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I extend the neoclassical 2x2x2 trade model to general preferences over a variety of goods supplied under monopolistic competition in a sector while the other sector is perfectly competitive. Non-homothetic preferences deliver pricing to market, incomplete pass-through and market size effects. Under realistic conditions, the differentiated goods are sold at a higher price in the capital-abundant country.

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Monopolistic competition, Heckscher-Ohlin Model, non-homothetic preferences, international trade

JEL Codes

F11, F12

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The Heckscher-Ohlin Model with Monopolistic Competition and General Preferences

by Federico Etro¹

June 2017

Abstract

I extend the neoclassical 2x2x2 trade model to general preferences over a variety of goods supplied under monopolistic competition in a sector while the other sector is perfectly competitive. Non-homothetic preferences deliver pricing to market, incomplete pass-through and market size effects. Under realistic conditions, the differentiated goods are sold at a higher price in the capital-abundant country.

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Recent progress in the analysis of monopolistic and imperfect competition under general microfoundations allows us to reconsider some of its main general equilibrium applications.² One of these applications is to international trade, as in the works of Dixit and Norman (1980) and Helpman and Krugman (1985), that have extended the Heckscher-Ohlin model to intraindustry trade. These works analyze trade between two countries producing goods in two sectors, one characterized by perfect competition and one by monopolistic competition. A key assumption is that a CES aggregator (as in Krugman, 1980) summarizes preferences over differentiated goods, which generates constant and equal markups in both countries, excluding realistic aspects of intra-industry trade such as pricing to market, incomplete pass-through of cost changes and market size effects on markups. We show how these aspects emerge after departing from CES preferences, and in particular, we show that each differentiated variety tends to be sold at a higher price in the capital abundant country.

Our work is related to one-sector models of trade with directly additive preferences (from Krugman, 1979, to the recent works of Bertolotti and Epifani, 2014 and Arkolakis *et al.*, 2015), homothetic preferences (Feenstra, 2014) and indirectly additive preferences (Bertolotti and Etro, 2017a; Bertolotti *et al.*, 2018), but our focus is on a two-sector framework with any (symmetric) preferences and identical firms. We also sketch how to extend the Heckscher-Ohlin model to imperfect competition in the differentiated sector (in the spirit of Brander and Krugman, 1983), as already done under homogenous goods and Cournot competition by Lahiri and Ono (1995) and under CES preferences with both Cournot and Bertrand competition by Etro (2015).³

Let us consider two countries engaged in free trade. Identical consumers are present in each country, with the following preferences:

$$\tilde{U} = U(\mathbf{x}, N)^{1-\omega} Y^\omega \quad (1)$$

where $U(\mathbf{x}, N)$ is an increasing, quasiconcave and symmetric aggregator of the consumption vector $\mathbf{x} \equiv [x_1, x_2, \dots, x_N]$ of N differentiated goods sold under monopolistic competition at prices $\mathbf{p} \equiv [p_1, p_2, \dots, p_N]$, Y is the consumption of the homogenous good, which is a *numeraire* produced under perfect competition, and $\omega \in (0, 1)$ represents the relative preference for the homogenous good.⁴

A CES consumption index $U = [\sum_j x_j^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}$ with $\theta > 1$ was adopted in Helpman and Krugman (1985) and the subsequent literature, but our formulation allows us to use any well behaved symmetric preference aggregator. This

²On monopolistic competition with general symmetric preferences in a closed economy see Bertolotti and Etro (2016). The *pendant* to this article, Etro (2016), provides the application to macroeconomics. Markups are variable over time there and between countries here. In Cavallari and Etro (2017) markups are variable both over time and across countries.

³An excellent overview of recent trade theory is in Feenstra (2015).

⁴Alternatively one can consider an indirect utility $\tilde{U} = V(\frac{\mathbf{p}}{E}, N)^{1-\omega} E^\omega$ decreasing and convex in each price, and obtain the direct demand functions $x_i(\mathbf{p}/E)$ from the Roy's identities. What is important for our characterization of monopolistic competition is not the Cobb-Douglas specification but the separability between the homogenous good and the differentiated ones.

includes a directly additive aggregator *à la* Dixit and Stiglitz (1977) and Krugman (1979) with $U = \sum_j u(x_j)$ for some increasing concave function $u(x)$, an implicitly additive aggregator *à la* Hanoch (1974) such that $\sum_j u(x_j, U) = 1$, a homothetic aggregator, as in the implicitly additive specification *à la* Kimball (1995) which satisfies $\sum_j u(x_j/U) = 1$, or non-separable aggregators, as for instance a quadratic aggregator *à la* Melitz and Ottaviano (2008) with $U = \alpha \sum_j x_j - \frac{\gamma}{2} \sum_j x_j^2 - \frac{\eta}{2} (\sum_j x_j)^2$ where $\alpha, \gamma > 0$.⁵

Capital and labor inputs are immobile and with endowments K (K^*) and L (L^*) for the Home (Foreign) country. Each worker supplies a unit of labor and holds an amount $k \equiv K/L$ ($k^* \equiv K^*/L^*$) of capital in the Home (Foreign) country. Technology is identical in both countries. A linearly homogenous production function for the competitive sector is associated with a constant marginal cost $c_Y(w, r)$, while the production of each differentiated variety requires a fixed cost of η units of labor and a constant marginal cost $c_X(w, r)$, where w and r are the wage and the rental rate. Final goods in both sectors are freely tradable, but producers of differentiated goods can set different prices in the two countries (markets are segmented). We exclude factor intensity reversal and we focus on an equilibrium with diversification between sectors in both countries. This requires identical marginal costs in the competitive sector and zero profits for firms in both countries, which in turn insures factor price equalization.

A consumer with income E allocates spending to maximize utility under the budget constraint $E = \mathbf{p}\mathbf{x} + Y$. Maximizing $\log \bar{U}$ we obtain the FOCs:

$$p_i = \frac{(1 - \omega)U_i(\mathbf{x}, N)}{U(\mathbf{x}, N)\lambda} \quad \text{for any } i, \quad \text{and} \quad \lambda = \frac{\omega}{Y}$$

where $U_i(\mathbf{x}, N) \equiv \partial U(\mathbf{x}, N)/\partial x_i$ is the marginal utility of consumption of good i and λ is the Lagrange multiplier.

Multiplying the first set of conditions by individual consumption and summing over all goods, using the budget constraint and the last FOC, one can solve for $\lambda = [(1 - \omega) \sum_j x_j U_j/U + \omega]/E$ and obtain the inverse demand system:

$$p_i(\mathbf{x}, E) = \frac{U_i(\mathbf{x}, N)E}{\sum_{j=1}^N x_j U_j(\mathbf{x}, N) + \frac{\omega}{1-\omega} U(\mathbf{x}, N)} \quad i = 1, \dots, n \quad (2)$$

where individual income is $w + rk$ in the Home country and $w + rk^*$ in the Foreign country. Accordingly, income is higher in the capital abundant country.

Under monopolistic competition, each firm producing a variety i maximizes its profits in the Home country:

$$\pi_i = [p_i(\mathbf{x}, w + rk) - c_X(w, r)] x_i L$$

choosing the consumption level x_i taking into account only its direct effects on the demand. The same is done for the profits in the Foreign country:

$$\pi_i^* = [p_i^*(\mathbf{x}^*, w + rk^*) - c_X(w, r)] x_i^* L^*$$

⁵The case of indirect additivity (Bertoletti and Etro, 2017a) requires $V(\mathbf{p}/E, N) = \sum_j v(p_j/E)$ for some decreasing convex function $v(s)$.

where the choice variable is now the consumption level x_i^* . The total profits of firm i across both markets are $\pi_i + \pi_i^* - \eta w$. The symmetric equilibrium price in the two countries can be derived as $p = \mu(x, N)c_X(w, r)$ and $p^* = \mu(x^*, N)c_X(w, r)$ with the markup rule:

$$\mu^{MC}(x, N) = \frac{1}{1 - \epsilon(x, N)} \quad (3)$$

where the perceived elasticity of the inverse demand is the Morishima elasticity of complementarity between goods (see Bertolotti and Etro, 2016), that is $\epsilon(x, N) = -d \ln(p_i/p_j)/d \ln x_i$ for $p_i = p_j$. Direct computation from $p_i/p_j = U_i(\mathbf{x}, N)/U_j(\mathbf{x}, N)$ allows one to obtain an expression for this elasticity:

$$\epsilon(x, N) = \frac{x U_{ij}(x, N)}{U_j(x, N)} - \frac{x U_{ii}(x, N)}{U_i(x, N)} \quad (4)$$

which is assumed less than unitary to guarantee a well defined price and is the reciprocal of the Morishima elasticity of substitution.⁶ Needless to say, in the CES case we have $\epsilon(x, N) = 1/\theta$ and the markup $\mu(x, N) = \frac{\theta}{\theta-1}$ is independent from income, marginal cost and factor endowments. However the general elasticity (4) changes (and not necessarily monotonically) with the number of goods and individual consumption. This implies that in general prices can be different between countries due to differences in per-capita income that affect individual consumption. Moreover, there is no full pass-through of cost changes on prices: a change in the marginal cost affects markups through its general equilibrium effect on the number of firms and on local consumption. Finally, also an increase in factor endowments affects markups through changes in income and local consumption as well as in the number of varieties produced.

It is by now well known that in a symmetric equilibrium, the monopolistic competition markup depends under a homothetic aggregator only on the number of firms N , under a directly additive aggregator only on consumption x , and under an indirectly additive aggregator only on the normalized price p/E .⁷ This already shows that under homothetic preferences, as indeed in the Helpman-Krugman model with CES preferences, the price of each differentiated variety will be identical in both countries (because agents in both countries consume always all the N goods even if in different quantities). Instead, when preferences are not homothetic the price will be higher in the country with higher consumption of each variety if and only if $\epsilon(x, N)$ is increasing in x .

⁶Given the symmetric utility $U(x, N) \equiv U(x\mathbf{u}, N)$, where $\mathbf{u}$ is the N -dimensional unit vector, we defined the marginal utility of consumption of one good under symmetry as $U_i(x, N) \equiv \partial U(x\mathbf{u}, N)/\partial x_i$, and the second derivative as $U_{ij}(x, N) \equiv \partial^2 U(x\mathbf{u}, N)/\partial x_i \partial x_j$. Starting from the indirect utility, profit maximization with respect to prices would lead to an equivalent pricing rule depending on the Morishima elasticity of substitution. On the definition of the Morishima elasticities see Blackorby and Russell (1981) and the application in Bertolotti and Etro (2016) for more details and examples in a closed economy.

⁷In the case of a symmetric implicitly additive aggregator one can show that the monopolistic competition markup depends on both consumption x and the same aggregator $U(x, N)$. For extensions of the analysis of monopolistic competition equilibria to asymmetric preferences and firms see Bertolotti and Etro (2017b).

And the elasticity $\epsilon(x, N)$ is indeed increasing in consumption under common directly additive aggregators, for instance when subutilities from consumption are exponential ($u(x) = 1 - e^{-bx}$ with $b > 0$) or quadratic ($u(x) = \alpha x - \frac{\gamma}{2}x^2$), as well as with the Melitz-Ottaviano aggregator. In all these cases, the country with higher consumption of each differentiated variety must face higher prices. The same happens with indirectly additive preferences used in Bertolletti and Etro (2017a) as long as the demand elasticity is increasing in the price-income ratio.

The equilibrium market structure is characterized by the following pricing and free entry conditions in the two sectors:

$$c_Y(w, r) = 1 \quad (5)$$

$$[p - c_X(w, r)]xL + [p^* - c_X(w, r)]x^*L^* = \eta w \quad (6)$$

$$p = \mu(x, N)c_X(w, r) \quad (7)$$

$$p^* = \mu(x^*, N)c_X(w, r) \quad (8)$$

The demand side generates the symmetric equilibrium conditions for the differentiated goods:

$$p = \frac{w + rk}{x \left[N + \frac{\omega}{(1-\omega)\varphi(x, N)} \right]} \quad (9)$$

$$p^* = \frac{w + rk^*}{x^* \left[N + \frac{\omega}{(1-\omega)\varphi(x^*, N)} \right]} \quad (10)$$

where $\varphi(x, N) \equiv xU_i(x, N)/U(x, N)$ is the elasticity of the utility to the consumption of a good. Mathematically, given the equilibrium number of firms N , the six conditions above determine prices and consumption of the varieties in the two countries.

The case of homothetic preferences implies that the capital abundant (richer) country must have higher consumption of each variety at identical prices, and a simple extension of the Stolper-Samuelson theorem holds: what increases prices in the labor intensive sector has to increase the wage. The case of non-homothetic preferences, instead, generates different prices between countries as long as these countries differ in the capital-labor endowments. It is natural that the capital abundant (richer) country has higher consumption of each variety, and therefore faces higher prices if $\epsilon(x, N)$ is increasing in consumption. It is immediate to verify from the above conditions that a sufficient condition for this is that $\varphi(x, N)$ is decreasing in consumption, as it holds in all our examples above,⁸ as well as with indirect additivity with a demand elasticity that is increasing in prices.⁹

⁸Indeed, combining (3), (7) and (9) provides an implicit expression for x as increasing in income when $\epsilon(x, N)$ is increasing and $\varphi(x, N)$ is decreasing in x .

⁹Following Bertolletti *et al.* (2018), consider indirect additivity with subutility $v(s) = \frac{(a-s)^{1+\gamma}}{1+\gamma}$ for $a, \gamma > 0$. Equilibrium prices are $p = \frac{a(w+rk) + \gamma c_X(w, r)}{1+\gamma}$ in the Home country

The four market clearing conditions for the factor markets in both countries employ the Shephard Lemma for which the labor and capital requirements for a unitary production in sector $j = Y, X$ are the same in both countries and given by $a_{Lj} = \partial c_j(w, r) / \partial w$ and $a_{Kj} = \partial c_j(w, r) / \partial r$. Denoting with n the number of domestic varieties and with y and y^* the domestic and foreign production of homogenous goods, we have:

$$a_{LY}y + a_{LX}n(xL + x^*L^*) + \eta n = L \quad (11)$$

$$a_{LY}y^* + a_{LX}(N - n)(xL + x^*L^*) + \eta(N - n) = L^* \quad (12)$$

$$a_{KY}y + a_{KX}n(xL + x^*L^*) = K \quad (13)$$

$$a_{KY}y^* + a_{KX}(N - n)(xL + x^*L^*) = K^* \quad (14)$$

The equilibrium in the market for the homogenous goods holds by Walras' law. The system of ten equations (5)-(14) can be solved for the ten unknowns w , r , p , p^* , x , x^* , n , N , y and y^* . This concludes the description of the equilibrium (of course, with CES preferences we have $\mu(x, N) = \frac{\theta}{\theta-1}$ and the equilibrium corresponds to the one in Helpman and Krugman, 1985).

Homothetic preferences allow us to generalize easily the traditional Heckscher-Ohlin analysis due to the identical prices across countries: assuming that the differentiated goods are capital intensive, the capital abundant country is a net exporter of differentiated goods and a net importer of homogenous goods. Nevertheless, the free trade equilibrium loses the efficiency properties holding under CES preferences, that is the ability to reproduce the efficient equilibrium of an integrated economy (Helpman and Krugman, 1985). This happens because the equilibrium number of firms depends on monopolistic pricing (and the Morishima elasticity) rather than on efficient pricing (and the elasticities of the utility function with respect to consumption and the number of goods).¹⁰

Non-homothetic preferences add to inefficient business creation also an inefficient wedge between prices of the differentiated goods when they are sold in the Home country and in the Foreign country. Again, the equilibrium markup reflects the elasticity of the demand functions in each country rather than efficiency conditions. Restoring constrained optimality requires a system of subsidies to business creation and production (or import tariffs) to target the optimal number of varieties and the optimal consumption of differentiated goods in each country.

The analysis of the decentralized equilibrium can be extended from monopolistic competition to Bertrand and Cournot competition. The symmetric markup $\mu(x, N)$ must be amended. For illustrative purposes, consider the case

and $p^* = \frac{a(w+rk^*) + \gamma c_X(w, r)}{1+\gamma}$ in the Foreign country. The price is higher in the capital abundant country, pass-through of cost changes on prices is incomplete and changes in factor endowments affect pricing through the local capital-labor ratio and a general equilibrium impact on the factor prices.

¹⁰Under factor price equalization, the difference between equilibrium and optimal market structures for homothetic preferences is the same as in a closed economy (Bertoletti and Etro, 2016).

of CES aggregator mentioned above. This implies $U_i(\mathbf{x}, N) = x_i^{-1/\theta} U(\mathbf{x}, N)^{1/\theta}$, therefore the inverse demand system can be derived from (2) and inverted to obtain the familiar forms:

$$p_i(\mathbf{x}, E) = \frac{(1 - \omega) x_i^{-1/\theta} E}{\sum_{j=1}^N x_j^{\frac{\theta-1}{\theta}}} \quad \text{and} \quad x_i(\mathbf{p}/E) = \frac{(1 - \omega) \left(\frac{p_i}{E}\right)^{-\theta}}{\sum_{j=1}^N \left(\frac{p_j}{E}\right)^{1-\theta}}$$

Replacing in the profit functions, this allows to solve for the equilibrium under monopolistic, Bertrand and Cournot competition with the symmetric markups:

$$\mu^{MC} = \frac{\theta}{\theta - 1}, \quad \mu^B = \frac{N\theta - \theta - 1}{(N - 1)(\theta - 1)} \quad \text{and} \quad \mu^C = \frac{N\theta}{(N - 1)(\theta - 1)}$$

to be used in the system (5)-(14). In case of Cobb-Douglas production functions, the model can be even solved in closed-form solutions for the endogenous number of firms and all the other variables (see Etro, 2015). In this environment, as under any symmetric homothetic preferences, the markups remain functions only of the number of firms under all the forms of competition, which preserves the Heckscher-Ohlin theorem. Under non-homotheticity, the characterization of markups under imperfect competition is more complex analytically. We hope that further research will investigate trade models of this kind taking into account asymmetric preferences as well as heterogeneous firms.

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