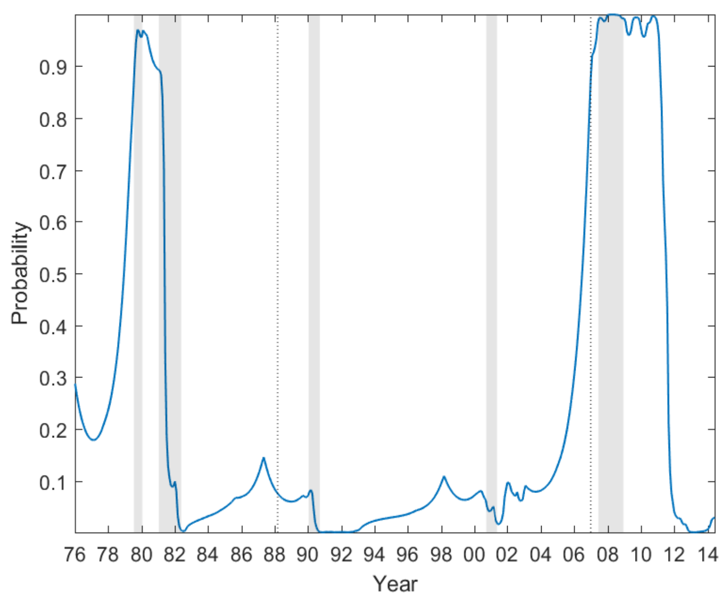
Note: Grey shaded areas correspond to NBER recessions, dotted vertical lines mark the beginning of systemic banking crises as identified by Laeven and Valencia (2013)) and Reinhart (2009).

Figure 5: Smoothed probability of financial downturn



Note: Grey shaded areas correspond to NBER recessions, while dotted vertical lines mark the beginning of systemic banking crises as identified by Laeven and Valencia (2013)) and Reinhart (2009).

Figure 6: Smoothed probability of the “Interdependent cycles” regime



Note: Grey shaded areas correspond to NBER recessions, dotted vertical lines mark the beginning of systemic banking crises as identified by Laeven and Valencia (2013)) and Reinhart (2009).

4.4 Robustness check

The dynamics of interaction suggested by the estimates of DI-MS-DFM, generally coherent with observations on the comovement of financial and business cycles in the US in the beginning and in the end of the sample, contributes to the discussion on the degree of interaction between 1982 and 2008. Currently, there is no consensus on it in the literature. For example, Gilchrist et al. (2009) find that "credit market shocks have contributed significantly to U.S. economic fluctuations during the 1990-2008 period". Gertler and Lown (1999) and Mody and Taylor (2004) suggest that yield spreads based on indexes of high yield corporate bonds perform well in forecasting of output during 1980s-1990s. Meeks (2012) find that "adverse credit shocks have contributed to declining output in every post-1982 recession". On the other hand, Stock and Watson (2003), find mixed evidence for the high-yield spread as a leading indicator of the business cycle. Rachdi and Ben Mbark (2013) find that the link between the cycles is bi-directional. Our own findings are in line with those of Rousseau and Watchel (2011), Valickova et al. (2015) who show that the link between financial sector and output growth has weakened worldwide and especially in the developed countries.

Given the ambiguity of findings on the interaction between the cycles, we check the robustness of our results by performing two auxiliary exercises: use of other indicators for the financial and business cycles and estimation of the model on subsamples. In this section we briefly present the main results of these exercises. More details can be found in Appendix C.1 and Appendix C.2.

We verify the validity of use of business and financial cycle indicators RF_t and FF_t by replacing them by two other proxies commonly used in the literature. According to Leamer (2015), the number of housing starts (New Privately Owned Housing Units Started) is a "critical part of the U.S. business cycle" and is therefore a good proxy for the business cycle[18] used by Conrad and Loch (2015), Ferrara and Vigna (2010), Luciani (2015) and others. In the same time, Claessens et al. (2012), Runstler and Vlekke (2015) and Drehmann et al. (2012) suggest that house prices, on a par with credit and equity markets, characterize the financial cycle.

To evaluate the impact of each of the indicators, for our robustness check we consider three alternative datasets: (RC1) RF_t and house price index;[19] (RC2) number of house starts[20] and FF_t; (RC3) number of house starts and house price index. The three cases are compared to the results obtained with the baseline scenario (BL).

Table C.1 and Figure C.1 show that the estimates obtained with four datasets are very similar. Importantly, the recessions and financial downturns identified with alternative proxies match the ones previously obtained very closely, with two exceptions. The number of house starts completely misses

[18]Certainly, other series could have been used to approximate the business cycle, either univariate (such as index of industrial production, for example) or composite indexes (such as Conference Board business cycle indicators), as well as the enhanced versions of the factors (with time-varying weights, for instance). We prefer to perform the robustness check on a single (but not composite) indicator in order to eliminate a possible additional impact of the method used for the construction of the aggregate indicator. The choice has been made in favor of housing starts since the industrial production index appeared to be not informative enough to capture all the business cycle recessions.

[19]We use NAREIT Composite Index as a measure of house price.

[20]Although the Conference Board considers this indicator as leading with respect to the cycle, the correlation with the RF_t and index of industrial production is the highest when the series are considered simultaneously, i.e. with zero lag. This observation has been also considered by Kydland et al. (2016).

out the dot-com bubble crisis; so does the house price index, as it ignores the stress evoked on the equity market. These observations allow us to conclude that RF_t and FF_t approximate the business and financial cycles at least as good as single series indicators, and, moreover, provide a more comprehensive view on each of the sectors.

Even more importantly, in all cases the two identified regimes of interaction correspond to independence and interdependence, as in the baseline case. While RC1 and RC3 confirm the independence between 1982 and 2008 crises, thus bringing another evidence on the weakening of the finance-growth nexus after 1980s. Also, the use of house price for financial cycle tends to exacerbate the degree of dependence of the financial cycle on the business cycle. The result of case RC2 is more ambiguous: the dependence is weaker and present between 1982 and 2008 as well, indicating that the results on this period should be considered with caution.

Table C.2 and Figure C.3 demonstrate the results obtained on the right and left subsamples, i.e. omitting the first and the last 100 observations (the double dip recession and the Great recession). The results indicate the interaction regimes is robustly identified as "Independent cycles" and "Interdependent cycles". However, the type of interdependence in terms of causality appears to be dependent on the subsample: while in the beginning the financial cycle seems to lead the business cycle, which is in line with the literature, later the causality inverses the direction. This finding suggests that the hypothesis of just two regimes of interaction is somewhat restrictive. Figure C.3 demonstrates that the level of systemic risk during the period of Great recession is comparable only to the double-dip recession, as the other two critical periods - the early 1990s recessions and the dot-com bubble - are classified as periods of "Independent cycle" regimes when sample spans the Great recession.

4.5 Transition probabilities and smoothed probabilities of future states

Table (4.2) contains the estimated one-step ahead transition probabilites for the business cycle and the financial cycle ($P(S_t^i|S_{t-1}^i, S_{t-1}^k, r_{t-1})$, $i, k \in \{1, 2\}$, $i \neq k$) calculated using equation (49). These estimates are important since they provide a description of the individual characteristics of each of the cycles. So save space, we report only the probability to switch to expansion (financial boom) $P(S_t^i = 1|S_{t-1}^i, S_{t-1}^k, r_{t-1})$, the probability of recession (financial downturn) being ($P(S_t^i = 2|S_{t-1}^i, S_{t-1}^k, r_{t-1}) = 1 - P(S_t^i = 1|S_{t-1}^i, S_{t-1}^k, r_{t-1})$. Table (4.2) contains the forecasts for all possible combinations of the past values of the chains S_{t-1}^1, S_{t-1}^2, r_{t-1} known at $t-1$.

Table 4.2: Estimated one-step ahead probability of expansion and financial boom ($P(S_t^i = 1|S_{t-1}^i, S_{t-1}^k, r_{t-1})$)

	Probability of expansion in the business cycle $P(S_t^1 = 1\|S_{t-1}^1, S_{t-1}^2, r_{t-1})$	
	"Independent chains"	"Interdependent chains"
$S_{t-1}^1 = 1,\ S_{t-1}^2 = 1$	0.99	0.95
$S_{t-1}^1 = 1,\ S_{t-1}^2 = 2$	0.97	0.85
$S_{t-1}^1 = 2,\ S_{t-1}^2 = 1$	0.02	0.11
$S_{t-1}^1 = 2,\ S_{t-1}^2 = 2$	0.01	0.01

	Probability of financial boom $P(S_t^2 = 1\|S_{t-1}^1, S_{t-1}^2, r_{t-1})$	
	"Independent chains"	"Interdependent chains"
$S_{t-1}^2 = 1,\ S_{t-1}^1 = 1$	0.98	0.92
$S_{t-1}^2 = 1,\ S_{t-1}^1 = 2$	0.93	0.21
$S_{t-1}^2 = 2,\ S_{t-1}^1 = 1$	0.75	0.87
$S_{t-1}^2 = 2,\ S_{t-1}^1 = 2$	0.69	0.16

For the business cycle, the probability to switch to expansion depends on the previous state of the business cycle to a large extent. Both expansion and recession states are very persistent (the probability to stay in expansion for any past conditions is above 0.85; similarly, the probability to stay in recession is above 0.89). However, when the "interdependent cycles regime" is active, the impact of the financial cycle is not negligible: financial downturn decreases the probability that the business cycle switches from recession to expansion (from 0.11 to 0.01) confirming the findings of Claessens et al. (2012) who found that downturns in financial sector tend to make recessions longer. In the same manner, financial downturn reduces chances to stay in expansion in the business cycle (the probability decreases from 0.95 to 0.85).

The probability of financial boom depends both on its past and on the past influence regime. In the "Independent cycles" regime the boom state is very persistent contrary to the downturn state (with the probability to stay in the state above 0.93 and 0.25 (under any past conditions) correspondingly). In the "Interdependent cycles" regime, the past state of the business cycle plays a decisive role. When the business cycle is in expansion, the probability to stay in financial boom is high and is close to the corresponding one in the "Independent cycles" regime. However, a recession in the business cycle decreases this probability dramatically: from 0.92 to 0.21 (for the probability of staying in financial boom), and from 0.87 down to 0.16 (for the probability to switch from financial downturn to boom).

The findings above indicate that the downturns in the financial cycles are temporary by their nature, as the financial market in the developed economies is flexible enough to absorb the shocks relatively quickly. For this reason, on Figure 5 the episodes of financial instability are presented just as spikes in the smoothed probability during the "Independent cycles" regime. To the contrary, when the financial cycle enters into interaction with the business cycle, the downturn state becomes much more persistent.

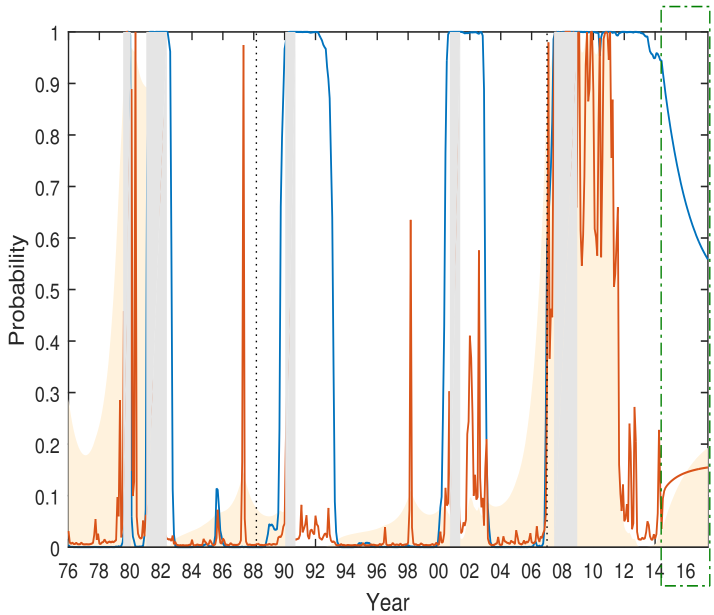
What are the projections of the model for the future? Figure 7 gathers the 36 months ahead forecasts of the smoothed probability of recession (blue line), financial downturn (red line) and "Interdependent cycle" regime (yellow area). The model thus predicts that by 2018 the period of low growth rates in the real sector will be over, financial sector will be stable and the "Independent cycles" regime will dominate.

What sort of implication can this have for policy-makers? Even though at this moment theoretical models do not have an unequivocal answer to the question on linkages between financial and business cycles, the impact of certain instruments of monetary, macro- and microprudential policy, and so do not provide an optimal policy rule, the knowledge of the current state of both cycles as well as the level of their interaction can be helpful for policy adjustments. For example, when the cycles are independent, the spillover effects documented by Zdzienicka et al. (2015), such as the impact of monetary policy on the stability of the financial sector, can be quite limited, which may allow to run more aggressive policies to stimulate either of the cycles. Similarly, the trade-off between financial stability and economic prosperity in the environment of the low interest rates discussed by Coimbra and Rey (2017) and Heider and Schepens (2017) can be less pronounced. On the contrary, when the cycles are interdependent, the regulator should be prepared to implement large interventions since the the recessions appear to be longer and more severe (Claessens et al. (2012)), and the financial sector needs increased support to stabilize.

Given the aggravated character of recessions during the periods of high interaction between the cycles, the set of monetary, fiscal and macroprudential measures should be directed towards the reduction of the procyclicality of the financial sector. Cerutti et al. (2015) find that macroprudential policy is an effective instument for this purpose and works better during the bust phase of the financial cycle and are more efficient in emergent economies rather than advanced ones. Blanchard et al. (2010) suggest that the monetary policy should take into account the assets price movements, too, however, by now is it not clear how to operationalize this. Another solution for mitigating credit cycles and dramatically reducing the level of government and public debt, proposed by Fischer (1936) and recently rediscovered by Kumhof and Benes (2014), is the radical idea of separation of monetary and credit functions of the banking system, also known as Chicago plan.

Whatever the relevant policy is, given the usual lag between the moment when a problem in an economy is recognized and the moment when the undertaken policy starts giving the first effects, timing is very important. In this concern, the probabilities of the influence regimes and states of individual cycles are of a great use since they provide an operative measure of the current state of the economy and future tendencies, and can be updated as soon as new information arrives. Moreover, once the causality direction is identified for each of the influence regimes, the leading cycle can serve as an early-warning indicator.

Figure 7: 36 months ahead forecast of smoothed probability



Note: Blue line and red line correspond to the smoothed probability of recession in the business cycle and the downturn state in the financial cycle, respectively. Yellow area marks the smoothed probability of being in the "Interdependent cycle" regime. Grey shaded areas correspond to NBER recessions.

5 Conclusion

Previous findings in the literature on business and financial cycles have shown that the cycles evolve, and so does the interaction between them. In this paper we suggest a flexible econometric framework, the Dynamical Influence Markov Switching Dynamic Factor model (DI-MS-FM), which allows to capture the changes in this interaction. Contrary to the existing models of the joint dynamics of business and financial cycles, we allow the interaction to be intrinsically dynamical, which implies that there is no need to search for an exogenous variable which could serve as as a proxy for the process governing the interaction. Based on the mix of the Dynamical influence model from computer science and the classical Markov-Switching model, the DI-MS-FM produces a wide range of statistical tools which can be very useful to design a relevant policy mix for mitigating the effects of downturns in both cycles as well as for reduction of the procyclicality of the financial cycle. More precisely, besides the individual characteristics of the cycles, the model allows to characterize the existing influence regimes in terms of leading-lagging relation between them as well as the degree of their interdependence, and to provide a probabilistic indicator of being in a particular regime of interaction at each point of time. Forecasts of the future states and future influence regimes can also be calculated.

We applied the model to the macroeconomic and financial series of the US for the period 1976m06 to 2014m12. The obtained estimates complement the findings in the previous literature. The model clearly identifies two distinct influence regimes, “Independent cycles” and “Interdependent cycles”, the second being active during the double-dip recession in July 1979-November 1981 and the Great Recession in January 2007-January 2012. The periods of higher interaction are well detected, although the results may be even more telling if one allows for three influence regimes.

As any other model, the DI-MS-FM has several limitations. First, it requires the time span to be long enough in order to make sure that all the regimes of all chains are observed at least once. This implies that the more flexibility one introduces into the model (by increasing the number of chains, individual states, influence regimes), the more data is needed, which can obviously be a problem especially for the analysis of developing countries. Secondly, the simulations show that the influence regimes are well identified only when they are different enough, however, this does not deteriorate the quality of the estimates the individual behavior of each cycle. Nevertheless, this issue can be solved by a more accurate selection of initial values for the optimization process.

The model can be extended in several ways. The one mentioned in this paper concerns the introduction of policy-dependence into the parameters responsible for state transitions in order to evaluate the effect of government policy on the duration of recessions, financial downturns and regime of high interaction between business and financial cycles. The other straightforward direction is the generalization of the model for a larger number of influence regimes and states of each of the cycles. Secondly, it seems appealing to engage more chains into the dynamical interaction, for example, by letting the credit and equity part of the financial market each follow their individual chain. Another interesting application of this kind concerns the interaction of business and financial cycles of several countries (for example, the core countries of the Euro area) which would allow to asses the contribution of each country to the cross-country systemic risk, identify the clusters of interdependence, and construct an indicator of systemic risk in the region. Third, we can let the cycles to interact not only on the level of underlying latent finite-state processes, but also on the level of observations by allowing for cross-correlation in the error terms of the DGPs of the cycles and/or by introducing a VAR structure in equations (1) and (2), which might improve the forecasting ability of the model. In this case the identification issues concerning the distinction between the observation-level and chain-level interaction should be resolved, as well as the causality definition is to be reconsidered.

The Dynamic Influence Markov-Switching Dynamic Factor Model, to our knowledge, is the first instrument for objective and reproducible empirical identification of the regimes of interaction between the real and the financial sectors. Even in its basic form, it appears to produce meaningful inference on individual features of cycles as well as the dynamics of their interaction. All this information can be useful for policy-makers as it enables to adjust the fiscal, monetary and macroprudential policy according to the current influence regime.

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A Composition of factors RF_t and FF_t

Table A.1: List of financial variables used for describing the financial cycle in the US

Series	**Source**
Series from Guidolin et al. (2013) database	
Monthly SP500 portfolio returns	FREDII
3mtb, monthly rate	FREDII
10-Year Treasury Constant Maturity Rate	FREDII
2-Year Treasury Constant Maturity Rate	FREDII
Moody's Seasoned Baa Corporate Bond Yield (to change, see Shiller)	FREDII
Composite NAREIT	NAREIT
Equity REITs	NAREIT
Mortgage REITs	NAREIT
Excess return on a value-weighted market	FREDII
S&P 500 dividend yield — (12 month dividend per share)/price.	FREDII
Moody's Seasoned Baa Corp. Bond Yield to Yield on 10-Year Treasury Const. Maturity	FREDII
10-Year Treasury Constant Maturity Minus 3-Month Treasury Constant Maturity	FREDII
10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant Maturity Rate	FREDII

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Table A.1 – continued from previous page

Series	Source
Unexpected inflation rate	FREDII
Industrial production index	FREDII
Real personal consumption expenditures	FREDII
Other series	
3-month Tbill rate of return minus CPI	FREDII
SP500 PE ratio	FREDII
Federal funds effective rate	FREDII
Monetary Base; Total	FREDII
Total Reserve Balances Maintained with Federal Reserve Banks	FREDII
M1 Money Stock	FREDII
M2 Money Stock	FREDII
Federal Debt: Total Public Debt as Percent of Gross Domestic Product	FREDII
Median Sales Price for New Houses Sold in the United States	FREDII
Total Assets, All Commercial Banks	FREDII
Commercial and Industrial Loans, All Commercial Banks	FREDII
Loans and Leases in Bank Credit, All Commercial Banks	FREDII
Total Savings Deposits at all Depository Institutions	FREDII
Loans to deposits ratio	FREDII
Consumer Credit Outstanding (Levels)	FREDII

Table A.2: Components of RF_t and their factor loadings

Abbreviation	Indicator	Loading
RPI	Real Personal Income	0.0413
W875RX1	Real personal income ex transfer receipts	0.0569
DPCERA3M086SBEA	Real personal consumption expenditures	0.0366
CMRMTSPLx	Real Manu. and Trade Industries Sales	0.0725
RETAILx	Retail and Food Services Sales	0.0578
INDPRO	IP Index	0.0127
IPFPNSS	IP: Final Products and Nonindustrial Supplies	0.0122
IPFINAL	IP: Final Products (Market Group)	0.0109
IPCONGD	IP: Consumer Goods	0.0754
IPDCONGD	IP: Durable Consumer Goods	0.0673
IPNCONGD	IP: Nondurable Consumer Goods	0.0502
IPBUSEQ	IP: Business Equipment	0.0117
IPMAT	IP: Materials	0.0111
IPDMAT	IP: Durable Materials	0.0119
IPNMAT	IP: Nondurable Materials	0.0853
IPMANSICS	IP: Manufacturing (SIC)	0.0132
IPB51222S	IP: Residential Utilities	0.0167
IPFUELS	IP: Fuels	0.0128
NAPMPI	ISM Manufacturing: Production Index	0.0138
CUMFNS	Capacity Utilization: Manufacturing	0.0142
HWI	Help-Wanted Index for United States	0.0633
HWIURATIO	Ratio of Help Wanted/No. Unemployed	0.0112
CLF16OV	Civilian Labor Force	0.0517
CE16OV	Civilian Employment	0.0110
UNRATE	Civilian Unemployment Rate	0.0858
UEMPMEAN	Average Duration of Unemployment (Weeks)	0.0562
UEMPLT5	Civilians Unemployed - Less Than 5 Weeks	0.0827
UEMP5TO14	Civilians Unemployed for 5-14 Weeks	0.0415
UEMP15OV	Civilians Unemployed - 15 Weeks & Over	0.0899
UEMP15T26	Civilians Unemployed for 15-26 Weeks	0.0547
UEMP27OV	Civilians Unemployed for 27 Weeks and Over	0.0716
CLAIMSx	Initial Claims	0.0143
PAYEMS	All Employees: Total nonfarm	0.0170
USGOOD	All Employees: Goods-Producing Industries	0.0792
CES1021000001	All Employees: Mining and Logging: Mining	0.0218
USCONS	All Employees: Construction	0.0107
MANEMP	All Employees: Manufacturing	0.0153
DMANEMP	All Employees: Durable goods	0.0147
NDMANEMP	All Employees: Nondurable goods	0.0123
SRVPRD	All Employees: Service-Providing Industries	0.0151
USTPU	All Employees: Trade, Transportation & Utilities	0.0156
USWTRADE	All Employees: Wholesale Trade	0.0160
USTRADE	All Employees: Retail Trade	0.0132
USFIRE	All Employees: Financial Activities	0.0122
USGOVT	All Employees: Government	0.0432
CES0600000007	Avg Weekly Hours : Goods-Producing	0.0491
AWOTMAN	Avg Weekly Overtime Hours : Manufacturing	0.0339
AWHMAN	Avg Weekly Hours : Manufacturing	0.0499
NAPMEI	ISM Manufacturing: Employment Index	0.0144
HOUST	Housing Starts: Total New Privately Owned	0.0140
HOUSTNE	Housing Starts, Northeast	0.0122
HOUSTMW	Housing Starts, Midwest	0.0135
HOUSTS	Housing Starts, South	0.0121
HOUSTW	Housing Starts, West	0.0122
NAPM	ISM : PMI Composite Index	0.0124
NAPMNOI	ISM : New Orders Index	0.0125
NAPMSDI	ISM : Supplier Deliveries Index	0.0129
NAPMII	ISM : Inventories Index	0.0870
AMDMNOx	New Orders for Durable Goods	0.0118
AMDMUOx	Unfilled Orders for Durable Goods	0.0158
BUSINVx	Total Business Inventories	0.0135
ISRATIOx	Total Business: Inventories to Sales Ratio	0.0119
M1SL	M1 Money Stock	0.0128

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Abbreviation	Indicator	Loading
M2SL	M2 Money Stock	0.0535
M2REAL	Real M2 Money Stock	0.0118
AMBSL	St. Louis Adjusted Monetary Base	0.0105
TOTRESNS	Total Reserves of Depository Institutions	0.0536
NONBORRES	Reserves Of Depository Institutions	0.0461
BUSLOANS	Commercial and Industrial Loans	0.0113
REALLN	Real Estate Loans at All Commercial Banks	0.0661
NONREVSL	Total Nonrevolving Credit	0.0964
CONSPI	Nonrevolving consumer credit to Personal Income	0.0982
S&P 500	S&P's Common Stock Price Index: Composite	0.0792
S&P: indust	S&P's Common Stock Price Index: Industrials	0.0748
S&P div yield	S&P's Composite Common Stock: Dividend Yield	0.0103
S&P PE ratio	S&P's Composite Common Stock: Price-Earnings Ratio	0.0108
FEDFUNDS	Effective Federal Funds Rate	0.0204
CP3Mx	3-Month AA Financial Commercial Paper Rate	0.0551
TB3MS	3-Month Treasury Bill:	0.0103
TB6MS	6-Month Treasury Bill:	0.0102
GS1	1-Year Treasury Rate	0.0628
GS5	5-Year Treasury Rate	0.0838
GS10	10-Year Treasury Rate	0.0899
AAA	Moody's Seasoned Aaa Corporate Bond Yield	0.0590
BAA	Moody's Seasoned Baa Corporate Bond Yield	0.0576
COMPAPFFx	3-Month Commercial Paper Minus FEDFUNDS	0.0625
TB3SMFFM	3-Month Treasury C Minus FEDFUNDS	0.0492
TB6SMFFM	6-Month Treasury C Minus FEDFUNDS	0.0228
T1YFFM	1-Year Treasury C Minus FEDFUNDS	0.0210
T5YFFM	5-Year Treasury C Minus FEDFUNDS	0.0138
T10YFFM	10-Year Treasury C Minus FEDFUNDS	0.0452
AAAFFM	Moody's Aaa Corporate Bond Minus FEDFUNDS	0.0430
BAAFFM	Moody's Baa Corporate Bond Minus FEDFUNDS	0.0583
TWEXMMTH	Trade Weighted U.S. Dollar Index: Major Currencies	0.0806
EXSZUSx	Switzerland / U.S. Foreign Exchange Rate	0.0101
EXJPUSx	Japan / U.S. Foreign Exchange Rate	0.0657
EXUSUKx	U.S. / U.K. Foreign Exchange Rate	0.0456
EXCAUSx	Canada / U.S. Foreign Exchange Rate	0.0672
PPIFGS	PPI: Finished Goods	0.0194
PPIFCG	PPI: Finished Consumer Goods	0.0724
PPIITM	PPI: Intermediate Materials	0.0713
PPICRM	PPI: Crude Materials	0.0959
OILPRICEx	Crude Oil, spliced WTI and Cushing	0.0573
PPICMM	PPI: Metals and metal products:	0.0441
NAPMPRI	ISM Manufacturing: Prices Index	0.0789
CPIAUCSL	CPI : All Items	0.0126
CPIAPPSL	CPI : Apparel	0.0879
CPITRNSL	CPI : Transportation	0.0437
CPIMEDSL	CPI : Medical Care	0.0646
CUSR0000SAC	CPI : Commodities	0.0120
CUUR0000SAD	CPI : Durables	0.0828
CUSR0000SAS	CPI : Services	0.0399
CPIULFSL	CPI : All Items Less Food	0.0537
CUUR0000SA0L2	CPI : All items less shelter	0.0772
CUSR0000SA0L5	CPI : All items less medical care	0.0773
PCEPI	Personal Cons. Expend.: Chain Index	0.0894
DDURRG3M086SBEA	Personal Cons. Exp: Durable goods	0.0873
DNDGRG3M086SBEA	Personal Cons. Exp: Nondurable goods	0.0516
DSERRG3M086SBEA	Personal Cons. Exp: Services	0.0818
CES0600000008	Avg Hourly Earnings : Goods-Producing	0.0573
CES2000000008	Avg Hourly Earnings : Construction	0.0541
CES3000000008	Avg Hourly Earnings : Manufacturing	0.0153
UMCSENTx	Consumer Sentiment Index	0.0569
MZMSL	MZM Money Stock	0.0554
DTCOLNVHFNM	Consumer Motor Vehicle Loans Outstanding	0.0214
DTCTHFNM	Total Consumer Loans and Leases Outstanding	0.0439
INVEST	Securities in Bank Credit at All Commercial Banks	0.0223

Continued on next page

Table A.2 – continued from previous page

Abbreviation	Indicator	Loading

Data source: Federal Reserve Bank of St. Louis, https://research.stlouisfed.org/econ/mccracken/fred-databases/

Figure A.1: Top 50 factor loadings of RF_t

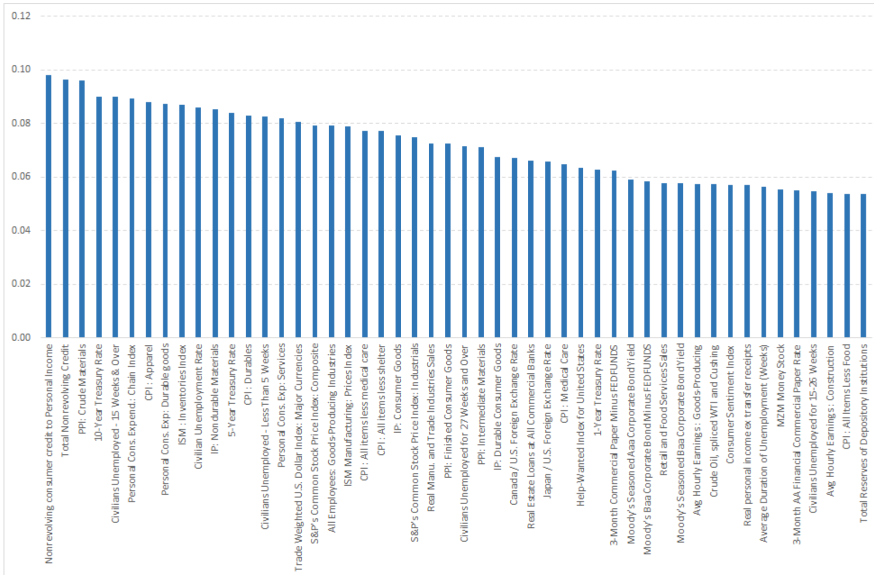
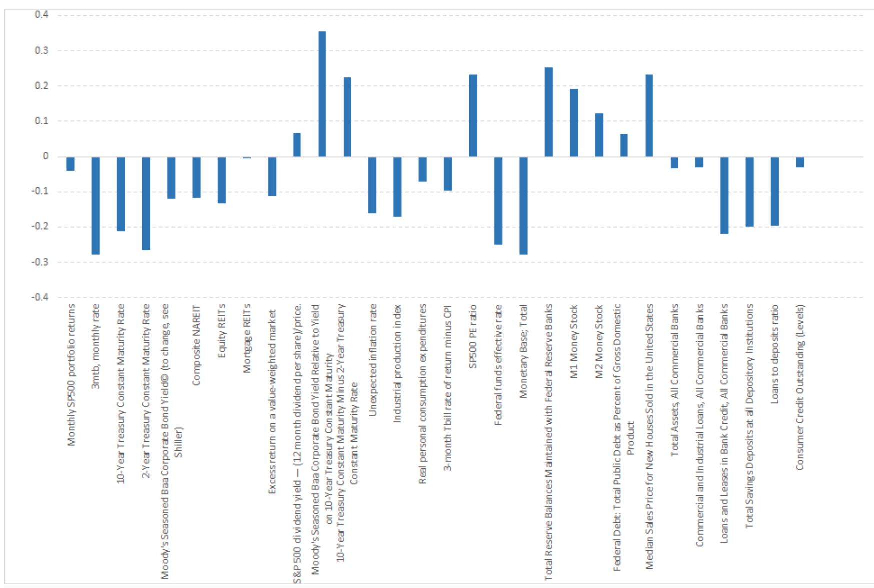


Figure A.2: Factor loadings of FF_t

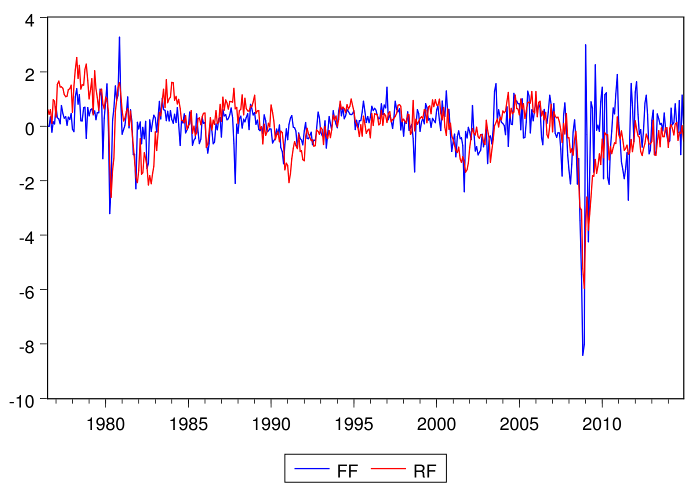


B Dynamics of RF_t and FF_t

Figure B.3 shows the dynamics of the extracted factors RF_t and FF_t. Not surprisingly, it differs from the cycles proxies presented in Figure 1 since they correspond to the growth cycle rather than business cycle (defined as deviation from the trend).

As the state of financial downturn is usually characterized not only by low levels of the corresponding financial cycle indicator but also by its high volatility,[21] the usual correlation estimated on a moving window is not very informative as it would ignore the volatility aspect of the financial cycle. For this reason, to make a preliminary assessment of the interaction between the cycles approximated by RF_t and FF_t, we estimate the correlation between the signals of recession and financial downturn extracted from the factors[22] (see Figure B.4). As in Figure 2, the correlation is lower in the middle of the sample, although the period of high interaction starts a much earlier, indicating a possibility of a slightly different pattern of interaction for the growth cycle with respect to the business cycle.

Figure B.3: Dynamics of RF_t and FF_t

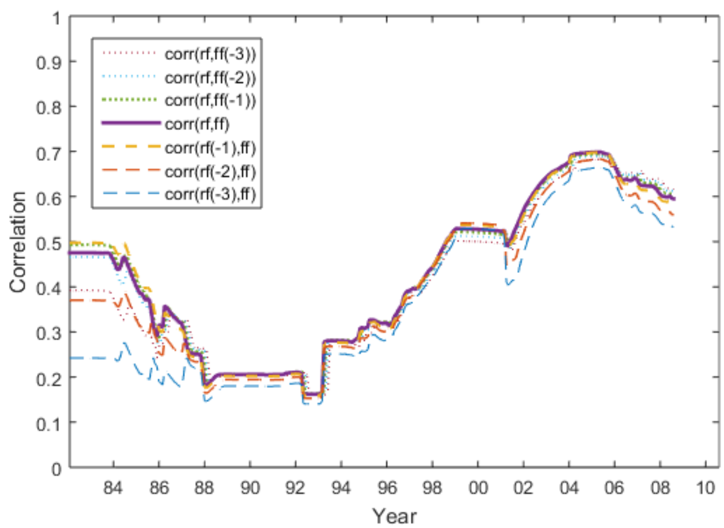


Note: blue and red line correspond to RF_t and FF_t respectively

[21] When the cycle is characterized by the growth rate series.

[22] we use smoothed probability of recession (financial downturn) estimated on RF_t (FF_t) with a standard Markov-Switching model à la Hamilton (1989).

Figure B.4: Cross-correlations between RF_t and FF_t



Note: Cross-correlations between the smoothed probability of recession (estimated on RF_t with a Markov-Switching model by Hamilton (1989)) and the smoothed probability of financial downturn (estimated on FF_t) computed on a moving window with width $w = 141$, i.e. a estimate for a date t is obtained using observations from $t - 70$ to $t + 70$.

C Robustness check

C.1 Alternative dataset

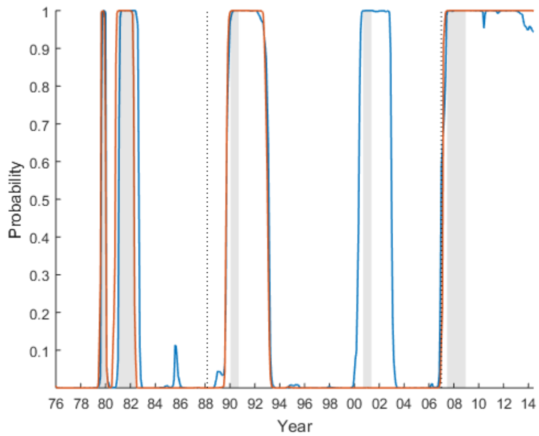
Table C.1: DI-MS-FM estimates on alternative datasets

	BL	RC1	RC2	RC3
Business cycle				
$\hat{\mu}_1$	0.57***	0.57***	0.58***	0.59***
$\hat{\mu}_2$	-0.85***	-0.85***	-1.19***	-1.19***
$\hat{\sigma}_1^2$	0.31***	0.31***	0.30***	0.29***
$\hat{\sigma}_2^2$	0.81***	0.81***	0.31***	0.31***
$\hat{D}_{11}^1$	0.99***	0.99***	0.99***	0.99***
$\hat{D}_{22}^1$	0.98***	0.98***	0.99**	0.99***
$\hat{C}_{11}^{12}$	0.91***	0.72***	0.93***	0.73**
$\hat{C}_{22}^{12}$	0.99*	0.85***	0.99***	0.87***
Financial cycle				
$\hat{\beta}_1$	0.16***	0.06***	0.15***	0.06***
$\hat{\beta}_2$	-0.86***	-0.39***	-0.97***	-0.37***
$\hat{\theta}_1^2$	0.27***	0.46***	0.29***	0.46***
$\hat{\theta}_2^2$	4.05***	4.39***	4.47***	4.26***
$\hat{D}_{11}^2$	0.99***	0.98***	0.99***	0.99***
$\hat{D}_{22}^2$	0.88***	0.36***	0.59**	0.39***
$\hat{C}_{11}^{21}$	0.75***	0.55***	0.98***	0.99***
$\hat{C}_{22}^{21}$	0.99***	0.99***	0.99***	0.99***
Influence regimes				
$\hat{R}^1$	$\begin{bmatrix} 0.98 & 0.06 \\ 0.02 & 0.94 \end{bmatrix}$	$\begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix}$	$\begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix}$	$\begin{bmatrix} 0.99 & 0.04 \\ 0.01 & 0.96 \end{bmatrix}$
$\hat{R}^2$	$\begin{bmatrix} 0.86 & 0.81 \\ 0.14 & 0.19 \end{bmatrix}$	$\begin{bmatrix} 0.92 & 0.44 \\ 0.08 & 0.56 \end{bmatrix}$	$\begin{bmatrix} 0.91 & 1.00 \\ 0.09 & 0.00 \end{bmatrix}$	$\begin{bmatrix} 0.86 & 1.00 \\ 0.14 & 0.00 \end{bmatrix}$
$\hat{q}_{11}$	0.99	0.99	0.99	0.99
$\hat{q}_{22}$	0.97	0.95	0.99	0.95

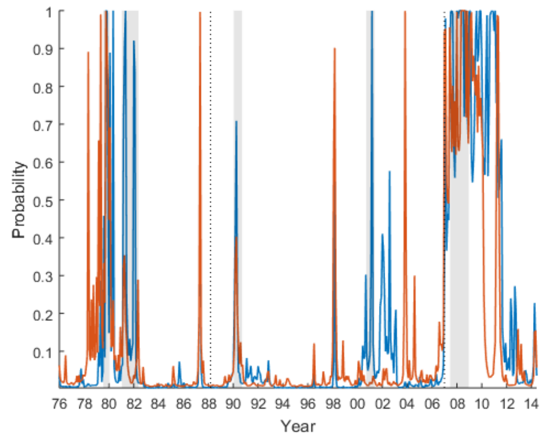
Note: The parameters significant at 15%, 10% and 5% are marked with *, ** and ***, correspondingly. BL stands for baseline dataset, the description of cases RC1, RC2 and RC3 is given in Section 4.4.

In all figures below, grey shaded areas correspond to NBER recessions, dotted vertical lines mark the beginning of systemic banking crises as identified by Laeven and Valencia (2008 and 2010) and Reinhart and Rogoff (2008).

Figure C.1: Smoothed probability of recession and financial downturn



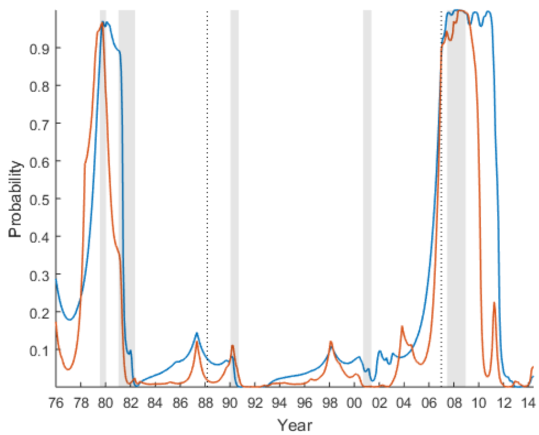
(a) Business cycle: RF_t vs HS



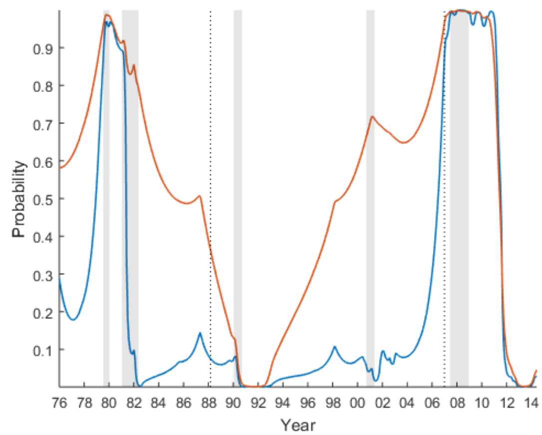
(b) Financial cycle: FF_t vs $NAREIT$

Note: Blue line corresponds to the estimate of the smoothed probability of recession (financial downturn) in the baseline case (using RF_t and FF_t). The red line corresponds to the estimates obtained with alternative data (Number of housing starts HS_t for the business cycle, House price $NAREIT_t$ (NAREIT Composite index) for the financial cycle)

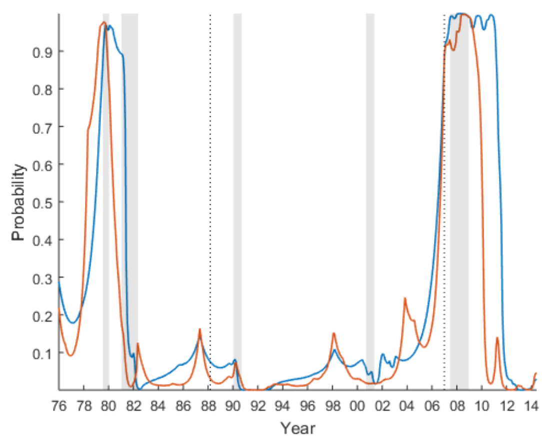
Figure C.2: Smoothed probability of high interaction regime



(a) Baseline case vs RC1 (RF + NAREIT)



(b) Baseline case vs RC2 (HS + FF)



(c) Baseline case vs RC3 (HS + NAREIT)

Note: Blue line corresponds to the estimate of the smoothed probability of "Interdependent cycle" regime in the baseline case (using RF_t and FF_t). The red line corresponds to the estimates obtained with alternative data (Number of housing starts HS_t for the business cycle, House price $NAREIT_t$ (NAREIT Composite index) for the financial cycle)

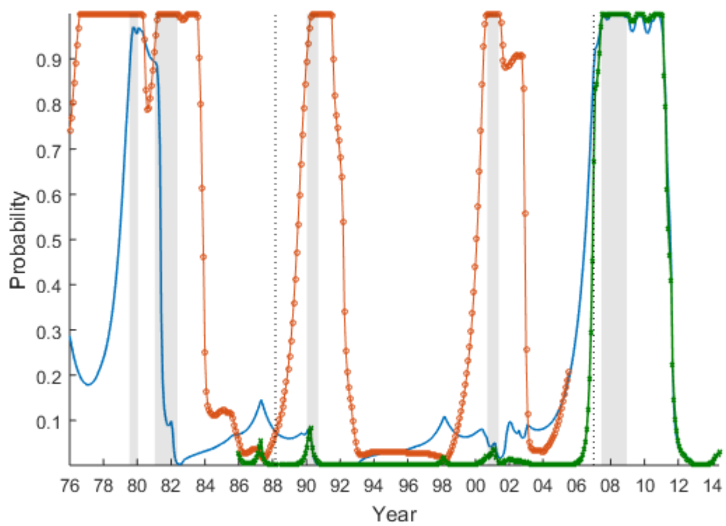
C.2 Estimation on subsets

Table C.2: DI-MS-FM estimates on alternative datasets

	Jul 1976-Dec 2014	Jul 1976-Aug 2006	Nov 1984-Dec 2014
Business cycle			
$\hat{\mu}_1$	0.57***	0.00***	0.00***
$\hat{\mu}_2$	-0.85***	-1.77***	-1.14***
$\hat{\sigma}_1^2$	0.31***	2.23**	5.74**
$\hat{\sigma}_2^2$	0.81***	0.30**	0.35**
$\hat{D}_{11}^1$	0.99***	0.99***	0.96***
$\hat{D}_{22}^1$	0.98***	0.87***	0.99***
$\hat{C}_{11}^{12}$	0.91***	0.62**	0.89***
$\hat{C}_{22}^{12}$	0.99*	0.13***	0.01***
Financial cycle			
$\hat{\beta}_1$	0.16***	0.37***	0.11***
$\hat{\beta}_2$	-0.86***	-0.52***	-1.20***
$\hat{\theta}_1^2$	0.27***	0.32***	0.31***
$\hat{\theta}_2^2$	4.05***	1.47***	5.15***
$\hat{D}_{11}^2$	0.99***	0.96***	0.52***
$\hat{D}_{22}^2$	0.88***	0.99***	0.99***
$\hat{C}_{11}^{21}$	0.75***	0.00***	0.00***
$\hat{C}_{22}^{21}$	0.99***	0.99***	0.00***
Influence regimes			
$\hat{R}^1$	$\begin{bmatrix} 0.98 & 0.06 \\ 0.02 & 0.94 \end{bmatrix}$	$\begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix}$	$\begin{bmatrix} 0.95 & 0.16 \\ 0.05 & 0.84 \end{bmatrix}$
$\hat{R}^2$	$\begin{bmatrix} 0.86 & 0.81 \\ 0.14 & 0.19 \end{bmatrix}$	$\begin{bmatrix} 0.00 & 0.20 \\ 1.00 & 0.80 \end{bmatrix}$	$\begin{bmatrix} 0.79 & 1.00 \\ 0.21 & 0.00 \end{bmatrix}$
$\hat{q}_{11}$	0.99	0.98	0.99
$\hat{q}_{22}$	0.97	0.98	0.97

Note: The parameters significant at 15%, 10% and 5% are marked with *, ** and ***, correspondingly.

Figure C.3: Smoothed probability of "Interdependent cycles" regime



Note: Blue line corresponds to the smoothed probability "Interdependent cycle" regime in the baseline case (estimated on the whole period). Red and green lines correspond to the estimates obtained on the right and left subsamples (Nov 1984 - Dec 2014 and Jul 1976 - Aug 2006), respectively.