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Abstract

Internet and near-instant communication services have had a strong impact on the way people communicate and exchange information since the end of the 1990s. The COVID-19 pandemic shock is likely to provide a dramatic boost to this phenomenon. In the world that will emerge from lockdown, digital communication and videoconferencing will be integral part of daily working and social life to a much higher extent than before. This will have a disproportionately strong impact on knowledge based activities, and therefore on research. This paper presents a theoretical model of endogenous economic growth with the aim of providing hints on the possible consequences of the COVID-19 pandemic shock on income per capita growth and disparities within a system of integrated urban areas. In the model, abstract technological knowledge flows at no cost across space. In contrast, flows of tacit knowledge arise from the interaction between researchers and hence tends to be hampered by distance. Within this framework, the diffusion of broadband technology, high-speed connections and videoconferencing takes the form of a reduction of the “cost of distance” for flows of tacit knowledge. This reinforces productive specialization that, in turn, yields an increase in the system-wide, common growth rate and an increase in income per capita disparities across areas. On the other hand, social distancing is likely to yield an asymmetric negative impact on the productivity of urban research systems thus leading to a decrease in the growth rate.

Keywords : COVID-19, social distancing, videoconferencing, innovation, growth, disparities

JEL Codes: O31, O41, O51, R12

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1 Introduction

The idea of a network allowing users from different nodes to communicate through their PCs dates back to the 1950s. The first message was sent over the ARPANET (funded by the U.S. Department of Defense) in 1969 from a laboratory at UCLA to the second network node at Stanford. National Science Foundation began to commercialize the Internet in 1992. Popularity of the net becomes massive during the 1990s thanks to the introduction of the World Wide Web.

Since the end of the 1990s, broadband technology and high-speed connections has allowed the rise of near-instant communication (electronic mail, instant messaging, voice over Internet Protocol (VoIP) telephone calls, videoconferencing). While near-instant communication services have certainly had a strong impact on the way people live and work over the last two decades, the recent events are likely to revolutionize it. As most of the world experienced the lockdown due to the COVID-19 pandemic, people had no choice but to resort to digital communications in order to keep their working and social lives running. In particular, this situation pushed videoconferencing into the limelight, as people all over the world are forced to work and study from home.

The COVID-19 pandemic shock is likely to produce deep and permanent consequences. In the world that will emerge when the lockdown period is eventually over, digital communication in general, and videoconferencing in particular, will most probably be integral part of daily working and social life to a much higher extent than before. And this will have a disproportionally strong impact on all those activities in which knowledge represents a fundamental element. Research is certainly one of these. From this viewpoint, therefore, the stable diffusion of videoconferencing is likely to affect productivity of researchers and their ability to innovate as well as the spatial distribution of research and innovation activities. At the same time, confinement and social distancing, by reducing the ability of transmitting tacit knowledge, are likely to produce a negative impact on the productivity of research systems.

Specifically, the research question of the paper is: what could be the long-run impact of COVID-19 pandemic shock on research activities and, through this, on per capita income levels and growth rates? To answer, we start from recognizing that innovation is essentially a clustered, urban phenomenon:

- the clustering of R&D labs in the US is greater than the clustering of manufacturing facilities (Buzard et al., 2017)
- the top 50 US metros account for 97 percent of all venture capital investment, a key driver of innovation (correlation with patents is 0.588, significant at the 1%

level, between 2005 and 2009) (Florida and King, 2016)

As for the role of broadband diffusion on knowledge intensive activities, Mack and Rey (2014) report a generally positive relationship broadband adoption and the level of knowledge intensive activities across US metropolitan areas but also that specialization in traditional manufacturing has a negative impact on this relationship. In addition, they suggest that in large hubs of knowledge firm agglomerations broadband may represent a complement to face-to-face contacts; while the latter allows the exchange of complex, tacit knowledge, the former support the exchange of information with the rest of the world.

Hence, we develop a theoretical model in which economic growth is driven by innovations realized within urban research sectors whose productivity depends on the ability of researchers to communicate.

2 A theoretical Model of Urban Growth

The main features of the model are as follows. Consider two urban areas at some distance from each other; each urban area engages in three production sectors (research, capital goods, final good):

- the research sector produces patents for differentiated intermediate inputs (and knowledge) using human capital and knowledge
- differentiated intermediate inputs are produced by profit maximizing entrepreneurs using forgone final good and a patent
- the final sector produces a homogeneous good employing unskilled labor, human capital, and physical capital made up of the set of specialized intermediate inputs

Within the system, workers move freely across areas and, in the case of human capital, across sectors evaluating locations and sectors solely in terms of wage rates. Capital goods and final good are traded at no cost across urban areas.

Production in the research sector involves two forms of knowledge spillovers:

- abstract knowledge spillovers: abstract, codified knowledge spills over freely to all researchers, in all urban areas
- tacit knowledge spillovers: individual, tacit knowledge spills over as a result of interaction between individuals but its transmission is hampered by distance

In their role as workers, the inhabitants of the system are endowed with fixed quantities of unskilled labour and human capital that are supplied inelastically. While unskilled labour is employed only in the final good sector, human capital is used both in research

and manufacturing. The overall fixed supply of human capital H is therefore equal to:

$$H = Hr_i + Hm_i + Hr_j + Hm_j$$

while the overall fixed supply of unskilled labour is equal to:

$$L = L_i + L_j$$

Within this framework, the diffusion of broadband technology, hi-speed connection and videoconferencing reduces friction exerted by geographical distance. Social distancing, on the other hand, has an urban-specific, negative impact on productivity in the research sector.

2.1 The Research sector

Following the large literature originated from Romer (1990a,b), the research sector produces knowledge, in the form of designs for new intermediate inputs, using human capital and existing knowledge.

Knowledge takes two forms in the model: abstract and tacit. The former is represented by codifiable knowledge created during the research effort. Embodied in a patent document for a new intermediate input or published in a scientific journal, this form of knowledge becomes potentially available to everyone, independent of their location. In other words, as in the original Romer's setup, once created, abstract knowledge spreads freely throughout the system enhancing the productivity of every researcher. Tacit knowledge is instead all that body of knowledge that cannot be codified, being the non-written heritage of individuals or groups (Polanyi, 1967). Also this form of knowledge can be transmitted, and positively affects the productivity of other researchers; but the flows of tacit knowledge occur essentially through direct, face-to-face, contacts rather than through impersonal means such as patent documents or scientific papers. From this distinction, it stems a further distinction between system-wide and spatially bounded external spillovers on the basis of the type of knowledge being transmitted.

To clarify this, let us consider the following formal representation of the research activity. The flow of new knowledge – *i.e.*, the number of new designs – created in urban area i at any point in time is given by:

$$\dot{A}_i = \delta_i Hr_i^\eta Sr_{ij} A \tag{1}$$

in which Hr_i is the amount of human capital employed in the research sector of i , η

$(0 \leq \eta < 1)$ is a parameter inducing decreasing returns in its stock¹ and δ_i is a parameter characterizing the productivity of the local research system. As for spillovers, A is the number of intermediate inputs already existing in the system (*i.e.*, $A = A_i + A_j$) as well as the overall level of abstract knowledge created so far and available to all researcher due to a-spatial spillovers of knowledge; Sr_{ij} reflects the size of inter-urban spillovers originating from j . Formally, spillovers have a gravity-type representation:

$$Sr_{ij} = (Hr_i Hr_j \nu_{ij})^\psi \quad (2)$$

with ψ governing the strength of their impact on researcher in i . Assuming that d_{ij} represents geographical distance between i and j , the function ν_{ij} expressing the effectiveness of interaction to the benefit if i is modeled as:

$$\nu_{ij} = 1 + \frac{\log \delta_i}{\log d_{ij}} \quad (3)$$

and hence depends positively on the productivity of researchers in the recipient urban area² and negatively on geographical distance. In order to concentrate on meaningful cases, assume that distance metric is parametrized so that $d_{ij} > 1$; similarly, the productivity of local research systems is parametrized so that $\delta_i > 1$. Finally, assume $0 \leq \psi \leq 1 - \eta$.³

2.2 The Manufacturing Production of Intermediate Inputs

Intermediate input producers engage in monopolistic competition, earning monopoly rents. These entrepreneurs will devote resources to finance the research effort only if the present discounted value of future profits exceeds the current cost of development.

As for production costs, intermediate input producers face a variable cost of production for the use of physical capital on top of the fixed cost represented by the development cost of a patent. In particular, it is assumed that one unit of intermediate input requires one unit of physical capital if the intermediate input is manufactured in the same region in which the relative patent has been developed or $1 + \kappa$ (with $\kappa > 0$) units of physical capital, if the intermediate input is manufactured in the other region. This structure of

¹ A similar feature can be found in Kortum (1993) and Jones (1995).

² Hence, the effectiveness of interaction is not symmetric as $\nu_{ij} \neq \nu_{ji}$ whenever $\delta_i \neq \delta_j$.

³ This assumption on the size of ψ is not strictly necessary but, as clarified in Appendix A.2, it eases calculations being a sufficient condition for the stability of the equilibrium allocation of researchers across urban areas.

production costs hence excludes the possibility of inter-regional trade of patents.

Finally, intermediate inputs do not deteriorate over time, while physical capital is simply a measure of forgone output so that:

$$\dot{K}_i = M_i - C_i$$

where M stands for the level of output of the homogeneous final good and C stands for the level of consumption.

2.3 Production and Consumption of the Homogeneous Final Good

The residents within the system work either in a research sector or in a manufacturing one, and consume a final good. As consumers, they maximise intertemporal utility with savings devoted to the acquisition of physical capital. In particular, assuming the absence of transport costs for the consumption good, all consumption arising from the system can be aggregated in the system-wide variable $C(t)$.

Let us assume that all consumers in the system maximise a Ramsey-type, constant elasticity of substitution utility function:

$$U[C] = \int_0^\infty e^{-\rho t} \frac{C^{1-\sigma}}{1-\sigma} dt \quad (4)$$

where ρ is the intertemporal rate of discount and σ^{-1} (with $0 < \sigma < 1$) the willingness to substitute intertemporally. Preferences are then used to derive the intertemporal optimisation condition that relates interest rates, r , to the rate of growth of consumption:

$$r = \sigma \frac{\dot{C}}{C} + \rho \quad (5)$$

To simplify calculations, assume without loss of generality that the intertemporal rate of discount relates to the interest rate according to the following equation:

$$\rho = \Gamma r$$

Here, we concentrate on the case in which the growth rate of per capita consumption is positive, which implies $0 < \Gamma < 1$. As a result, equation (5) simplifies to

$$r(1 - \Gamma) = \sigma \frac{\dot{C}}{C} \quad (6)$$

To ensure that the integral in equation (4) converges, the rate of growth of current utility $(1 - \sigma)\frac{\dot{C}}{C}$ is assumed to be smaller than the rate of time preference, so that:

$$(1 - \sigma)\frac{\dot{C}}{C} < \Gamma r \quad (7)$$

The consumption good is produced using labour, human capital and physical capital in the form of intermediate inputs. The stock of intermediate inputs available in i at any point in time consists of the summation of inputs produced in the same area and inputs imported from the other urban area (Rivera-Batiz and Romer, 1991; Rivera-Batiz and Xie, 1993). In particular, as A_i and A_j indicate the number of intermediate inputs designed and produced respectively in i and j at any point in time, the quantity of any intermediate input produced in i and employed in the same urban area is $x_i(a_i)$, with $a_i \in A_i$; analogously, the quantity of any intermediate input produced in j and employed in i is $x_i(a_j)$, with $a_j \in A_j$.

Also the final good production may benefit from external effects. In this case, the benefits take the form of agglomeration economies arising from the interaction between researchers and human capital employed in manufacturing within the same urban area. Formally, these effects have a representation that resembles the one in equation (2) for the research sector. However, since they now develop within the same urban area, external effects in the final good production of urban area i simplify to:

$$Sm_i = (Hm_i Hr_i)^\phi \quad (8)$$

where Hm represents human capital employed in the manufacturing sector and ϕ ($0 \leq \phi < 1$) determines the strength of the economies arising from agglomeration of human capital; the overall production structure in this sector is represented by the following additively separable function:

$$M_i = L_i^\alpha Hm_i^\beta \left[\int_{A_i} x_i(a_i)^\gamma da + \int_{A_j} x_i(a_j)^\gamma da \right] Sm_i \quad (9)$$

The sector operates in a perfectly competitive setting, hence $\alpha + \beta + \gamma = 1$, and the ratio Hm/L is assumed to be lower than β/α in both urban areas.⁴

Finally, it is also assumed that both intermediate inputs and consumption goods are traded freely within the system in the absence of any transportation cost.

⁴ Given that wage rates are equal to factors' marginal product, this assumption implies that the wage rate earned by human capital is higher than the wage rate earned by labour.

2.4 Patents, Intermediate Inputs and Their Pricing

Let the price of the consumption good be the numeraire; then, let a differentiated capital good produced in urban area i have price $p_i(a_i)$ when sold in i , and price $p_j(a_i)$ when sold in j . Given the absence of transport costs involved in the inter-urban trade of these inputs, it follows $p_i(a_i) = p_j(a_i)$. Analogously, the price of an intermediate input produced in j and sold in the same urban area, $p_j(a_j)$, is equal to the price of the same input sold in i , $p_i(a_j)$.

In each area, given the prices for intermediate inputs, final good producers will determine the demand for each intermediate input solving their profit maximisation problem. These problems are described in the Appendix (Section A.1) where, from first-order conditions, it is also shown that the demand functions faced by producers of intermediate inputs located in the two urban areas are

$$\begin{aligned} X(a_i) &= x_i(a_i) + x_j(a_i) \\ X(a_j) &= x_j(a_j) + x_i(a_j) \end{aligned}$$

or, equivalently

$$\begin{aligned} X(a_i) &= \gamma^{\frac{1}{1-\gamma}} \left(L_i^{\frac{\alpha}{1-\gamma}} H m_i^{\frac{\beta}{1-\gamma}} S m_i^{\frac{1}{1-\gamma}} + L_j^{\frac{\alpha}{1-\gamma}} H m_j^{\frac{\beta}{1-\gamma}} S m_j^{\frac{1}{1-\gamma}} \right) p_i(a_i)^{-\frac{1}{1-\gamma}} \\ X(a_j) &= \gamma^{\frac{1}{1-\gamma}} \left(L_i^{\frac{\alpha}{1-\gamma}} H m_i^{\frac{\beta}{1-\gamma}} S m_i^{\frac{1}{1-\gamma}} + L_j^{\frac{\alpha}{1-\gamma}} H m_j^{\frac{\beta}{1-\gamma}} S m_j^{\frac{1}{1-\gamma}} \right) p_j(a_j)^{-\frac{1}{1-\gamma}} \end{aligned}$$

The equilibrium level for these demand functions and the corresponding prices can then be determined considering the profit maximisation problem faced by intermediate input producers. For one of these producers located in i , the profit maximisation problem can be expressed as

$$\max_{X(a_i)} \pi = \max_{X(a_i)} [p_i(a_i)X(a_i) - rX(a_i)] \quad (10)$$

The monopoly pricing problem introduced above is that of a firm facing a constant marginal cost equal to the interest rate, r , and a demand curve with constant elasticity $\epsilon_D = -\frac{1}{1-\gamma}$. The profit maximising price can then be determined using the relationship between marginal and average revenue

$$MR = AR (1 + \epsilon_D^{-1})$$

and the profit maximisation condition equating marginal revenue to marginal cost. These

relationships yield

$$r = AR \left[1 + \left(-\frac{1}{1-\gamma} \right)^{-1} \right]$$

from which it is easy to show that the profit maximising price⁵ for an intermediate input is the same in the two areas and equal to:

$$\hat{p}(a) = \hat{p}_i(a_i) = \hat{p}_j(a_j) = r\gamma^{-1} \quad (11)$$

From the symmetry existing in the model, it follows that the final good sector in i demands $\hat{x}_i(a_i)$ of each intermediate inputs produced in i and $\hat{x}_i(a_j)$ of each intermediate inputs produced in j . Similarly, the demand from j for each domestically produced intermediate input is $\hat{x}_j(a_j)$ while $\hat{x}_j(a_i)$ is the demand for each intermediate input shipped from i . As, at any point in time, the number of designed differentiated inputs is A_i in i and A_j in j , the amount of these inputs employed in i can be expressed as:

$$\int_{A_i} x_i(a_i)^\gamma da + \int_{A_j} x_i(a_j)^\gamma da = A_i \hat{x}_i(a_i)^\gamma + A_j \hat{x}_i(a_j)^\gamma$$

and, analogously, as:

$$\int_{A_j} x_j(a_j)^\gamma da + \int_{A_i} x_j(a_i)^\gamma da = A_j \hat{x}_j(a_j)^\gamma + A_i \hat{x}_j(a_i)^\gamma$$

in j . Given that the profit maximising price of an intermediate input is the same across areas, and using results in Appendix Section A.1, it can be shown that each urban area demands a specific, constant quantity of each intermediate input, irrespective of their origin:

$$\begin{aligned} \hat{x}_i(a_i) &= \hat{x}_i(a_j) = \hat{x}_i(a) = \gamma^{\frac{1}{1-\gamma}} L_i^{\frac{\alpha}{1-\gamma}} H m_i^{\frac{\beta}{1-\gamma}} S m_i^{\frac{1}{1-\gamma}} \hat{p}(a)^{-\frac{1}{1-\gamma}} \\ \hat{x}_j(a_j) &= \hat{x}_j(a_i) = \hat{x}_j(a) = \gamma^{\frac{1}{1-\gamma}} L_j^{\frac{\alpha}{1-\gamma}} H m_j^{\frac{\beta}{1-\gamma}} S m_j^{\frac{1}{1-\gamma}} \hat{p}(a)^{-\frac{1}{1-\gamma}} \end{aligned} \quad (12)$$

Using this result, the functions describing the production of final goods in the two urban areas can be re-written as:

$$\begin{aligned} M_i &= L_i^\alpha H m_i^\beta A \hat{x}_i(a)^\gamma S m_i \\ M_j &= L_j^\alpha H m_j^\beta A \hat{x}_j(a)^\gamma S m_j \end{aligned} \quad (13)$$

⁵ A hat over a variable indicates its equilibrium value.

In addition, also the overall demand faced by intermediate input producers located in the two different urban areas will be the same

$$\hat{X}(a) = \hat{X}(a_i) = \hat{X}(a_j) = \hat{x}_i(a_i) + \hat{x}_j(a_j) = \frac{1}{2} [\hat{x}_i(a) + \hat{x}_j(a)] \quad (14)$$

as well as the maximum instant profits they enjoy:

$$\hat{\pi}_i = \hat{\pi}_j = r \hat{X}(a) \frac{1-\gamma}{\gamma}$$

The decision about undertaking the production of a new intermediate input is taken comparing the discounted value of the flow of future profits to the cost of the initial investment in a patent or, more generally, in research. Assuming perfect competition in the patents' market, the price of a patent must equal the present value of the stream of future profits. Therefore, the cost of a patent is:

$$P(a_i) = P(a_j) = P(a) = \int_0^\infty \hat{\pi} e^{-rt} dt$$

Because patents are infinitely lived, if the interest rate is constant, the value of a patent in terms of final output is:

$$\hat{P}(a) = \hat{\pi} r^{-1} = \hat{X}(a) \frac{1-\gamma}{\gamma} \quad (15)$$

2.5 The Steady-state Equilibrium

We have just seen that, when the interest rate is constant, the equilibrium price for the intermediate inputs is also constant. Assuming for the moment that the labour market is characterised by a stable allocation of both unskilled labour and human capital across regions and sectors, then it is clear from equations (12) that the employed quantities of intermediate inputs $\hat{x}_i(a)$ and $\hat{x}_j(a)$ would also be constant. Moreover, if such a constant allocation of resources exists, it follows from equations (13) that the production of final goods grows in both urban areas at the same constant rate at which system-wide abstract knowledge grows.

As for the accumulation of physical capital, since it takes one unit of forgone consumption to create one unit of any type of intermediate input, at any point in time the overall level of physical capital available in the system, K , is related to the intermediate

inputs employed in production by the following rule:

$$K = K_i + K_j = \int_A \hat{x}_i(a) da + \int_A \hat{x}_j(a) da = A\hat{x}_i(a) + A\hat{x}_j(a)$$

implying that, when quantities $\hat{x}_i(a)$ and $\hat{x}_j(a)$ are constant, also K grows at the same constant rate as output and the ratio M/K stays constant. Consequently, the ratio

$$\frac{C}{M} = 1 - \frac{\dot{K}}{M} = 1 - \frac{\dot{K}}{K} \frac{K}{M}$$

must be constant as well and C grows at the same rate of all other variables. Finally, equation (6) ensures that the interest rate is also constant.

To sum up, if the system is characterised by a constant allocation of unskilled labour and human capital across areas and sectors, a steady-state equilibrium solution exists in which

$$\frac{\dot{M}}{M} = \frac{\dot{M}_i}{M_i} = \frac{\dot{M}_j}{M_j} = \frac{\dot{K}}{K} = \frac{\dot{C}}{C} = \frac{\dot{A}}{A}$$

To solve the model for this balanced growth equilibrium it is therefore necessary to determine the equilibrium allocation of workers through the equalization of wage rates across areas and sectors.

2.6 Equilibrium Allocation of Workers

2.6.1 Inter-urban Allocation of Researchers

Let us assume for the moment that Hr , the overall level of human capital employed in research, is given. As stated above, workers evaluate locations and sectors solely in terms of wage rates.

The wage rate for human capital employed in the research sector of urban area i is

$$w(Hr)_i = \frac{\partial}{\partial Hr_i} \dot{A}_i P(a)$$

or, using equations (1) and (15):

$$w(Hr)_i = \eta \delta_i Hr_i^{\eta-1} Sr_{ij} \hat{X}(a) \frac{1-\gamma}{\gamma} A \quad (16)$$

Analogously, the wage rate for human capital employed in the research sector of j is

$$w(Hr)_j = \eta \delta_j Hr_j^{\eta-1} Sr_{ji} \hat{X}(a) \frac{1-\gamma}{\gamma} A \quad (17)$$

By equating the wage rates in (16) and (17), it is easy to get the following stable equilibrium allocation

$$\frac{\widehat{H}r_i}{\widehat{H}r_j} = \left(\frac{\delta_i}{\delta_j} \right)^{\frac{1}{1-\eta}} \left(\frac{\nu_{ij}}{\nu_{ji}} \right)^{\frac{\psi}{1-\eta}} \quad (18)$$

where the terms describing the effectiveness of interaction between researchers located in different areas (ν_{ij} and ν_{ji}) depend on distance and on the urban-specific productivity parameters (δ_i and δ_j) as described in equation (3). For a given distance, therefore, the equilibrium spatial allocation of human capital in research stays constant.

It is also possible to evaluate the relationship between this equilibrium allocation and the relative size of the parameters characterizing the productivity in research in the two urban areas. Substituting the functions expressing the effectiveness of interaction between urban research systems into equation (18) we get

$$\frac{\widehat{H}r_i}{\widehat{H}r_j} = \left(\frac{\delta_i}{\delta_j} \right)^{\frac{1}{1-\eta}} \left[\frac{\log d_{ij} + \log \delta_i}{\log d_{ij} + \log \delta_j} \right]^{\frac{\psi}{1-\eta}} \quad (19)$$

It is clear from equation (19) that there is a direct relationship between the relative size of the parameters characterizing the productivity in research and the relative concentration of research activities: an urban area characterized by a higher productivity in research (*i.e.*, having an higher value of δ) ends up having a larger share of researchers.

Finally, we can assess the implication of the equilibrium allocation of researchers on the growth rate of abstract knowledge

$$\frac{\dot{A}}{A} = \frac{\dot{A}_i}{A} + \frac{\dot{A}_j}{A} = \delta_i \widehat{H}r_i^\eta S r_{ij} + \delta_j \widehat{H}r_j^\eta S r_{ji}$$

Substituting the definition of spillovers, the rate can be written as

$$\frac{\dot{A}}{A} = \delta_j \nu_{ji}^\psi \widehat{H}r_i^\psi \widehat{H}r_j^\psi \left[\frac{\delta_i \nu_{ij}^\psi}{\delta_j \nu_{ji}^\psi} \widehat{H}r_i^\eta + \widehat{H}r_j^\eta \right]$$

Using the equilibrium allocation of researchers in equation (19), the steady-state growth rate of abstract knowledge becomes

$$\frac{\dot{A}}{A} = \delta_j \widehat{H}r_i^\psi \widehat{H}r_j^{\eta+\psi-1} \nu_{ji}^\psi \widehat{H}r = \delta_i \widehat{H}r_j^\psi \widehat{H}r_i^{\eta+\psi-1} \nu_{ij}^\psi \widehat{H}r \quad (20)$$

where $\widehat{H}r$ is the equilibrium level of human capital devoted to research activities within the system.

2.6.2 Inter-urban Allocation of Workers in the Final Sector

The condition determining the allocation of labour and human capital between the final good sectors says that, for each type of worker, wages are the same across urban areas:

$$\begin{cases} w(L)_i &= w(L)_j \\ w(Hm)_i &= w(Hm)_j \end{cases}$$

where

$$\begin{aligned} w(L)_i &= \frac{\partial M_i}{\partial L_i} = \alpha L_i^{\alpha-1} H m_i^\beta A \hat{x}_i(a)^\gamma S m_i \\ w(L)_j &= \frac{\partial M_j}{\partial L_j} = \alpha L_j^{\alpha-1} H m_j^\beta A \hat{x}_j(a)^\gamma S m_j \end{aligned} \quad (21)$$

and

$$\begin{aligned} w(H)m_i &= \frac{\partial M_i}{\partial H m_i} = \beta L_i^\alpha H m_i^{\beta-1} A \hat{x}_i(a)^\gamma S m_i \\ w(H)m_j &= \frac{\partial M_j}{\partial H m_j} = \beta L_j^\alpha H m_j^{\beta-1} A \hat{x}_j(a)^\gamma S m_j \end{aligned} \quad (22)$$

Using equation (12), it is easy to show that the system's solution is

$$\frac{\hat{L}_i}{\hat{L}_j} = \frac{\hat{H} m_i}{\hat{H} m_j} = \frac{\hat{x}_i(a)}{\hat{x}_j(a)} \quad (23)$$

stating that the equilibrium ratios between the urban endowments of the production factors are the same, and by an additional condition on the equalization of external effects:

$$S m_i = S m_j$$

Given the definition of these effects (equation 8 for $S m_i$ and an analogous one for $S m_j$), the external effects equalization condition becomes:

$$\frac{\hat{H} m_i}{\hat{H} m_j} = \frac{\hat{H} r_j}{\hat{H} r_i} \quad (24)$$

implying that, in equilibrium, the allocation of human capital in manufacturing between i and j is simply the inverse the corresponding allocation of human capital in research.

2.6.3 Urban Allocation of Human Capital

The urban equilibria for human capital require inter-sectoral wage equalisation:

$$\begin{cases} w(Hm)_i = w(Hr)_i \\ w(Hm)_j = w(Hr)_j \end{cases}$$

When the inter-urban equilibrium allocation of researchers examined in Section 2.6.1 holds, the condition becomes

$$w(\hat{H}m)_i + w(\hat{H}m)_j = 2w(\hat{H}r)_i$$

We show in Section A.3 in the Appendix that, in steady-state, this condition is met when

$$\hat{H}r = \frac{\eta(1-\Gamma)(1-\gamma)\gamma}{2\sigma\beta} \hat{H}m \quad (25)$$

In other words, the allocation of the given stock of human capital stays constant as it depends on the consumers' preferences, via σ and Γ , and on factors' shares in final good production. As a consequence, the equilibrium allocation of human capital in the research sector (equation 19) and in the final good sector (equation 24) remain constant.

2.7 General Features of the Steady-state Equilibrium

We saw in Section 2.6 that the system is characterised by a constant allocation of workers across sectors and urban areas. Given that such a constant allocation exists, the system is characterized by a steady-state equilibrium in which output, consumption, physical capital and abstract knowledge grow at the same constant growth rate represented in equation (20). Moreover, it is clear from equations (16) and (17) – for the research sector – and from equations (21) and (22) – for the manufacturing sector – that along the steady-state growth path all wages grow at the same constant rate as the other variables, while the price of a patent, the price of each intermediate input, the price of the final good and the interest rate stay constant.

The next step is the determination of the rates at which abstract knowledge grows in the urban areas. To do so, we need first to evaluate the equilibrium value of the ratio between the flows of new patents in the two areas

$$\frac{\dot{A}_i}{\dot{A}_j} = \frac{\delta_i H r_i^\eta S r_{ij} A}{\delta_j H r_j^\eta S r_{ji} A}$$

that, using the equilibrium allocation of researchers in equation (18), simply becomes

$$\frac{\dot{A}_i}{\dot{A}_j} = \frac{\hat{H}r_i}{\hat{H}r_j} \quad (26)$$

Then, we can work out the asymptotic value of the ratio between the stocks of patents created in the two areas A_i/A_j . Using L'Hôpital's rule and equation (26), it turns out that the ratio is asymptotically constant:

$$\lim_{t \rightarrow \infty} \frac{A_i}{A_j} = \lim_{t \rightarrow \infty} \frac{\dot{A}_i}{\dot{A}_j} = \frac{\hat{H}r_i}{\hat{H}r_j}$$

In addition, the ratios A_i/A and A_j/A tend respectively to $\hat{H}r_i/\hat{H}r$ and $\hat{H}r_j/\hat{H}r$. Using the steady-state growth rate in equation (20), the upshot is that the asymptotic growth rate of abstract knowledge is the same in the two areas and equal to

$$\begin{aligned} \frac{\dot{A}_i}{A_i} &= \delta_i \hat{H}r_i^\eta S r_{ij} \frac{A}{A_i} = \delta_i \hat{H}r_j^\psi \hat{H}r_i^{\eta+\psi-1} \nu_{ij}^\psi \hat{H}r = \frac{\dot{A}}{A} \\ \frac{\dot{A}_j}{A_j} &= \delta_j \hat{H}r_j^\eta S r_{ji} \frac{A}{A_j} = \delta_j \hat{H}r_i^\psi \hat{H}r_j^{\eta+\psi-1} \nu_{ji}^\psi \hat{H}r = \frac{\dot{A}}{A} \end{aligned} \quad (27)$$

2.8 Disparities in GDP Per Capita Levels and Growth Rates among Urban Areas

It is now possible to evaluate the presence of GDP per capita disparities among urban areas, both in terms of levels and growth rates, along the BGP. Without loss of generality, let us assume that area i has developed a more productive research system; hence, $\delta_i > \delta_j$.

Proposition 1. *Along the BGP, the urban area with a relatively more productive research system is characterised by a relative specialization in research activities.*

Proof. From Section 2.6.1, equation (19), the assumption $\delta_i > \delta_j$ implies that research activities tend to concentrate in region i , i.e. $\hat{H}r_i > \hat{H}r_j$, while from Section 2.6.2, equation (24) the opposite occurs for manufacturing and $\hat{H}m_i < \hat{H}m_j$. $\square$

Having established the relative productive specialization of the urban areas we can turn to income disparities in levels. The level of GDP in each urban area can be calculated as the summation of the value added generated in its sectors as shown in Appendix A.4.

Proposition 2. *Along the BGP, a relative specialization in research activities is a sufficient condition for a constantly higher level of GDP per capita.*

Proof. See Appendix A.5 □

On the basis of these propositions, therefore, the urban area whose research systems is more productive features a relative specialization in research activities compared to the other urban area and enjoys a permanently higher level of GDP per capita. Next, we turn to growth rates.

Proposition 3. *Along the BGP, per capita GDP grows in both urban areas at the same constant rate as all other variables, irrespective of the areas' specialization.*

Proof. See Appendix A.6 □

3 Long-run Consequences of COVID-19 Pandemic Shock

As explained at the outset, the COVID-19 pandemic shock is likely to yield effects that extend well above the short term. In the R&D sector, in addition to labour and materials, a firm needs knowledge and information. Beside keeping up with scientific and technological advancements, an R&D firm requires timely access to problems, needs and requests to direct its activity. Clearly, confinement and social distancing put distance between people and this inevitably produces a disruption in this flow of knowledge and information.

Resorting to near-instant communication and videoconferencing is a way to counteract this. However, these services are likely to produce different results depending on the features of the network of relations in which they are employed.

Within the local, urban network of relations in which the R&D firm is embedded, their effectiveness is likely to be limited since an important component of information and knowledge flows generated at this level usually originate from casual sources. The ability of near-instant communication and videoconferencing to counteract the consequences of lockdown and social distancing may instead be stronger when these tools are adopted within an already established network of relations. Under this perspective, it is within global, inter-urban networks of relations that these tools are more likely to be successful in reducing distances, possibly giving a boost to the pre-existing phenomenon towards a digitalization of communications.

In model's terms, the COVID-19 shock may take essentially two forms: *i.* a reduction of the parameter summarizing the productivity of an urban research system (δ); *ii.* a reduction in the distance involved in inter-urban relations between researchers (d_{ij}). As for

their duration, while productivity reduction might be eliminated, at least partially, when social distancing restrictions are eventually lifted, the effect induced by videoconferencing on inter-urban distances is likely to persist.

The analysis of the possible long-run consequences of the COVID-19 pandemic shock requires examining the effects on the steady state solution of a change in d_{ij} and in δ . Let us start from the former.

Proposition 4. *A reduction in the distance between urban areas, d_{ij} , determines a reallocation of researchers that fosters the previously existing pattern of specialization.*

Proof. See Appendix A.7 □

Corollary 1. *The reallocation of researches in Proposition 4 implies an increase in the number of researchers employed in the more productive urban research system and a corresponding reduction in the less productive urban research system.*

Proof. See Appendix A.7 □

Aside from the effect on the allocation of researchers between urban areas, there is also an effect on the common long-run growth rate.

Proposition 5. *A reduction in the distance between urban areas, d_{ij} , determines an increase in the growth rate of the system along the BGP.*

Proof. See Appendix A.8 □

Now, attention can be moved to the long-term effects of a variation in the productivity of urban research systems. We start from the effects on the allocation of researchers

Proposition 6. *A reduction in the productivity of an urban research systems determines an outflow of researchers towards the research system of the other urban area .*

Proof. See Appendix A.9 □

Finally, we consider the effects on the growth rate of the entire economy.

Proposition 7. *A reduction in the productivity of an urban research systems determines a reduction of the system's growth rate along the BGP.*

Proof. See Appendix A.10 □

4 Discussion and Conclusions

We can now characterize the possible outcomes of the COVID-19 pandemic shock. As before, let us assume that, before the shock, urban area i has a relatively more productive research system. Proposition 1 then states that this area is also characterized by a relative specialization in research activities while urban area j has a relative specialization in manufacturing activities. Proposition 2 shows that GDP per capita is higher in i compared to j but Proposition 3 ensures the urban areas grow at the same constant rate.

Starting from this characterization, let us isolate the effects of a reduction in distance brought in by the diffusion of broadband technology in general, and videoconferencing in particular. Other things being equal, a reduction in distance re-allocates research activities in favour of the urban regional system with the higher level of productivity in research (Proposition 4) and hence increases urban disparities in GDP per capita. On the other hand, the increased concentration of researchers in the more productive research system has a positive effect on the common growth rate.

Turn now to the consequences of social distancing on productivity in research. A reduction in the level of productivity of an urban research system determines an outflow of researchers (Proposition 6), and a parallel reduction in GDP per capita, to the benefit of the other area. However, when social distancing measures are imposed to the entire system, the end result in terms of productive specialization and GDP per capita levels depends on the differential local incidence of this dimension of the shock. Social distancing measures have instead an unambiguously negative effect on the growth rate of the system due to Proposition 7. In our framework, therefore, the two dimensions of the COVID-19 shock have opposite implications on the growth rate of the system and the final outcome will depend on how they impact on the parameters of the economy.

A Appendix

A.1 Demand for Intermediate Inputs

The demand for each intermediate input is determined from the profit maximisation problems faced by final good producers. For a final good producer located in i , the problem is:

$$\max_x \pi = (\text{Revenues}_i - \text{Costs}_i)$$

where

$$\begin{aligned} \text{Revenues}_i &= L_i^\alpha H m_i^\beta \left[\int_{A_i} x_i(a_i)^\gamma da + \int_{A_j} x_i(a_j)^\gamma da \right] S m_i \\ \text{Costs}_i &= p_i(a_i) \int_{A_i} x_i(a_i)^\gamma da + p_i(a_j) \int_{A_j} x_i(a_j)^\gamma da + w(L)_i L_i + w(H m)_i H m_i \end{aligned}$$

From first-order conditions it is straightforward to derive the demand functions for intermediate inputs arising from final good producers located in region i :

$$\begin{aligned} x_i(a_i) &= \gamma^{\frac{1}{1-\gamma}} L_i^{\frac{\alpha}{1-\gamma}} H m_i^{\frac{\beta}{1-\gamma}} S m_i^{\frac{1}{1-\gamma}} p_i(a_i)^{-\frac{1}{1-\gamma}} \\ x_i(a_j) &= \gamma^{\frac{1}{1-\gamma}} L_i^{\frac{\alpha}{1-\gamma}} H m_i^{\frac{\beta}{1-\gamma}} S m_i^{\frac{1}{1-\gamma}} p_j(a_j)^{-\frac{1}{1-\gamma}} \end{aligned}$$

Analogously, from the profit maximization problem faced by final good producers in j it is possible to derive:

$$\begin{aligned} x_j(a_j) &= \gamma^{\frac{1}{1-\gamma}} L_j^{\frac{\alpha}{1-\gamma}} H m_j^{\frac{\beta}{1-\gamma}} S m_j^{\frac{1}{1-\gamma}} p_j(a_j)^{-\frac{1}{1-\gamma}} \\ x_i(a_j) &= \gamma^{\frac{1}{1-\gamma}} L_j^{\frac{\alpha}{1-\gamma}} H m_j^{\frac{\beta}{1-\gamma}} S m_j^{\frac{1}{1-\gamma}} p_i(a_i)^{-\frac{1}{1-\gamma}} \end{aligned}$$

Since

$$\begin{aligned} X(a_i) &= x_i(a_i) + x_j(a_j) \\ X(a_j) &= x_j(a_j) + x_i(a_j) \end{aligned}$$

it is immediate to show that the total demand functions faced by intermediate input producers are:

$$\begin{aligned} X(a_i) &= \gamma^{\frac{1}{1-\gamma}} \left(L_i^{\frac{\alpha}{1-\gamma}} H m_i^{\frac{\beta}{1-\gamma}} S m_i^{\frac{1}{1-\gamma}} + L_j^{\frac{\alpha}{1-\gamma}} H m_j^{\frac{\beta}{1-\gamma}} S m_j^{\frac{1}{1-\gamma}} \right) p_i(a_i)^{-\frac{1}{1-\gamma}} \\ X(a_j) &= \gamma^{\frac{1}{1-\gamma}} \left(L_i^{\frac{\alpha}{1-\gamma}} H m_i^{\frac{\beta}{1-\gamma}} S m_i^{\frac{1}{1-\gamma}} + L_j^{\frac{\alpha}{1-\gamma}} H m_j^{\frac{\beta}{1-\gamma}} S m_j^{\frac{1}{1-\gamma}} \right) p_j(a_j)^{-\frac{1}{1-\gamma}} \end{aligned}$$

while the corresponding prices are:

$$\begin{aligned} p_i(a_i) &= \gamma \left(L_i^{\frac{\alpha}{1-\gamma}} H m_i^{\frac{\beta}{1-\gamma}} S m_i^{\frac{1}{1-\gamma}} + L_j^{\frac{\alpha}{1-\gamma}} H m_j^{\frac{\beta}{1-\gamma}} S m_j^{\frac{1}{1-\gamma}} \right)^{1-\gamma} X(a_i)^{-(1-\gamma)} \\ p_j(a_j) &= \gamma \left(L_i^{\frac{\alpha}{1-\gamma}} H m_i^{\frac{\beta}{1-\gamma}} S m_i^{\frac{1}{1-\gamma}} + L_j^{\frac{\alpha}{1-\gamma}} H m_j^{\frac{\beta}{1-\gamma}} S m_j^{\frac{1}{1-\gamma}} \right)^{1-\gamma} X(a_j)^{-(1-\gamma)} \end{aligned}$$

A.2 Stability of Equilibrium Allocation of Researchers across Urban Areas

As shown in the text (equations 16 and 17), the wage rates for human capital employed in the two research sectors are:

$$w(Hr)_i = \eta \delta_i H r_i^{\eta-1} S r_{ij} \hat{X}(a) \frac{1-\gamma}{\gamma} A \quad (\text{A.1})$$

and

$$w(Hr)_j = \eta \delta_j H r_j^{\eta-1} S r_{ji} \hat{X}(a) \frac{1-\gamma}{\gamma} A \quad (\text{A.2})$$

The equilibrium allocation is achieved when the gap between the local wages is eliminated, *i.e.* when:

$$\frac{\hat{H}r_i}{\hat{H}r_j} = \left(\frac{\delta_i}{\delta_j} \right)^{\frac{1}{1-\eta}} \left(\frac{\nu_{ij}}{\nu_{ji}} \right)^{\frac{\psi}{1-\eta}} \quad (\text{A.3})$$

as shown in equation (18). To check the stability of the equilibrium, it is necessary to see whether the wage gap declines are researchers move towards the area offering a higher wage. Put it differently, and given the symmetry in the model, the equilibrium is stable when, in its correspondence, the higher wage declines (and the lower wage increases) as researcher migrate towards the area with the higher wage. More formally, assume the overall number of researchers is given⁶ and equal to $\hat{H}r$. The condition for the stability

⁶ See A.3 for a demonstration that this assumption is met in the steady-state equilibrium.

of the equilibrium in equation (A.3) thus is

$$\frac{\partial w(Hr)_i}{\partial Hr_i} \Big|_{Hr_i = \widehat{H}r_i} < 0 \quad (\text{A.4})$$

Using equation (16) in the text, the definition of the spatial spillovers in equation (2), and taking note that $Hr_j = \widehat{H}r - Hr_i$, after some manipulation it is possible to show that equilibrium stability requires

$$\left[\frac{\psi + \eta - 1}{\widehat{H}r_i} - \frac{\psi}{\widehat{H}r_j} \right] \left[\widehat{H}r_i^{\psi+\eta-1} \widehat{H}r_j^\psi \right] \nu_{ij}^\psi \eta \delta_i X(a) \frac{1-\gamma}{\gamma} A < 0$$

The sign of the quantity on the left hand side simply depends on the sign of its first term; hence, the stability condition simplifies to:

$$\frac{\psi}{\psi + \eta - 1} < \frac{\widehat{H}r_i}{\widehat{H}r_j} \quad (\text{A.5})$$

Clearly, if the restriction $\psi < 1 - \eta$ holds, the condition (A.5) for the stability of the equilibrium spatial allocation of researchers is always satisfied. It must be noted, however, that the above restriction is sufficient but not necessary to ensure stability. For instance, when $\delta_i > \delta_j$, the equilibrium allocation leads to $\widehat{H}r_i/\widehat{H}r_j > 1$ and a more general restriction $0 \leq \psi < 1$ would suffice. It is only when $\delta_i < \delta_j$ and $\widehat{H}r_i/\widehat{H}r_j < 1$ that $\psi < 1 - \eta$ might be necessary to ensure stability.

A.3 Equilibrium Allocation of Human Capital across Sectors

As shown in the text, the equilibrium condition for the allocation of human capital across all sectors and urban areas can be written as

$$w(\widehat{H}m)_i + w(\widehat{H}m)_j = 2w(\widehat{H}r)_i \quad (\text{A.6})$$

The wage rates that human capital receives in final goods sectors can be derived from production functions in equation (13)

$$\begin{aligned} w(\widehat{H}m)_i &= \frac{\partial M_i}{\partial Hm_i} = \beta L_i^\alpha Hm_i^{\beta-1} A \hat{x}_i(a)^\gamma Sm_i \\ w(\widehat{H}m)_j &= \frac{\partial M_j}{\partial Hm_j} = \beta L_j^\alpha Hm_j^{\beta-1} A \hat{x}_j(a)^\gamma Sm_j \end{aligned}$$

From demand functions in equations 12 it is then immediate to derive

$$\begin{aligned}\hat{p}_i(a_i)\hat{x}_i(a) &= \gamma L_i^\alpha H m_i^\beta S m_i \hat{x}_i(a)^\gamma \\ \hat{p}_j(a_j)\hat{x}_j(a) &= \gamma L_j^\alpha H m_j^\beta S m_j \hat{x}_j(a)^\gamma\end{aligned}$$

Substituting these into the previous equations, and recalling that $\hat{p}(a) = \hat{p}_i(a_i) = \hat{p}_j(a_j) = r\gamma^{-1}$, wage rates in manufacturing become

$$\begin{aligned}w(\hat{H}m)_i &= \beta r \gamma^{-2} A \hat{x}_i(a) \hat{H}m_i^{-1} \\ w(\hat{H}m)_j &= \beta r \gamma^{-2} A \hat{x}_j(a) \hat{H}m_j^{-1}\end{aligned}$$

As shown in equation (17), the wage rate for researchers in j is

$$w(Hr)_j = \eta \delta_j H r_j^{\eta-1} S r_{ji} \hat{X}(a) \frac{1-\gamma}{\gamma} A$$

thus, after simple manipulation, condition (A.6) becomes

$$\frac{r\beta\gamma^{-2}}{\gamma^{-1}(1-\gamma)} = \frac{2\hat{X}(a)}{\left[\hat{x}_i(a)\hat{H}m_i^{-1} + \hat{x}_j(a)\hat{H}m_j^{-1}\right]} \eta \delta_j \hat{H}r_j^{\psi+\eta-1} \hat{H}r_i^\psi \nu_{ji}^\psi$$

Given that the overall demand faced by intermediate input producers is

$$\hat{X}(a) = \frac{1}{2} [\hat{x}_i(a) + \hat{x}_j(a)]$$

that $\hat{H}m = \hat{H}m_i + \hat{H}m_j$ and that in equilibrium the ratio $\hat{x}_i(a)/\hat{x}_j(a) = \hat{H}m_i/\hat{H}m_j$, the condition simplifies to

$$2r\beta\gamma^{-1}(1-\gamma)^{-1} = \eta \delta_j \hat{H}r_i^\psi \hat{H}r_j^{\eta+\psi-1} \nu_{ji}^\psi \hat{H}m \quad (\text{A.7})$$

Using the intertemporal optimisation condition that relates r to the rate of growth of consumption in equation (5), and given that the growth rate of consumption must be equal to the growth rate of abstract knowledge along a steady-state growth path shown in equation (20), it is possible to express the interest rate as

$$r = \sigma(1-\Gamma)^{-1} \delta_j \hat{H}r_i^\psi \hat{H}r_j^{\eta+\psi-1} \nu_{ji}^\psi \hat{H}r = \sigma(1-\Gamma)^{-1} \delta_i \hat{H}r_j^\psi \hat{H}r_i^{\eta+\psi-1} \nu_{ij}^\psi \hat{H}r \quad (\text{A.8})$$

By substituting this expression into the equilibrium condition (A.7), it is easy to show

that the equilibrium in the market for human capital requires

$$\widehat{H}r = \frac{\eta(1-\Gamma)(1-\gamma)\gamma}{2\sigma\beta} \widehat{H}m$$

In steady-state, the allocation of the existing stock of human capital stays constant as it depends on the consumers' preferences, via σ and Γ , and on factors' shares in final good production. Since $\widehat{H}r$ is constant, also $\widehat{H}r_i$ and $\widehat{H}r_j$, the equilibrium levels of researchers in the urban areas, stay constant.

A.4 Urban GDP levels

The level of GDP in each urban area is calculated as the summation of the value added generated in its sectors. Consider the case of urban area i . The value added in its research sector is just the wage rate multiplied by the level of human capital being employed or, equivalently, the number of patents realised multiplied by the patent's price

$$VA_r = w(H)\widehat{H}r_i = \widehat{P}(a)\dot{A}_j$$

Value added in the intermediate goods sector is equal to the total value of its output net of the fixed costs of patents

$$VA_{ig} = \hat{p}(a)A_i [\hat{x}_i(a_i) + \hat{x}_j(a_i)] - \widehat{P}(a)\dot{A}_j$$

Value added in the final goods sector is the total value of output net of the cost of intermediate goods employed in production

$$VA_m = \widehat{M}_i - \hat{p}(a)A_i\hat{x}_i(a_i) - \hat{p}(a)A_j\hat{x}_i(a_j)$$

Given that the final goods sector is perfectly competitive and the remuneration of the production factors exhausts M_i , the level of GDP in i can be expressed as

$$\widehat{Y}_i = \hat{w}(L)\widehat{L}_i + \hat{w}(H)\widehat{H}m_i + \hat{p}(a)A_i\widehat{X}(a)$$

Substituting $\hat{p}(a)$ from equation (11), $\widehat{X}(a)$ from (15), and then using the definition of the interest rate along the steady-state path in equation (A.8), the overall GDP level in i becomes

$$\widehat{Y}_i = \hat{w}(L)\widehat{L}_i + \hat{w}(H)\widehat{H}m_i + \hat{w}(H)\widehat{H}r_i\sigma(1-\gamma)^{-1}(1-\Gamma)^{-1} \quad (\text{A.9})$$

With an analogous procedure, the level of GDP in j can be shown to be

$$\hat{Y}_j = \hat{w}(L)\hat{L}_j + \hat{w}(H)\hat{H}m_j + \hat{w}(H)\hat{H}r_j\sigma(1-\gamma)^{-1}(1-\Gamma)^{-1} \quad (\text{A.10})$$

A.5 Proof of Proposition 2

To evaluate disparities, we can focus on the ratio between GDP per capita levels:

$$\frac{\hat{y}_i}{\hat{y}_j} = \frac{\hat{w}(L)\hat{L}_i + \hat{w}(H)\hat{H}m_i + \hat{w}(H)\hat{H}r_i\sigma(1-\gamma)^{-1}(1-\Gamma)^{-1}}{\hat{w}(L)\hat{L}_j + \hat{w}(H)\hat{H}m_j + \hat{w}(H)\hat{H}r_j\sigma(1-\gamma)^{-1}(1-\Gamma)^{-1}} \frac{\hat{L}_j + \hat{H}m_j + \hat{H}r_j}{\hat{L}_i + \hat{H}m_i + \hat{H}r_i} \quad (\text{A.11})$$

Dividing numerator and denominator by $w(L)$ and rewriting gives

$$\frac{\hat{y}_i}{\hat{y}_j} = \frac{\hat{L}_i \left(1 + w \frac{\hat{H}m_i}{\hat{L}_i} + w \frac{\hat{H}r_i}{\hat{L}_i} \delta_i \Omega\right)}{\hat{L}_j \left(1 + w \frac{\hat{H}m_j}{\hat{L}_j} + w \frac{\hat{H}r_j}{\hat{L}_j} \delta_j \Omega\right)} \frac{\hat{L}_j \left(1 + \frac{\hat{H}m_j}{\hat{L}_j} + \frac{\hat{H}r_j}{\hat{L}_j}\right)}{\hat{L}_i \left(1 + \frac{\hat{H}m_i}{\hat{L}_i} + \frac{\hat{H}r_i}{\hat{L}_i}\right)}$$

where $w \equiv w(H)/w(L)$ and $\Omega \equiv \sigma(1-\gamma)^{-1}(1-\Gamma)^{-1}$. Recalling that in equilibrium $\hat{H}m_i/\hat{L}_i = \hat{H}m/\hat{L} = \hat{H}m/\hat{L}$ and rearranging yields

$$\frac{\hat{y}_i}{\hat{y}_j} = \frac{\left[\left(1 + w \frac{\hat{H}m}{\hat{L}}\right) + \left(w\Omega \frac{\hat{H}r_i}{\hat{L}_i} \delta_i\right)\right] \left[\left(1 + \frac{\hat{H}m}{\hat{L}}\right) + \frac{\hat{H}r_j}{\hat{L}_j}\right]}{\left[\left(1 + w \frac{\hat{H}m}{\hat{L}}\right) + \left(w\Omega \frac{\hat{H}r_j}{\hat{L}_j} \delta_j\right)\right] \left[\left(1 + \frac{\hat{H}m}{\hat{L}}\right) + \frac{\hat{H}r_i}{\hat{L}_i}\right]}$$

Multiplying terms and rearranging finally gives

$$\frac{\hat{y}_i}{\hat{y}_j} = \frac{\left(1 + w \frac{\hat{H}m}{\hat{L}}\right) \left(1 + \frac{\hat{H}m}{\hat{L}}\right) + \Omega w \frac{\hat{H}r_i \hat{H}r_j}{\hat{L}_i \hat{L}_j} \delta_i + \left(1 + w \frac{\hat{H}m}{\hat{L}}\right) \frac{\hat{H}r_j}{\hat{L}_j} + \Omega w \left(1 + w \frac{\hat{H}m}{\hat{L}}\right) \frac{\hat{H}r_i}{\hat{L}_i} \delta_i}{\left(1 + w \frac{\hat{H}m}{\hat{L}}\right) \left(1 + \frac{\hat{H}m}{\hat{L}}\right) + \Omega w \frac{\hat{H}r_i \hat{H}r_j}{\hat{L}_i \hat{L}_j} \delta_j + \left(1 + w \frac{\hat{H}m}{\hat{L}}\right) \frac{\hat{H}r_i}{\hat{L}_i} + \Omega w \left(1 + w \frac{\hat{H}m}{\hat{L}}\right) \frac{\hat{H}r_j}{\hat{L}_j} \delta_j}$$

Since $\delta_i > \delta_j$ it is clear that a sufficient condition for $\hat{y}_i > \hat{y}_j$ is

$$\frac{\left(1 + w \frac{\hat{H}m}{\hat{L}}\right) \frac{\hat{H}r_j}{\hat{L}_j} + \Omega w \left(1 + w \frac{\hat{H}m}{\hat{L}}\right) \frac{\hat{H}r_i}{\hat{L}_i} \delta_i}{\left(1 + w \frac{\hat{H}m}{\hat{L}}\right) \frac{\hat{H}r_i}{\hat{L}_i} + \Omega w \left(1 + w \frac{\hat{H}m}{\hat{L}}\right) \frac{\hat{H}r_j}{\hat{L}_j} \delta_j} > 1$$

Rearranging terms, the sufficient can be written as

$$\Omega \frac{w + w \frac{\hat{H}m}{\hat{L}}}{1 + w \frac{\hat{H}m}{\hat{L}}} \frac{\frac{\hat{H}r_i}{\hat{L}_i} \delta_i - \frac{\hat{H}r_j}{\hat{L}_j} \delta_j}{\frac{\hat{H}r_i}{\hat{L}_i} - \frac{\hat{H}r_j}{\hat{L}_j}} > 1 \quad (\text{A.12})$$

The proof now proceeds by showing that each of the three terms on the left hand side is larger than 1. The first, Ω , has been defined above and is equivalent to $\sigma(1-\gamma)^{-1}(1-\Gamma)^{-1}$. Since γ is the capital share of final output, $(1-\gamma)^{-1} > 1$. Putting the interest rate r from equation (6) into the condition (7) directly yields

$$\sigma(1-\Gamma)^{-1} > 1 \quad (\text{A.13})$$

demonstrating that $\Omega > 1$. Moving to the second term, note that w is simply the ratio by the equilibrium wage rates of human capital and labour. Given the assumption $H_m/L < \beta/\alpha$ is assumed to be lower than made in Section 2.3, the ratio w is always greater than 1. As a consequence

$$\frac{w + w \frac{\hat{H}m}{\hat{L}}}{1 + w \frac{\hat{H}m}{\hat{L}}} > 1$$

Finally, the denominator in the last term is always larger than the denominator whenever $\delta_i > \delta_j$ as assumed in the text.

A.6 Proof of Proposition 3

Divide GDP levels in equations (A.9) and (A.10) by corresponding work forces to get GDP per capita levels in the two urban areas

$$\hat{y}_i = \frac{\hat{w}(L)\hat{L}_i + \hat{w}(H)\hat{H}m_i + \hat{w}(H)\hat{H}r_i\Omega}{\hat{L}_i + \hat{H}m_i + \hat{H}r_i}$$

$$\hat{y}_j = \frac{\hat{w}(L)\hat{L}_j + \hat{w}(H)\hat{H}m_j + \hat{w}(H)\hat{H}r_j\Omega}{\hat{L}_j + \hat{H}m_j + \hat{H}r_j}$$

where $\Omega \equiv \sigma(1-\gamma)^{-1}(1-\Gamma)^{-1}$. From here, it is now possible to derive the following proportional growth rates:

$$\frac{\dot{\hat{y}}_i}{\hat{y}_i} = \frac{\frac{\dot{\hat{w}}(L)}{\hat{w}(L)} \frac{\hat{L}_i}{\hat{w}(H)} + \frac{\dot{\hat{w}}(H)}{\hat{w}(H)} \frac{\hat{H}m_i}{\hat{w}(L)} + \Omega \frac{\dot{\hat{w}}(H)}{\hat{w}(H)} \frac{\hat{H}r_i}{\hat{w}(L)}}{\frac{\hat{L}_i}{\hat{w}(H)} + \frac{\hat{H}m_i}{\hat{w}(L)} + \frac{\hat{H}r_i}{\hat{w}(L)}}$$

$$\frac{\dot{\hat{y}}_j}{\hat{y}_j} = \frac{\frac{\dot{\hat{w}}(L)}{\hat{w}(L)} \frac{\hat{L}_j}{\hat{w}(H)} + \frac{\dot{\hat{w}}(H)}{\hat{w}(H)} \frac{\hat{H}m_j}{\hat{w}(L)} + \Omega \frac{\dot{\hat{w}}(H)}{\hat{w}(H)} \frac{\hat{H}r_j}{\hat{w}(L)}}{\frac{\hat{L}_j}{\hat{w}(H)} + \frac{\hat{H}m_j}{\hat{w}(L)} + \frac{\hat{H}r_j}{\hat{w}(L)}}$$

Since in the steady-state all wage rates grow at the same rate as abstract knowledge, it follows immediately that, along the BGP, GDP per capita grows in both urban areas at the same constant rate as all other variables:

$$\frac{\dot{\hat{y}}_i}{\hat{y}_i} = \frac{\dot{\hat{y}}_j}{\hat{y}_j} = \frac{\dot{A}}{A}$$

A.7 Proof of Proposition 4 and Corollary 1

Previously, we assumed that urban area i is characterized by a more productive research system, *i.e.*, $\delta_i > \delta_j$. Proposition 1 determined that, in the decentralized equilibrium, research activities tend to be relatively concentrated in this area: $\hat{H}r_i/\hat{H}r_j > 1$. In particular, equation (19) showed that the equilibrium allocation of researchers depends on productivity levels in the two research systems and on distance d_{ij} . Now, we evaluate the effect of a variation of distance on this allocation taking the partial derivative of $\hat{H}r_i/\hat{H}r_j > 1$ with respect to d_{ij} . After simple manipulation, we have

$$\frac{\partial \frac{\hat{H}r_i}{\hat{H}r_j}}{\partial d_{ij}} = \left(\frac{\delta_i}{\delta_j} \right)^{\frac{1}{1-\epsilon\eta\alpha}} \frac{\psi}{1-\eta} \left[\frac{\log d_{ij} + \log \delta_i}{\log d_{ij} + \log \delta_j} \right]^{\psi+\eta-1} (\log d_{ij} + \log \delta_j)^2 (\log \delta_j - \log \delta_i)$$

the sign of which depends on the sign of the last term on the right hand side. Given we are considering the case $\delta_i > \delta_j$, it follows

$$\frac{\partial \frac{\hat{H}r_i}{\hat{H}r_j}}{\partial d_{ij}} < 0$$

implying that a reduction of d_{ij} determines an increase in $\hat{H}r_i/\hat{H}r_j$. In addition, since equation (25) ensures that $\hat{H}r = \hat{H}r_i + \hat{H}r_j$ does not depend on d_{ij} , a variation of d_{ij} produces a variation in the same direction of $\hat{H}r_i$ and in the opposite direction of $\hat{H}r_j$:

$$\frac{\partial \hat{H}r_i}{\partial d_{ij}} < 0 \quad \text{and} \quad \frac{\partial \hat{H}r_j}{\partial d_{ij}} > 0$$

A.8 Proof of Proposition 5

Proposition (3) stated that along the BGP per capita GDP grows at the same, constant rate in both urban areas and that this rate is equal to $\dot{A}/A$. This common rate can

be expressed as

$$\frac{\dot{A}}{A} = \frac{\dot{A}_i}{A} + \frac{\dot{A}_j}{A} = \delta_i \hat{H}r_i^{\eta+\psi} \hat{H}r_j^\psi \nu_{ij}^\psi + \delta_j \hat{H}r_j^{\eta+\psi} \hat{H}r_i^\psi \nu_{ji}^\psi$$

After some manipulation it is possible to obtain

$$\frac{\dot{A}}{A} = \delta_i \hat{H}r \left(\hat{H}r_i^{\eta+\psi-1} \hat{H}r_j^\psi \right) \nu_{ji}^\psi$$

or, equivalently, as

$$\frac{\dot{A}}{A} = \delta_j \hat{H}r \left(\hat{H}r_j^{\eta+\psi-1} \hat{H}r_i^\psi \right) \nu_{ji}^\psi$$

Let us focus on the latter and consider

$$\frac{\partial \frac{\dot{A}}{A}}{\partial d_{ij}} = \left[\frac{\partial \left(\hat{H}r_j^{\eta+\psi-1} \hat{H}r_i^\psi \right)}{\partial d_{ij}} \nu_{ji}^\psi + \frac{\partial \nu_{ji}^\psi}{\partial d_{ij}} \left(\hat{H}r_j^{\eta+\psi-1} \hat{H}r_i^\psi \right) \right] \delta_j \hat{H}r \quad (\text{A.14})$$

The first partial derivative within the square bracket can be expanded as

$$\frac{\partial \left(\hat{H}r_j^{\eta+\psi-1} \hat{H}r_i^\psi \right)}{\partial d_{ij}} = \hat{H}r_j^{\eta+\psi-1} \hat{H}r_i^\psi \left[\psi \hat{H}r_i \frac{\partial \hat{H}r_i}{\partial d_{ij}} - (1 - \psi - \eta) \hat{H}r_j^{-1} \frac{\partial \hat{H}r_j}{\partial d_{ij}} \right]$$

Corollary 1 showed that a variation of d_{ij} produces a variation in the same direction of $\hat{H}r_i$ and in the opposite direction of $\hat{H}r_j$. Consequently, the first partial derivative in equation (A.14) has a negative sign. As for the sign of the second partial derivative in (A.14), this is also certainly negative since

$$\nu_{ji} = 1 + \frac{\log \delta_j}{\log d_{ij}}$$

Hence, a reduction of d_{ij} determines an increase in the growth rate of A along the BGP.

A.9 Proof of Proposition 6

From the equilibrium allocation of researchers in equation (18), we easily find

$$\frac{\partial \frac{\hat{H}r_i}{\hat{H}r_j}}{\partial \delta_i} = \frac{\partial \frac{\delta_i}{\delta_j}}{\partial \delta_i} \left(\frac{\nu_{ij}}{\nu_{ji}} \right)^{\frac{\psi}{1-\eta}} + \frac{\partial \frac{\nu_{ij}}{\nu_{ji}}}{\partial \delta_i} \left(\frac{\delta_i}{\delta_j} \right)^{\frac{1}{1-\eta}} > 0$$

as

$$\frac{\partial \frac{\delta_i}{\delta_j}}{\partial \delta_i} > 0 \quad \text{and} \quad \frac{\partial \frac{\nu_{ij}}{\nu_{ji}}}{\partial \delta_i} = \delta_i^{-1} (\log d_{ij} + \log \delta_j)^{-1} > 0$$

In a similar fashion,

$$\frac{\partial \frac{\hat{H}r_i}{\hat{H}r_j}}{\partial \delta_j} = \frac{\partial \frac{\delta_i}{\delta_j}}{\partial \delta_j} \left(\frac{\nu_{ij}}{\nu_{ji}} \right)^{\frac{\psi}{1-\eta}} + \frac{\partial \frac{\nu_{ij}}{\nu_{ji}}}{\partial \delta_j} \left(\frac{\delta_i}{\delta_j} \right)^{\frac{1}{1-\eta}} < 0$$

as

$$\frac{\partial \frac{\delta_i}{\delta_j}}{\partial \delta_j} < 0 \quad \text{and} \quad \frac{\partial \frac{\nu_{ij}}{\nu_{ji}}}{\partial \delta_j} = -\delta_j^{-1} \frac{\log d_{ij} + \log \delta_i}{(\log d_{ij} + \log \delta_j)^{-2}} < 0$$

In analogy with Corollary 1, given that equation (25) ensures that $\hat{H}r = \hat{H}r_i + \hat{H}r_j$ does not depend on neither δ_i nor δ_j , we have

$$\frac{\partial \hat{H}r_i}{\partial \delta_i} > 0 \quad \text{and} \quad \frac{\partial \hat{H}r_j}{\partial \delta_i} < 0$$

as well as

$$\frac{\partial \hat{H}r_i}{\partial \delta_j} < 0 \quad \text{and} \quad \frac{\partial \hat{H}r_j}{\partial \delta_j} > 0$$

A.10 Proof of Proposition 7

The effect of a variation of δ_i on the growth rate along the BGP is

$$\frac{\partial \dot{\frac{A}{A}}}{\partial \delta_i} = \delta_j \hat{H}r \hat{H}r_i^\psi \hat{H}r_j^{(\psi+\eta)-1} \nu_{ji}^\psi \psi \left[\frac{\partial \hat{H}r_i}{\partial \delta_i} \hat{H}r_i^{-1} + \frac{\partial \hat{H}r_j}{\partial \delta_i} \frac{\psi + \eta - 1}{\psi} \hat{H}r_j^{-1} \right] > 0$$

given that

$$\frac{\partial \hat{H}r_i}{\partial \delta_i} > 0, \quad \frac{\partial \hat{H}r_j}{\partial \delta_i} < 0 \quad \text{and} \quad \psi + \eta < 1$$

In a similar fashion, the effect of a variation of δ_j is

$$\frac{\partial \dot{\frac{A}{A}}}{\partial \delta_j} = \delta_i \hat{H}r \hat{H}r_j^\psi \hat{H}r_i^{(\psi+\eta)-1} \nu_{ij}^\psi \psi \left[\frac{\partial \hat{H}r_j}{\partial \delta_j} \hat{H}r_j^{-1} + \frac{\partial \hat{H}r_i}{\partial \delta_j} \frac{\psi + \eta - 1}{\psi} \hat{H}r_i^{-1} \right] > 0$$

given that

$$\frac{\partial \hat{H}r_j}{\partial \delta_j} > 0, \quad \frac{\partial \hat{H}r_i}{\partial \delta_j} < 0 \quad \text{and} \quad \psi + \eta < 1$$

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