Summer 2018

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Math 1b Calculus, Series and Differential Equations
MATH S-1Bv

Course Description
About four hundred years ago, Galileo wrote "The book of the universe is written in the language of mathematics." Although the language of mathematics has evolved over time, the statement has as much validity today as it did when it was written. In this course you will become more well-versed in the language of modern mathematics and learn more about its applications to other disciplines. Over the summer we will study three (related) topics, topics that form a central part of the language of modern science:

- Applications and methods of integration
- Infinite series and the representation of functions by infinite polynomials known as power series,
- Differential equations.

What better place is there to study integration and differential equations than in Italy, the land of Galileo, Cavalieri, Agnesi, Riccati and Volterra! The material we take up in this course has applications in physics, chemistry, biology, environmental science, astronomy, economics, and statistics.

Prerequisites
An introduction to calculus is the expected prerequisite. Students are expected to be fluent with trigonometry, inverse trig, exponentials, and logarithms, understand the notion of a derivative, be able to differentiate using the Product, Quotient, and Chain Rules, and know what a definite and indefinite integral are. Students are expected to have been introduced to the Fundamental Theorem of Calculus and be able to calculate definite integrals via an anti-derivative. (If you have all but the latter, we can work that out if you have some spare time!)

Readings
A textbook and course notes will be provided free of charge on our course website.

Course requirements
Problem Sets will be assigned daily to be turned in at the following class. Students are encouraged to work with one-another on these assignments. Talking about your ideas and solutions is a great way to improve your understanding and doing mathematics is really about communicating ideas and questions. However, all work you submit must be written up individually in your own words, and you shouldn't ever submit work that you wouldn't be comfortable explaining clearly to another student or to your instructor.

At the end of each assignment, please acknowledge any help that you have received; doing so will not affect your homework score in any way. Of course, you should not under any circumstances turn in work that you have copied from the internet, from a solution manual, from another student or from any other source.

Class meetings and section
Classes for this course run for **8 weeks** in total - June 18th - August 8th
The first two sessions are Monday June 18th and Tuesday June 19th, 2-5pm

Then from June 25th until August 2nd:
Mondays 8:45-9:45
Tuesdays 9:45-12:15
Thursdays 8:45-12:15

Finally, there will be an extra session on Friday August 3rd and the two final sessions in the last week are:
Tuesday August 7th, 9-12
Wednesday August 8th , 9-12

Five one-hour sections will be held on Wednesdays in weeks 3-7.

Field trips
There is an optional trip to the DaVinci Museum in Florence, tentatively July 20th.

Exams: There will be two midterms, one on integration and another on series, and a final exam. They will be given in the evening:

Midterm I: Tuesday, July 3rd
Midterm II: Tuesday, July 24th
Final: Wednesday, August 7th, 9 am - 12

Grading
Expectations: We conduct this class in an "active learning" style. This means the following:

1. Your attendance is mandatory. On the days when we have three hours of class we are covering the material covered in an entire week term-time, so to miss a class is equivalent to missing a week of class.
2. You must actively participate. When faced with a problem, focus your attention on it and be willing to work. Mistakes are welcomed; that is how you learn. An unwillingness to makes errors hinders the progress of the entire class.
3. Make an attempt to communicate your reasoning. This can be challenging, but communicating mathematics is a goal of the course.

The grading policy is designed so that each student can represent his or her mastery of

the material in a favorable light. Your course grade will be determined as follows:

- Midterm score: Take the higher of
    - 45% First Exam + 55% Second Exam
    - 55% First Exam + 45% Second Exam
- Course score: Take the higher of
    - 45% Final Exam + 30% midterm score + 25% homework
    - 30% Final Exam + 45% midterm score + 25% homework
    - 40% Final Exam + 45% midterm score + 15% homework

Seminars

Each bullet point indicates more or less an hour of class time, with some flexibility built into the schedule.

Note: Techniques of integration will be taught in a "flipped classroom" style. There are videotaped lessons on our course website. You should start working through them independently, outside of class, so that you have completed that by the first week.

WEEK	Topics	Date	Time
1	• Slicing and Approximation: Total mass from density, total population from population density. • Slicing and the definite integral as limit of Riemann sums. • Recap of the Fundamental Theorem of Calculus and areas – perhaps start volumes	6/18 Monday	**2-5pm**
	• Volumes • Volumes of revolution • Techniques of Integration: - Substitution: the integration analogue of the Chain Rule; - Integration by Parts: the integration counterpart of the Product Rule	6/19 Tuesday	**2-5pm**
2	• Numerical Integration Methods and Error Part I	6/25 Monday	**8:45 – 9:45**
	• Numerical Integration Methods and Error Part II • Improper Integrals: how to evaluate, comparison techniques: Take I	6/26 Tuesday	**9:45-12:15**
	• Improper integrals • 3-D Density problems • Work or Present value of a continuous	6/28 Thursday	**8:45-12:15**

	income stream		
3	• **Taylor approximation**	7/2Monday	**8:45-9:45**
	• Taylor series • Review of Integration unit	7/3 Tuesday	**9:45 –12:15**
	Mid-term exam	7/3 Tuesday	**5-7:30pm**
	• Define Convergence • Nth Term Test • Monotonic Bounded Sequences of Partial Sums, and Comparison	7/5Thursday	**8:45 – 12:15**
4	• p-series	7/9 Monday	**8:45-9:45**
	• Geometric Series • Ratio Test	7/10 Tuesday	**9:45 –12:15**
	• Introduction to Power Series • Start Representation of Functions by Power Series: with substitution • Power series representations of functions. Interval of Convergence	7/12Thursday	**8:45 – 12:15**
5	• Getting new power series from old ones by substitution, differentiation and integration.	7/16 Monday	**8:45-9:45**
	• Alternating Series and Error Bound • Bounding error by a geometric series, by an improper integral.	7/17 Tuesday	**9:45 –12:15**
	• Asymptotics: How to think intuitively about whether or not a series converges • More series, Review	719 Thursday	**8:45 – 12:15**
6	• Introduction to differential equations, solutions, modeling	7/23 Monday	**8:45-9:45**
	• A Qualitative Look at Differential Equations – Slope fields: dy/dt= 1 dy/dt = t dy/dt= y and dy/dt = -t/y • Guess and check solutions. Qualitative discussion of Euler's method. • Begin Autonomous first order differential equations	7/24 Tuesday	**9:45 –12:15**
	Mid-term exam (Series)	7/24 Tuesday	**5-7:30pm**
	• Autonomous first order differential equations: qualitative analysis of solutions and mixture problems • Solving Separable Differential Equations; simple substitution • Maybe a quiz in class	7/26 Thursday	**8:45 – 12:15**

7	• Intro to second order linear homogeneous diff'leq'ns via springs-	7/30 Monday	**8:45-9:45**
	• Springs: part II • Springs: part III	7/31 Tuesday	**9:45 –12:15**
	• Systems of Differential Equations • Intro to systems of differential equations • Phase plane analysis : Systems and shapes of trajectories	8/2 Thursday	**8:45 – 12:15**
	• Modeling Epidemics Using Systems of Differential Equations • Systems wrap-up	8/3 Friday	**4-7pm**
8	**Review**	8/6 Tuesday	**9-12**
	Final Exam	8/7 Wednesday	**9-12**