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INFORMATION, SOCIAL MOBILITY AND THE DEMAND FOR REDISTRIBUTION

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Abstract

We study the consensus for redistribution in a framework where the voters consider their income Perspectives Of Upward Mobility (POUM effect) and test it to the Italian case.

We propose a theoretical model where individuals, according to their personal characteristics, may discriminate to belong to a particular population group. These individual characteristics affect the future perspectives of mobility through different income dynamics. This general framework let us to explain how, individuals with the same current income, may have different preferences for redistribution. We show, by examples and estimations, that in the case people have better information on their future perspectives, an assumption of concavity on the aggregate income transition function as required by Ok and Benabou [2000], is neither sufficient nor necessary for a Poum effect.

Keywords: Social Mobility, Income Distribution, Political Economy, Inequality, Taxation.

1. Introduction

There is empirical evidence that in most countries income distribution is asymmetric (with the median income below the average income; i.e. the majority of the population has an income smaller than the mean). If this situation were crystallized, an absolute majority of the population would gain from complete income redistribution. From this observation a natural question arises: why didn’t it happen? Therefore, why does a relatively poor majority not impose complete income redistribution on a relatively rich minority? Often social mobility has been used to give an answer: given that today’s poor may be the wealthy of tomorrow, social mobility should affect individual preferences for redistributive policies. This effect is denoted as POUM effect: Prospect Of Upward Mobility.

The first author that used this argument was Tocqueville (1835). His idea was that the differences between the redistributive policies of Europe and United States could be explained by presumed differences in social mobility.

In recent years, the literature on the link between social mobility and redistributive politics is very scant. Indeed, there are many papers about the links between income inequalities, redistribution and growth but they don’t consider social mobility to be an important factor. For example, in the models of Alesina and Rodrik (1994), Persson and Tabellini (1994), the fundamental idea is that redistribution, by discouraging investments, results in a lower growth rate.

The first work that considers the link between social mobility and redistribution is Hirschmann’s (1973). More recently, we have Piketty’s study (1995), which addresses intergenerational mobility as a way to explaining the different preferences towards redistribution. Lastly, a recent paper of Bènabou and Ok (2001) considers intra-generational mobility as to explain the low level of redistribution in modern democracies. The basic idea is that the agents know the true mobility process and maximize their actual value of the expected incomes in future years. That is, agents evaluate their mobility perspectives with regard to each other. In this model the fundamental assumption is that the expected individual income is a strictly concave function of the current income. It follows that the distribution of the expected incomes involves a smaller fraction of people with an expected income below the expected income mean (with respect to the current income distribution). The consequence is
obviously a lower consensus for a more redistributive policy. A basic assumption of this model is the homogeneity of the population with respect to mobility process. Indeed the income process is unique and doesn’t take into account the different origins of income implicit in the individual differences. From another point of view, the individuals have not a “good information”: they cannot discriminate among different income processes in according to social differences.

Is the assumption of homogeneity of individuals (with regard the income process) sustainable? Or, do individuals have only an aggregate information regarding their income process? The answer to first question is not, while the answer to the second one is probably not. For example, we may consider three different groups of individuals: employees, the self-employed and the pensioners. The pensioners have an income determined by law, the income of self- employed individuals depends on the market’s conditions and the employees stay in an intermediate position. If these individuals could discriminate between different income dynamics, their expected income would be different, as well as, the uncertainty on their future (given they belong to a specific social group). It follows that, individuals preferences in the evaluation of the future redistributive policies can change accordingly to the level of information about group characteristics.

In support to this idea of heterogeneity, we have a growing empirical literature that considers how individual characteristics might affect income dynamics. For example, Stewart and Swaffield (1999) estimate, according to individual characteristics, the transition probabilities to remaining in the low income group. Whereas, Alvarez, Browning and Ejrnæs (2002) find the presence of a latent heterogeneity in the income dynamics.

In this paper we construct, and verify, a model that links preferences towards the redistributive policies with a specific mobility process of some considered different groups. This model let us to explain the differences in preferences (towards the redistributive policies) between different social groups, also in the case that income differences explain nothing. That is, it explains, for example, why two individuals, with the same income, could have different preferences. Finally, we show, by simple simulations that, if individuals have a better information on the income processes, the
The concavity of the aggregate transition function is neither necessary nor sufficient condition for the Poum effect.

The paper is organized as follows: in section 2 we delineate the hypotheses of the model; in section 3 we study the change in the size of the consensus for redistribution when individuals consider the mobility process. For this we suggest a simple model where the population is divided into two groups. In section 4 we discuss the role of the information and perform simulations. In section 5 we offer some empirical analysis. Finally, section 6 concludes. Appendix 1 provides the proofs and appendix 2 reports our estimation results.

2. The model

Since the model we present in this section follows Benaboù and Ok (2001) (onwards BO), we confine to indicate the hypothesis that depart from BO.

We assume that there is a population of individuals (voters) represented by the same utility function, but with different individual characteristics \( h \); these characteristics can be income, age, job, educational degree and so on.

There are two political parties, 1 and 2, characterized by a utility function

\[
\Pi^q : T \times T \rightarrow R
\]

where \( T \) is the policy space and \( q \in \{1, 2\} \). That is, the utility level of the \( q \)-th party is both a function of its announced policy and of its competitor.

Every citizen orders the policies following his preferences, that are represented by a utility function \( \nu : T \rightarrow R \). This utility function is affected on his given personal characteristics. When a voter must choose between several policies, he votes for the preferred one. If the voter is indifferent, he votes each party by equal probability.

The electoral mechanism is based on majority rule: the party that obtains more than the fifty percent of the votes, is the winner. The winning party keeps its promises and the policy comes into effect.

In this framework a political equilibrium is a Nash equilibrium in the non cooperative game that takes place between the two parties. In this game the strategy space is the same as the policy space, and the strategy is the announced policy.

As in the model of BO, the parties compete on the issue of the redistribution but, different from it, we assume that the population can be divided in 2 groups, \( a \) and \( b \), characterized by different dynamics and distributions. Let \( n_a \) the number of individuals
in the group $a$ and $n_b$ the number of individuals in the group $b$. In the following subsection we explain further assumptions of the model.

2.1 Income distribution

As in the BO’ model, we assume that the individual income $y$ lies in some interval $Y \equiv [0, \hat{y}]$, $0 \leq \hat{y} \leq \infty$. We define the income distribution function $F : Y \to [0,1]$ such that $F[0] = 0$, $F[\hat{y}] = 1$ and $\int_0^{\hat{y}} ydF = \mu_F$. We denote as $\Phi$ the set of all distributions that satisfy the above conditions and the subset of those with median value, $m_{F} = F^{-1}(0.5)$, below the mean by $\Phi_{m}$. Let $F$ be continuous and strictly increasing. Differently from the BO, we denote the income distribution of the group $a$ by $F_a$ and by $\mu_a$ its mean income. The income of the group $b$ is distributed by $F_b$ with mean $\mu_b$.

Without loss of generalization we assume $\mu_b > \mu_a$. It follows that the mean income of the whole population is given by $\mu_F = \frac{n_a \cdot \mu_a + n_b \cdot \mu_b}{n_a + n_b}$. Finally, we assume the income distributions to be invariant through the time. In this way the mean income of each group (and that of the whole population) remain unchanged among periods.

2.2 Income dynamics.

Individuals are characterized by uncertainty on their future income but, differently from BO, we assume that they are characterized from a income transition process depending on their belonging social group. We assume the transition processes to be given by:

\[ y_{t+1} = \rho_{t+1} + \rho_{t+1} \mu_{t+1} + \epsilon_{t+1} \]

where $k \in \{a,b\}$ and $\epsilon_{t+1}$ is the error term that is identically and independently distributed among individuals with zero mean. We assume the income process to be stationary.

2.3 The fiscal design

As in the BO’ model there is a redistributive scheme described by a function $r : Y \to R_+$ that associates to each gross income a level of disposable income $r(y; F)$ preserving the total income (there are no losses due to redistribution).
We consider a proportional scheme where all incomes are taxed by a rate $\tau$ and the revenue is distributed equally to all individuals. This scheme is defined by the following expression:

$$[2.2] \quad r_i(y; F) = (1 - \tau) \cdot y + \tau \cdot \mu_F$$

Let $r_1$ be the complete redistribution ($\tau = 1$) and $r_0$ the absence of redistribution ($\tau = 0$).

2.4 Voter’s utility

Voters maximize their disposable income. They are rational (no systematic mistakes) and are risk neutral. The utility function of individual $i$ is given by:

$$[2.3] \quad U_i(r) = (1 - \tau) y_i + \tau \cdot \mu_F.$$ 

2.5 Party’s utility

As in the Wittman model the party 1 maximizes his utility function given by:

$$[2.4] \quad \Pi' = \pi(\tau_1, \tau_2) v(\tau_1; h_1) + (1 - \pi(\tau_1, \tau_2)) v(\tau_2; h_1)$$

where $\pi(\tau_1, \tau_2)$ is the probability of victory of the policy $\tau_1$ against the policy $\tau_2$, and $v(\cdot; h_i)$ is the utility function of the representative agent of party 1, and it is equal to that of a voter with characteristics $h_i$. It follows trivially the utility of party 2.

2.6 Information

Onwards we assume that voters and parties know, for each group, the transition function of the income and the income distribution. Every voter knows the probability to have at least a given level of income in a future period given the current income and the belonging group. The voters need to know this information only for herself and need to know that the system has an invariant distribution over the time. By this last assumption the voters expect in the following periods the same income’s distribution.

In next section we will consider two cases: the former where in each period people must vote for a policy to be applied in the same period; the latter, agents vote for a policy to be applied in a future period.

3. The coalition for the redistribution in the political equilibrium

In this section we formulate the size of the coalition favorable to redistribution and its change in size (and composition) when the voters consider their perspectives of income mobility.
We assume the case of party 1 being represented by an individual with zero income and party 2 being represented by an individual with maximum income, $\hat{y}$.

3.1 Static case: no social mobility

Here, we assume that in every period people vote for a redistribution policy that takes place in the same period. This assumption eliminates every mobility’s consideration.

**Proposition 1**: let the income distribution of the whole population belonging to $\Phi_+$, then, party 1 announces a policy $\tau_1 = 1$ and party 2 announces a policy $\tau_2 \in [0,1)$; the policy $\tau_1$ wins with certainty.

The proof is in appendix 1. There are many policy equilibria all payoff equivalent. Now we calculate the size of the coalition of voters favorable to redistribution. To do this, we choose a political equilibrium between those in proposition 1: party 1 announces a policy $\tau_1 = 1$ and party 2 chooses a policy $\tau_2 = 0$.

In the group $a$ the share of individuals that have a current income below $\mu_F$, is given by $F^a(\mu_F)$. In the group $b$, the share of individuals that have an income below $\mu_F$, is given by $F^b(\mu_F)$. The share of the whole population, $\delta$, that vote $\tau_1$ is given by:

$$[3.1] \quad \delta = \frac{n_a \cdot F^a(\mu_F) + n_b \cdot F^b(\mu_F)}{n_a + n_b}$$

3.2 Two period case.

As in BO, we study the effect of the mobility’s consideration on the demand of redistribution, by assuming that in each period individuals are voting for the following period policy.

Given the previous assumptions, this case is equal to situation where individuals play a lottery, and the payoffs are future incomes. The redistribution’s level indicates the share of the future income that is predetermined and the share that depends on the result of the lottery: by $\tau = 1$ the future income (in steady state) is equal to $\mu_F$ with probability 1; by $\tau = 0$ all the future income is fully determined by the lottery; by a value of $\tau$ between 0 and 1 there is a share ($\tau$) of the income that is predetermined while the remaining share ($1 - \tau$) depends on the result of the lottery.
When the voters must decide the following period redistributive policy, given the assumptions of rationality and risk indifference, they will choose the policy that give them the greater disposable expected income. So, all the individuals with an expected income greater that $\mu_F$ will vote against the redistribution, i.e. for $\tau_2$.

Arranging the stationarity condition (on the mean) of the income distribution with the income transition process [2.1], we can write:

$$
\frac{1}{n_k} \sum_i \left[ \rho_{\theta k} + \rho_{\theta, k} y_{i,t} + \epsilon_{k,i,t} \right] = \frac{1}{n_k} \sum_i \left[ y_{i,t} \right] \text{ where } k \in \{A, B\}
$$

that is equivalent to condition:

$$
\mu_k = \frac{\rho_{\theta k}}{1-\rho_{\theta k}}
$$

Note that all the individuals of group $k$ with current income below $\mu_k$ are characterized by an higher expected income. From the assumption $\mu_b > \mu_F$ follows that $\mathbb{E}[\rho_{\theta,b} + \rho_{\theta,b} \cdot \mu_F + \epsilon_{b,i,t}] > \mu_F$. That is, an individual of the group $b$, with a current income equal to $\mu_F$, has a higher expected income in the following period. Whereas for an individual of the group $a$ is, $\mathbb{E}[\rho_{\theta,a} + \rho_{\theta,a} \cdot \mu_F + \epsilon_{a,i,t}] < \mu_F$ given the assumption of $\mu_a < \mu_F$. Then, we can re-write the proposition 1 in BO in the following way:

**Proposition 2:** For any given value $\mu_a < \mu_b$, there exists, in each group $k \in \{a, b\}$, a unique threshold income given by $\tilde{y}_k = (\mu_F - \rho_{\theta k})/\rho_{\theta k}$, such that all the individuals with current income in $[0, \tilde{y}_k)$ vote $\tau_1$ over $\tau_2$, while all those with current income in $(\tilde{y}_k, \tilde{y})$ vote for $\tau_2$ over $\tau_1$.

The proof is in appendix 1. Figure 1 shows how group $a$ is characterized by $\tilde{y}_a > \mu_F$ and group $b$ by $\tilde{y}_b < \mu_F$. When the voters consider their mobility perspectives, the coalition in favour of redistributive policy $\tau_1$ is changing with respect to the static case for the defection of individuals of the group $b$ and the agreement of individuals of group

\[2\] That is equivalent to $\mathbb{E}[\rho_{\theta k} + \rho_{\theta,k} \cdot \tilde{y}_k + \epsilon_{k,i,t}] = \mu_F$. 

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a. In the group \( b \) the condition that ensures this effect is \( \bar{y}_b < \mu_F \), that arranged is \( \frac{\mu_F - \rho_{0b}}{\rho_{ib}} < \mu_F \) (that is equal to \( \mu_F < \frac{\rho_{0b}}{1 - \rho_{ib}} \) and \( \mu_F < \mu_b \)). By the same argument, it is easy to demonstrate that the condition that ensures this effect in group \( a \) is \( \mu_F > \mu_a \).

Figure 1: Proposition 2

The functions of expected income cross the 45° line at the current income mean of their group. The \( \bar{y}_k \) values are in correspondence of the cross between the functions of expected income with the level of next period income’s mean of whole population \( \mu_F \).

Unlike the BO’model we don’t pay attention to any specific property of the income transition function (the concavity in BO) but we consider the differences in the future income perspectives between the two groups of voters.

**Definition 1:** given \( \mu_k \), \( k \in \{a, b\} \), let \( \Omega_k \) be the set of autoregressive functions of order 1 \( y_{t+1} = \rho_{0k} + \rho_{1k} y_t + \varepsilon_{k,t} \) such that \( \rho_{0k} / (1 - \rho_{1k}) = \mu_k \).
Now we can re-write the proposition 2 of the BO’model in the following way:

**Proposition 3:** Let be \( \mu_a < \mu_b \), for any income transition function in \( \Omega_a \), the income threshold for group \( a \), \( \tilde{y}_a \), is decreasing in the parameter \( \rho_{1a} \); for any income transition function in \( \Omega_b \), the income threshold for group \( b \), \( \tilde{y}_b \), is increasing in the parameter \( \rho_{1b} \).

The proof is in appendix 1. The parameter \( \rho_j \) indicates the persistency of a deviation from the steady state mean. The maximum persistence is given by \( \rho_j = 1 \) (the expected incomes are distributed as the current incomes). In case of maximum volatility, given by \( \rho_j = 0 \), the expected incomes for groups \( a, b \) are, respectively, equal to \( \mu_a \) or \( \mu_b \).

Therefore, the flatter the transition function, the larger results to be the share of individuals in group \( a \) (group \( b \)) with an expected income below (above) \( \mu_f \). With respect to BO’model, this proposition explains the changes in the value of \( \tilde{y}_k \), simply by changes in the income shocks’volatility, with no assumptions on the returns to scale.

Again, the individuals share of the whole population that vote \( \tau_f \) is given by:

\[
\bar{\delta} = \frac{n_a \cdot F^a(\tilde{y}_a) + n_b \cdot F^b(\tilde{y}_b)}{n_a + n_b}
\]
The differences with respect to equation [3.1] are the arguments of $F^a(\cdot)$ and $F^b(\cdot)$.

Given that $F$ is strictly increasing for hypothesis and $\tilde{y}_b < \mu_F$, follows that $F^b(\tilde{y}_b) < F^b(\mu_F)$. Using the same argument, we have $F^a(\tilde{y}_a) > F^a(\mu_F)$.

**Definition 2:** let $P$ (the net effect on the redistributive coalition) be given by $P = \delta - \tilde{\delta}$.

$P$ is a measure of POUM effect (Prospect Of Upward Mobility). If $P$ is negative, the coalition for $\tau_1$ will be increasing whereas, if the $P$ is positive, the coalition will be decreasing. The sign of $P$ is depending on the parameters of the system as stated in the following theorem:

**Theorem 1:** Let be $\mu_a < \mu_b$, then for any function in $\Omega_a$ such that $n_a \cdot [F^a(\tilde{y}_a) - F^a(\mu_F)] < n_b \cdot F^b(\mu_F)$, it exists in $\Omega_b$ an income transition function $y_{1+i} = \tilde{\rho}_{0,b} + \tilde{\rho}_{1,b} y_i + \epsilon_{b,i+1}$, such that all the transition functions in $\Omega_b$ characterized by $\rho_{1,b} < \tilde{\rho}_{1,b}$ lead to $P > 0$.

From theorem 1 follows trivially the case with $P < 0$. The proof (see in appendix 1) is intuitive: if there are enough people in the group $b$ with current income below $\mu_F$ (with respect to the number of individuals in group $a$ expecting to change position with respect to $\mu_F$), we will find a income transition function in $\Omega_b$ sufficiently flat, such that the number of individuals in group $b$ expecting to change position with respect $\mu_F$ is greater than that in group $a$. An immediate corollary is:

**Corollary 1:** Given $\mu_a$ and $\mu_b$ then, for any function in $\Omega_a$ such that the number of individuals of the group $a$ with an expected income below $\mu_F$ is less than half population, it exists an income transition function in $\Omega_b$, $y_{1+i} = \tilde{\rho}_{0,b} + \tilde{\rho}_{1,b} y_i + \epsilon_{b,i+1}$ such that for all the transition functions in $\Omega_b$ characterized by $\rho_{1,b} < \tilde{\rho}_{1,b}$, policy $\tau_2$ beats policy $\tau_1$. 

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The proof is in appendix 1. Differently from theorem 3 in BO (that sets a level of concavity of the transition function such that for all the functions that are more concave, $\tau_2$ beats $\tau_1$), our model suggests, in the case agents are considering the future income perspectives, a set of parameters affecting voters’ decision. These parameters are the size of the two groups, the gap between their mean incomes and the income volatility shocks in the two groups. The effect of the size of the two groups is ambiguous. For example, a rise of $n_b$ can increases the number of individuals with a current income smaller than $\mu_F$ and an expected income above, but causes a rise in $\mu_F$ too, reducing the size of the interval $(\bar{y}_b, \mu_F)$. An increase in the gap between the mean incomes of the two groups, causes a rise in the size of the intervals $(\mu_F, \bar{y}_a)$ and $(\bar{y}_b, \mu_F)$, then, if the condition $F^b(\mu_F) - F^b(\mu_F - \Delta) > F^a(\mu_F + \Delta) - F^a(\mu_F)$ is true, the variable Poum will be greater. The same effect is caused by a rise in the level of income shock’s volatility in the group $b$ with respect to that in group $a$ (See prop. 3 and th.1).

3.3 Multiperiod redistribution.

Now we extend the model in a framework where the redistributive policy, chosen at the time $t=1$, is implemented until $T (> 2)$. In this setting the individuals consider the sum of discounted values of the expected incomes in the future $T-1$ periods. We assume that all individuals have the same discount rate $\beta \in (0,1)$ and that all the previous assumptions are still valid. At the time $t=1$ individuals vote for a redistributive policy that will be implemented from $t=2$ until $t=T$. An individual votes for the policy $\tau_j$ if the following condition is true:

$$[3.5] \quad \sum_{t=2}^{T} \beta^{t-1} E_y \leq \sum_{t=2}^{T} \beta^{t-1} \mu_F$$

As in proposition 2 we can find for each group $k \in \{a, b\}$ a threshold $\bar{y}_k$: all people with current income below the threshold are voting for redistribution. The following proposition formalizes this concept:

**Proposition 4:** Let $\beta \in (0,1)$ and $T \geq 2$. For any given value $\mu_a < \mu_b$, there exists, in each group $k \in \{a, b\}$, a unique threshold income given by $\bar{y}_k(T, \beta)$, such that all the
individuals with current income in $[0, \tilde{y}_k(T, \beta)]$ vote $\tau_1$ over $\tau_2$, while all those with current income in $(\tilde{y}_k(T, \beta), \tilde{y})$ vote for $\tau_2$ over $\tau_1$.

The proof (in appendix 1) is very simple: it is enough to resolve condition 3.5 by $y_{i,t}$. Differently from proposition 2, the income’s threshold depends on political horizon $T$ and discount factor $\beta$ too. For simplicity, in the following we denote $\tilde{y}_k(T, \beta)$ only by $\tilde{y}_k$. We note that $\tilde{y}_a > \mu_f$ and $\tilde{y}_b < \mu_f$. Moreover all individuals in group $k$ by $y_i < \mu_k$ have expected incomes above their current income and strictly increasing over time, that is: $y_{i,t} < E_i(y_{i,t+1}) < E_i(y_{i,t+2}) < \ldots < E_i(y_{i,T})$ for any value of $T$. Otherwise expected incomes are below the current income and strictly decreasing over time. The following proposition states a result respect to changes in the political horizon $T$.

**Proposition 5:** Let be $\beta \in (0,1)$ and $\mu_a < \mu_b$. Then, group b’s individual share voting $\tau_1$ decreases by the duration $T-1$. The reverse is true for the group a.

The proof is in appendix 1. In an intuitive way, we can see that in group $b$ an individual, who is indifferent between $\tau_1$ and $\tau_2$, when considers only the period $t+1$ (that is he has an expected income in $t+1$equal to $\mu_f$), becomes against to redistribution when he considers the period $t+2$. Because in this period he has an expected income above $\mu_f$. In similar way we can explain this effect for the group $a$. Note that in each group the share of individuals voting for a given policy changes monotonically on $T$, but considering all population, the variable $P$ can change non monotonically on $T$, simply depending it from the slopes of the income transition functions and on the income distributions. For example when the income transition function for group $b$ is very flat and the respective transition function in group $a$ has a slope closed to 1, we observe an increasing PoAm effect for low values of $T$ and a decreasing effect for high values (You can see another example in next section). Then, differently from the BO model, an extension of the horizon has an ambiguous effect (depending on the underlying parameter values). We can just say that for $T \to \infty$, the coalition for redistribution has only (and all) group $a$ individuals, and:
\[
\lim_{T \to \infty} P = \frac{n_a \cdot F^a(\mu_F) + n_b \cdot F^b(\mu_F) - n_a}{n_a + n_b}
\]

Now we can stated the equivalent to theorem 1 in the case of multiperiod redistribution.

**Theorem 2:** Let be \( \beta \in (0,1) \) and \( \mu_a < \mu_b \), then: a) given \( T \), for any function in \( \Omega_a \) such that \( n_a \cdot [F^a(\bar{y}_a) - F^a(\mu_F)] < n_b \cdot F^b(\mu_F) \), it exists an income transition function in \( \Omega_b \). \( y_{i+1} = \hat{\rho}_{i,b} + \tilde{\rho}_{i,b} y_i + \epsilon_{i+1} \) such that all the functions in \( \Omega_b \) characterized by \( \rho_{i,b} < \hat{\rho}_{i,b} \) lead to \( P > 0 \); b) if \( n_a < n_a F^a(\mu_F) + n_b F^b(\mu_F) \) it exists a value of \( T \), say \( T'' \), such that for any given transition functions in \( \Omega_a \) and \( \Omega_b \) we find \( P > 0 \) for all \( T > T'' \).

The proof is in appendix 1. The result in part a is very similar to theorem 1: given a transition function for group \( a \) we can find a sufficiently flat function for group \( b \) to cause a positive Poum. The second part puts a sufficient condition such that longer horizons magnify the poum effect for larger values of \( T \). Indeed, when the duration \( T \) increases, the Poum effect can change no monotonically, but if the condition in part b is true, we can find a value of \( T \), say \( T'' \), sufficiently large such that \( n_b F^b(\bar{y}_b) + n_a F^a(\bar{y}_a) < n_a F^a(\mu_F) + n_b F^b(\mu_F) \). Given that in group \( b \) the share of individuals voting for \( \tau_b \) is strictly decreasing on \( T \), we can find a value of \( T \), say \( T'' \), such that \( n_b F^b(\bar{y}_b) < n_a F^a(\mu_F) + n_b F^b(\mu_F) - n_a \). In this case, given that \( n_a F^a(\mu_F) \leq n_a \), the Poum is always positive for all values of \( T \) equal or longer than \( T'' \).

Intuitively, for any value of \( T \) greater that \( T'' \) there aren’t sufficient people in group \( a \) changing position to permit a no positive Poum. An immediate consequence of this theorem is stated in the following corollary:

**Corollary 2:** Let be \( \mu_a < \mu_b \), then: a) Given \( T \), for any functions in \( \Omega_a \) such that of individuals in group \( a \), to whom condition 3.5 is true are less than half population, it exists an income transition function in \( \Omega_b \). \( y_{i+1} = \hat{\rho}_{i,b} + \tilde{\rho}_{i,b} y_i + \epsilon_{i+1} \) such that all the functions in \( \Omega_b \) characterized by \( \rho_{i,b} < \hat{\rho}_{i,b} \) we find that policy \( \tau_2 \) beats policy \( \tau_1 \).
b) if \( n_a < n_b \), it exists a value of \( T \), say \( T'' \), such that for any given transition functions for the two groups, \( \tau_2 \) beats \( \tau_1 \) for all \( T > T'' \).

The proof is omitted because is very similar to one of theorem 2 and change only the conditions that are more strict. While the first part is very similar to corollary 1, the second part says us that duration of the policy goes to favour of the larger group’s perspectives if \( T \) is very large. Differently from BO the effect on the Poum effect of more forward looking voters (or more long lived the tax scheme) is not clear for short policy horizon and even for large values of \( T \) it depends on the share of population in each group.

4. The information role

A crucial assumption in BO is that individuals have information only on an aggregate income transition function and cannot distinguish between different groups according to social characteristics (i.e. family background, education, kind of job and so forth). In our model we depart from BO assuming individuals have better information up to distinguishing between groups according to some social characteristics. The main result of our model is that in the case individuals have better information with respect to aggregate transition function, the concavity of the aggregate transition function is neither sufficient nor necessary condition for a positive Poum. We demonstrate this through a couple of examples.

In the first example, we show that Poum can be positive even if aggregate transition function is convex. In the second example, we find that Poum can be negative even if aggregate transition function is concave. These findings demonstrate the different implications of our model relate to BO.

In our examples all we need is to assume that there are two groups with different income distributions and different income transition functions. The income is distributed in the interval \([0, 5]\). The coefficients of the income transition functions are taken to produce the same mean through periods. To aggregate the transition functions, we use the following weighed average:

\[
y_t = \frac{(\rho_{ja} + \rho_{ja} \cdot y_{t-1}) \cdot n_a \cdot f^a[y_{t-1}] + (\rho_{jb} + \rho_{jb} \cdot y_{t-1}) \cdot n_b \cdot f^b[y_{t-1}]}{n_a \cdot f^a[y_{t-1}] + n_b \cdot f^b[y_{t-1}]}
\]
Example 1:
Let be:

- \( n_a = n_b \) (the same number of individuals in each group)
- Two income density functions:  
  \( f^a[y] = 0.2 - 0.08 \cdot y \)  
  \( f^b[y] = 3 / 10 - 0.04 \cdot y \)
- Income means:  
  \( \mu_a = 1.66667 \)  
  \( \mu_b = 2.08333 \)  
  \( \mu_F = 1.875 \)
- Transition functions:
  \( y_{a,t} = 1.166667 + 0.3 \cdot y_{t-1} \)
  \( y_{b,t} = 1.45833 + 0.3 \cdot y_{t-1} \)
- Aggregate transition function (convex):
  \( y_t = \frac{45.21 + 2.92y_{t-1} - 1.8y^2_{t-1}}{35 - 6y_{t-1}} \)

We have approximately 11.2% of people in the group \( a \) with current income above \( \mu_F \) and expected income below. In the group \( b \) we have approximately 11.4% of people with current income below \( \mu_F \) and expected income above. Follows a positive Poum (In the figure 3 there are the transition functions)

**Figure 3: example 1**

Example 2:
Let be:

- \( n_a = 1 \) and \( n_b = 2.3 \).
- Two income density functions:  
  \( f^a[y] = 0.35 - 0.06 \cdot y \)  
  \( f^b[y] = 3 / 10 - 0.04 \cdot y \)
- Income means:  
  \( \mu_a = 1.875 \)  
  \( \mu_b = 2.08333 \)  
  \( \mu_F = 2.0202 \)
• Two transition functions:
  \[ y_{a,t} = 1,73958 + 0,1 \cdot y_{t-1} + 0,01 \cdot y_{t-1}^2 \]
  \[ y_{b,t} = 1,64583 + 0,3 \cdot y_{t-1} + 0,03 \cdot y_{t-1}^2 \]

• Aggregate transition function (concave):
  \[ y_t = \frac{1,7445 - 0,0138y_{t-1} - 0,0578y_{t-1}^2 + 0,00336y_{t-1}^3}{1,04 - 0,152y_{t-1}} \]

We have approximately 12.6% of people in group a with current income above \( \mu_F \) and expected income below. In group b we have approximately 9% of people with current income below \( \mu_F \) and expected income above. It follows a negative Poum (In figure 4 we represent the transition functions)

**Figure 4: example 2**

These examples, although done with probably unrealistic parameters, are useful to demonstrate how concavity is neither necessary nor sufficient condition for a positive Poum if individuals have more information. In this setting the sign of the Poum is depending on the kind of information used by individuals to discriminate their future perspectives. For example, if individuals discriminate groups according to one characteristic, this does not affect the income transition function (therefore groups end up to have the same transition functions), the sign of the Poum has to depend on the functional form of the transition function. On the contrary, if in the same society,
people discriminate groups according to some individual characteristic the story (showed above) may be different.

In this setting it appears how the parties are interested in manipulating the information with respect to their proposals. For example, in simulation 2, party 1 will inform the voters on the influence of social differences while party 2 will support the idea of equal perspectives for all individuals.

Finally, by a third example we demonstrate as our model permits a no-monotonic change of the Poum.

Example 3:

• Let be all assumptions in example 1 still valid. We change only the two transition functions that are:

\[ y_{a,t} = 0.83333 + 0.5 \cdot y_{t-1} \quad y_{b,t} = 0.83333 + 0.6 \cdot y_{t-1} \]

We have 55% of the population with current income below the mean. Computing the poum effect for different political horizons and a discount factor equal to 1 we obtain:

<table>
<thead>
<tr>
<th>T</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poum</td>
<td>-0.94</td>
<td>-1.51</td>
<td>-2.07</td>
<td>-2.56</td>
<td>-2.91</td>
<td>-3.05</td>
<td>-2.32</td>
<td>-0.70</td>
<td>1.80</td>
<td>5.16</td>
<td>5.08</td>
<td>5.08</td>
</tr>
</tbody>
</table>

Figure 5: example 3

We have a decreasing Poum effect in the first 8 years. After the Poum increases until to became positive. Finally the Poum is stable because all people have changed position respect to the mean: all individual in group \( a \) are voting for redistribution while all
people in group $b$ are against. Changing the parameters we can obtain many different scenarios.

5. Empirical analysis

In this section we propose, for the period 1987–1995, an empirical analysis to estimate the Poum effect by using the Bank of Italy Survey on Italian Households Income and Wealth (SHIW). We reproduce the BO analysis for the Italian case and, we extend it considering different social groups according to very general characteristic. Finally, we suggest an alternative way to measure the Poum effect. As in BO we compute the share of individuals with an expected future income below the future income mean. In order to do that, we compute (using individual incomes):

1. the interdecile two–years mobility matrix. This is nothing but a weighed average of matrices 87-89, 89-91, 91-93, 93-95 that we denote by $M$.

2. we also compute the mean individual income for each decile in 87, 89, 91, 93. To do this, we normalize incomes to one. Finally, we compute the mean individual income for each decile as a weighted average of all available years and denote it with $a$.

<table>
<thead>
<tr>
<th>Table 1: Current Income Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>1° decile 0,21</td>
</tr>
<tr>
<td>2° decile 0,39</td>
</tr>
<tr>
<td>3° decile 0,53</td>
</tr>
<tr>
<td>4° decile 0,67</td>
</tr>
<tr>
<td>5° decile 0,79</td>
</tr>
<tr>
<td>6° decile 0,91</td>
</tr>
<tr>
<td>7° decile 1,05</td>
</tr>
<tr>
<td>8° decile 1,22</td>
</tr>
<tr>
<td>9° decile 1,52</td>
</tr>
<tr>
<td>10° decile 2,72</td>
</tr>
<tr>
<td>% of individuals with income below the mean 61,9</td>
</tr>
</tbody>
</table>
We start by examining the income distribution in the table. The median group earns about 85% of mean income. Using the linear interpolation we compute the size of the redistributive coalition as proportion of individuals with income below the mean. We estimate it to be 61.9%. Then, using a transition matrix, we compute the two-years ahead vector of conditionally expected relative incomes, namely $M^* a$. We estimate, by linear interpolation, the proportion of individuals with an expected income below the mean (2 years ahead). The result is that 4% of individuals have a current income below the mean but an expected income above. As in BO the “Poum effect” seems to be a real feature of Italian society. But, as stated in the previous section, this measure is based on the assumption that people consider only the aggregate transition function. When we extend it to more informed individuals the result can be quite different.

*Table 2: Poum effect dividing Italian population in social groups*

<table>
<thead>
<tr>
<th>Groups by:</th>
<th>Poum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geographical location: north-south</td>
<td>3.78</td>
</tr>
<tr>
<td>Professional condition: yes-no</td>
<td>4.25</td>
</tr>
<tr>
<td>Education: low-high</td>
<td>4.00</td>
</tr>
<tr>
<td>Sex: male-female</td>
<td>3.75</td>
</tr>
</tbody>
</table>

For that, we compute interdecile transition matrices for different social groups and the frequencies in each decile. Next, we estimate the proportion of individuals with an expected income below the expected two – years ahead mean for each different social group and finally, we compute the Poum effect. The results show very similar Poum values. In case we separate individuals according to education, the Poum effect is equal to 4.3% while, if we separate individuals according to geographic location (north and south Italy) the Poum effect is equal to 3.7%. From this analysis, there is not yet an evidence of information affecting the Poum effect.

We note that the Poum effect measured using interdecile transition matrices has two drawbacks: i) it is very rough and, ii) it can be inconsistent. It is rough, for we have to use a linear interpolation to compute the share of individuals with income below (above) the mean. It can be inconsistent, for the income transition function, computed by $M^* a$, can be affected by individual effects possibly correlated with the (t-1) income. Then, we can have an over or underestimation of the slope. Furthermore, the concavity
(or convexity) of transition function can be the result of a non linear relation between income in (t-1) and (unobserved) individual effects.

To the aim of better measuring the Poum effect, we propose a methodology applied to all individuals belonging to the labor force (therefore we address only to labor incomes). The methodology is the following:

1) We produce an income transition function estimation for all individuals that keep employed during the period. We estimate, by GMM, a simple AR1 process with fixed effects:

\[ y_{i,t} = \rho_0 + \rho_1 y_{i,t-1} + u_i + \epsilon_{i,t} \]

where \( y \) is the log of labor income, \( u \) is the individual effect and \( \epsilon \) is the individual error term. We use this estimation to compute the expected income at time \( t \) for all individuals employed in (t-1), conditionally both to the event of working at time \( t \) and to the earned income in (t-1).

2) We produce an income estimation for all individuals employed in \( t \) and unemployed in previous period. We estimate, by OLS, the following equation:

\[ y_{i,t} = X_{i,t} \beta + \epsilon_{i,t} \]

where \( y \) is the log of labor income and \( X \) is a vector of individual characteristics. We use this estimation to compute an expected income at time \( t \) (conditioned to the event of being employed) for individuals unemployed at time (t-1).

3) We estimate the transition probability between the employed and unemployed status. We use a bivariate probit model with endogenous selection as to address the initial condition. That is:

\[ P[E_{i,t} = 1, E_{i,t-1} = 1] = \Phi_2(X_{i,t} \beta, Z_{i,t-1} \gamma; \rho) \]
\[ P[E_{i,t} = 1, E_{i,t-1} = 0] = \Phi_2(X_{i,t} \beta, Z_{i,t-1} \gamma; \rho) \]

where \( E=1 \) indicates employed status and \( E=0 \) unemployed, \( X \) and \( Z \) are vectors of individual characteristics.

4) Finally, we compute the expected (unconditioned) income in the year 95, for all individuals sampled both in 93 and 95, using the following expression:

\[ E_{95}[Y_{95}] = \Pr[E_{95} = 1|E_{95} = a] \cdot E_{95}[Y_{95}|E_{95} = 1] \]

where \( a \in \{0,1\} \). Note that, in the right side, expected income is computed with the estimated equation [5.1] for individuals employed in 93, while is computed by the [5.2]
for individuals unemployed in 93. The idea is that labor income is zero when the individual is unemployed.

To compute the expected income we assume the lognormality of the income distribution. Using [5.1] the expected income is given by:

\[ E_{y_95}[Y|E_{y_95} = l] = \exp \left[ \bar{u} + \rho_{0} + \rho_{1} y_{93} + \frac{\sigma_{ue}^2}{2} \right] \]

where \( \bar{u} \) is the mean of the individual effects and \( \sigma_{ue}^2 \) is the variance of the sum between error term and individual effect. To obtain \( \bar{u} \) and \( \sigma_{ue}^2 \) we compute:

\[ u_{i} + \epsilon_{i,95} = y_{95} - \rho_{0} - \rho_{1} y_{93} \]

Then \( \bar{u} \) is computed as the mean of the left side of [5.7] (given the independence of the error term with mean 0) and \( \sigma_{ue}^2 \) is computed directly. Using [5.2] the calculation is similar with the difference that we have not the individual effect \( u_{i} \).

We propose different estimations of the Poum effect:

1) we estimate [5.1] without considering heterogeneity.

2) we estimate [5.1] for all individuals, but, the mean of individual effects \( \bar{u} \) and the variance \( \sigma_{ue}^2 \) are estimated for six different groups of education.

3) we estimate [5.1] for three different classes of education, whereas \( \bar{u} \) and \( \sigma_{ue}^2 \) are estimated for six groups of education.

4) finally, we mimick the third estimation computing, however \( \bar{u} \) and \( \sigma_{ue}^2 \) are given by multiple groups according to education, geographical location and sex. Our estimation results are provided in appendix 2.

In what follows we show some measures of the Poum

Table 3: Different measures of Poum effect

<table>
<thead>
<tr>
<th>N° 1</th>
<th>E[y_{95}]</th>
<th>y_{95}</th>
<th>&lt; \mu_{F}</th>
<th>&gt; \mu_{F}</th>
<th>Total</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>y_{93}</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt; \mu_{F}</td>
<td>1009</td>
<td>878</td>
<td>1887</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt; \mu_{F}</td>
<td>1</td>
<td>1805</td>
<td>1806</td>
<td>49</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>1010</td>
<td>2683</td>
<td>3693</td>
<td>100</td>
</tr>
<tr>
<td>%</td>
<td></td>
<td></td>
<td>27</td>
<td>73</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>N° 2</th>
<th>E[y_{95}]</th>
<th>y_{95}</th>
<th>&lt; \mu_{F}</th>
<th>&gt; \mu_{F}</th>
<th>Total</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>y_{93}</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt; \mu_{F}</td>
<td>1369</td>
<td>518</td>
<td>1887</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt; \mu_{F}</td>
<td>91</td>
<td>1715</td>
<td>1806</td>
<td>49</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>1460</td>
<td>2233</td>
<td>3693</td>
<td>100</td>
</tr>
<tr>
<td>%</td>
<td></td>
<td></td>
<td>40</td>
<td>60</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
There is a clear, positive Poum effect. However the introduction of more heterogeneity reduces the Poum effect.

Finally, we propose a measure using the expected income as declared by each individual (information available only in year 1995 of our dataset). Each individual is asked:

- The expected max income 12 months later, in case he will be working (maxY)
- The expected min income 12 months later, in case he will be working (minY)
- The probability to earn an income less than (maxY+minY)/2 (pr1)
- The probability to maintain the job or, to find a job 12 months later (pr2).

Therefore, the expected income (unconditionated to the event to have or not a job) is:

\[ E(Y) = (\max Y \cdot (1 - pr1) + \min Y \cdot pr1) \cdot pr2 \]

In the following table, we can see how individuals are distributed around the current and expected income mean.

**Table 4: Poum effect using declared expected income**

<table>
<thead>
<tr>
<th></th>
<th>( E[y_{1995}] )</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_{1995} )</td>
<td>&lt; ( \mu_F )</td>
<td>&gt; ( \mu_F )</td>
<td>Total</td>
<td>%</td>
<td></td>
</tr>
<tr>
<td>&lt; ( \mu_F )</td>
<td>1840</td>
<td>250</td>
<td>2090</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>&gt; ( \mu_F )</td>
<td>379</td>
<td>1634</td>
<td>2013</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2219</td>
<td>1884</td>
<td>4103</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>%</td>
<td>54</td>
<td>46</td>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
This new case revert our previus results by finding a negative poum effect. That is, there is an increase in the number of individuals with an expected income below the mean.

6. Conclusions

We propose a model as to explaining the Poum effect in a situation where individuals care on different income perspectives among social groups. We obtain two main results: the former is that the model links individual preferences towards redistribution to individual memberships in a social group. The latter, is a noticeable evidence on the important role of information. The strict BO’s conditions for the aggregate transition functions are here naturally replaced by the role of information. In our examples and empirical estimations we find that more information strongly affects the Poum effect. Not only, our results show a lower Poum effect than in BO.

Of course our econometric results can catch only part of individual information but, nevertheless, they are quite satisfactory. An immediate lesson form our estimation results is that the Poum effect seems to be not a real characteristics for the Italian society.
7. References


Appendix 1

Proposition 1
The problem of party 1 is:

\[ \text{max}_{\tau_1, \tau_2} \pi(\tau_1, \tau_2) \tau_1 \mu_1 + (1 - \pi(\tau_1, \tau_2)) \tau_2 \mu_2. \]

the problem of party 2 is:

\[ \text{max}_{\tau_1, \tau_2} \pi(\tau_1, \tau_2) (\tau_1 \mu_1 + (1 - \tau_1) \hat{\gamma}) + (1 - \pi(\tau_1, \tau_2)) (\tau_2 \mu_2 + (1 - \tau_2) \hat{\gamma}). \]

In this case, always \( \tau_1 \geq \tau_2 \). Individuals with an income below the mean (therefore those that prefer \( \tau_1 \) with respect to \( \tau_2 \)) are the majority. It follows that policy \( \tau_1 \) wins with certainty versus \( \tau_2 \). Then, parties’ problem are: \( \text{max}_{\tau_1, \tau_2} \tau_1 \mu_1 \) (for party 1) and \( \text{max}_{\tau_1, \tau_2} (\tau_1 \mu_1 + (1 - \tau_1) \hat{\gamma}) \) (for party 2). The equilibrium computation is just immediate.

QED

Proposition 2
Given the income process \( y_{i,j+1} = \rho_0 k + \rho_{1,k} y_{i,j} + \epsilon_{k,i,t}, \) the expected income is given by:

\[ E[y_{i,j+1}] = \rho_0 k + \rho_{1,k} y_{i,j}. \]

Solving again the income inequality \( \rho_0 k + \rho_{1,k} y_{i,j} \geq \mu_F \), we find the threshold income \( \tilde{y}_k \) such that all individuals with a greater current income have an expected income above \( \mu_F \).

Proposition 3
Rearranging [3.3] we obtain \( \rho_0 k = \mu_k (l - \rho_{1,k}) \). Substituting it inside the expression for \( \tilde{y}_k \), and taking the first derivative in \( \rho_{1,k} \) we have:

\[ \frac{d\tilde{y}_k}{d\rho_{1,k}} = \frac{\mu_k - \mu_F}{\rho_{1,k}} \]

that is negative for \( k=a \) and positive for \( k=b \).QED.

Theorem 1
Arranging variable \( P \) by [3.1] and [3.4] we obtain:

\[ P = \frac{n_a \cdot [F^a(\mu_F) - F^a(\tilde{y}_a)] + n_b \cdot [F^b(\mu_F) - F^b(\tilde{y}_b)]}{n_a + n_b} \]

Its sign depends on the numerator’s sign. Arranging it with the condition \( P>0 \) we obtain:

\[ n_b \cdot [F^b(\mu_F)] - n_a \cdot [F^a(\tilde{y}_a) - F^a(\mu_F)] > F^b(\tilde{y}_b) \]
Given the assumption that $F$ is strictly increasing, we can find a value $y^*$ such that for all values of $\tilde{y}_h$ below $y^*$ the inequality in [A.5] is true (for a given value of the left side in [A.5]). Using the result in proposition 3 we can find the set of transition function in $\Omega_h$, that for a given transition function in $\Omega_a$, lead to a $P>0$. From our income distributions’ assumptions it follows that the necessary condition is the positive left side in [A.5]. QED.

Corollary 1
The condition such that $\tau_2$ beats $\tau_j$ is:

\[ n_a \cdot F^a(\tilde{y}_a) + n_b \cdot F^b(\tilde{y}_b) \leq 0.5 \]

Rearranging it, we obtain:

\[ n_b \cdot F^b(\tilde{y}_b) < 0.5 \cdot (N - n_a \cdot F^a(\tilde{y}_a)) \]

where $N$ is the number of individuals in the whole population. The proof follows the same steps in proposition 3. QED.

Proposition 4
Solving condition 3.5 by $y_{i,1}$ (income of individual $i$ at time $t=1$) we obtain the following inequality:

\[ y_{i,t} \leq \frac{1 - \rho_{ik} \beta}{1 - \rho_{ik} \beta} \frac{1}{\rho_{ik} \beta} \sum_{t=2}^{T} \beta^{t-1} \left( \mu_F - \rho_{0k} \frac{1 - \rho_{ik}^{t-1}}{1 - \rho_{ik}} \right) \]

The right side depends from the parameters of the system and represents the upper boundary of the current income for individuals belonging to group $k$ in favor of a redistributive policy $\tau_j$. We denote it by $\tilde{y}_k$. QED.

Proposition 5
Simplifying the expression for $\tilde{y}_k$ we obtain:

\[ \tilde{y}_k = \frac{1 - \rho_{ik} \beta}{1 - \rho_{ik} \beta} \frac{1}{\rho_{ik} \beta} \sum_{t=2}^{T} \beta^{t-1} \left( \mu_F - \mu_k (1 - \rho_{ik}^{t-1}) \right) \]

Rearranging the right side we obtain:

\[ \tilde{y}_k = \frac{1 - \rho_{ik} \beta}{\rho_{ik} \beta} \left[ \frac{(\mu_F - \mu_k) \beta (1 - \beta^{t-1})}{(1 - \beta)(1 - \beta^{t-1} \rho_{ik}^{t-1})} + \frac{\mu_k \beta \rho_{ik}}{1 - \beta \rho_{ik}} \right] \]

30
In this last expression, only the first addend inside the bracket depends on $T$ (decreasing by $T$). Indeed, dividing it in two factors, we can see that the first one, \( \frac{\mu_f - \mu_k}{(1 - \beta)} \), is negative for group $b$, because $\mu_f \leq \mu_h$, and positive for group $a$. The second factor, 
\[
\frac{(1 - \beta^{T-1})}{(1 - \beta^{T-1} \rho_{ib}^{T-1})},
\]
has a positive derivative with respect to $T$ for all $\beta \in (0,1)$ and $\rho_{ib} \in (0,1)$. It follows that a rise of $T$ causes a decrease of $\hat{y}_k$ for individuals belonging to group $b$, and an increasing for individuals belonging to group $b$. QED

**Theorem 2**

The derivative of the threshold $\hat{y}_k$ with respect $\rho_{ik}$ is:

\[ [A.11] \]
\[
\frac{d\hat{y}_{i,l}}{d\rho_{ik}} = -\sum_{t=2}^{T} (t - 1) \beta^{t-1} \rho_{ik}^{t-2} (\mu_f - \mu_k) \sum_{t=2}^{T} \beta^{t-1} \left( \sum_{i=2}^{T} \beta^{t-1} \rho_{ik}^{t-1} \right)^2
\]

If $\mu_f < \mu_k$ the derivative is strictly positive: a decrease in $\rho_{ik}$ causes a lower value of $\tilde{y}_{i,l}$, then the number of agents in group $k$ voting for $\tau_l$ is strictly decreasing. Then the proof is similar to one in theorem 1. QED
Appendix 2

Note on the variables.

Branch of activity = 1 if it is in agriculture, 0 otherwise.

Sex = 1 if male, 0 if female

Education = years of education

Town size = 1 if population is from 20,000 to 40,000, 0 otherwise

D4 and D5 are dummies, respectively, for years 1993 and 1995

2.1 Income’s processes for employed individuals

1) One process for whole population

| Coef.    | Std. Err.  | t-student | P>|t| |
|----------|------------|-----------|-----|
| Income at t-1 | .1745134  | .0616727  | 2.83 | 0.005 |
| Constant  | -.0319878  | .0089306  | -3.58 | 0.000 |

Sargan test of over-identifying restrictions:

\[ \chi^2(5) = 4.71 \quad \text{Prob} > \chi^2 = 0.4526 \]

Arellano-Bond test that average autocovariance in residuals of order 1 is 0:

H0: no autocorrelation \( z = -7.82 \) \( \text{Pr} > z = 0.0000 \)

Arellano-Bond test that average autocovariance in residuals of order 2 is 0:

H0: no autocorrelation \( z = -0.00 \) \( \text{Pr} > z = 0.9990 \)
2) For groups in according to education’s level

<table>
<thead>
<tr>
<th>Low education</th>
<th>Number of obs = 1194</th>
<th>Number of groups = 977</th>
<th>Wald chi2(1) = 1.27</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>Std. Err.</td>
<td>t-student</td>
</tr>
<tr>
<td>Income at t-1</td>
<td>.0943331</td>
<td>.0837238</td>
<td>1.13</td>
</tr>
<tr>
<td>Constant</td>
<td>-.0596979</td>
<td>.0145687</td>
<td>-4.10</td>
</tr>
<tr>
<td>Sargan test of over-identifying restrictions:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>chi2(5) = 8.83</td>
<td>Prob &gt; chi2 = 0.1160</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arellano-Bond test that average autocovariance in residuals of order 1 is 0:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H0: no autocorrelation</td>
<td>z = -5.47</td>
<td>Pr &gt; z = 0.0000</td>
<td></td>
</tr>
<tr>
<td>Arellano-Bond test that average autocovariance in residuals of order 2 is 0:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H0: no autocorrelation</td>
<td>z = -0.50</td>
<td>Pr &gt; z = 0.6137</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Intermediate education</th>
<th>Number of obs = 930</th>
<th>Number of groups = 737</th>
<th>Wald chi2(1) = 12.55</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>Std. Err.</td>
<td>t-student</td>
</tr>
<tr>
<td>Income at t-1</td>
<td>.3319929</td>
<td>.0936989</td>
<td>3.54</td>
</tr>
<tr>
<td>Constant</td>
<td>-.0202127</td>
<td>.0151651</td>
<td>-1.33</td>
</tr>
<tr>
<td>Sargan test of over-identifying restrictions:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>chi2(5) = 0.25</td>
<td>Prob &gt; chi2 = 0.9985</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arellano-Bond test that average autocovariance in residuals of order 1 is 0:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H0: no autocorrelation</td>
<td>z = -6.19</td>
<td>Pr &gt; z = 0.0000</td>
<td></td>
</tr>
<tr>
<td>Arellano-Bond test that average autocovariance in residuals of order 2 is 0:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H0: no autocorrelation</td>
<td>z = 0.29</td>
<td>Pr &gt; z = 0.7731</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High education</th>
<th>Number of obs = 315</th>
<th>Number of groups = 245</th>
<th>Wald chi2(1) = 2.70</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>Std. Err.</td>
<td>t-student</td>
</tr>
<tr>
<td>Income at t-1</td>
<td>.2114442</td>
<td>1.64</td>
<td>.1287905</td>
</tr>
<tr>
<td>Constant</td>
<td>-.0125533</td>
<td>.018936</td>
<td>-0.66</td>
</tr>
<tr>
<td>Sargan test of over-identifying restrictions:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>chi2(5) = 7.02</td>
<td>Prob &gt; chi2 = 0.2192</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arellano-Bond test that average autocovariance in residuals of order 1 is 0:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H0: no autocorrelation</td>
<td>z = -3.34</td>
<td>Pr &gt; z = 0.0008</td>
<td></td>
</tr>
<tr>
<td>Arellano-Bond test that average autocovariance in residuals of order 2 is 0:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H0: no autocorrelation</td>
<td>z = 1.04</td>
<td>Pr &gt; z = 0.2975</td>
<td></td>
</tr>
</tbody>
</table>
2.2 Probability to be employed at time t if employed at time t-1

|                     | Coef.    | Std. Err. | t-student | P>|t| |
|---------------------|----------|-----------|-----------|-----|
| **Employed**        |          |           |           |     |
| Sex                 | -.066775 | .0595374  | -1.12     | 0.262|
| Age                 | .0108893 | .0027529  | 3.96      | 0.000|
| Education           | .032812  | .0080095  | 4.10      | 0.000|
| Income at t-1       | .4707643 | .0432023  | 10.90     | 0.000|
| Branch of activity  | -.0324599| .115648   | -0.28     | 0.779|
| Constant            | -3.421934| .4151324  | -8.24     | 0.000|

| **Selection's model** | Coef.    | Std. Err. | t-student | P>|t| |
|------------------------|----------|-----------|-----------|-----|
| Sex                    | .1442276 | .0411248  | 3.51      | 0.000|
| Age                    | .0812238 | .0022514  | 36.08     | 0.000|
| Education              | .0114131 | .0053848  | 2.12      | 0.034|
| Residence North Italy  | .8894033 | .0453271  | 19.62     | 0.000|
| Residence Central Italy| .5075743 | .0543651  | 9.34      | 0.000|
| Constant               | -2.271132| .10044    | -22.61    | 0.000|

athrho: -6.640837 .1833914 -3.68 0.000
rho: -5.878597 .1200583

LR test of indep. eqns. (rho = 0):
chi2(1) = 21.21  Prob > chi2 = 0.0000
2.3 Probability to be unemployed at time t if unemployed at time t-1

<table>
<thead>
<tr>
<th>Employed</th>
<th>Log likelihood = -3310.47</th>
<th>Wald chi2(4) = 32.35</th>
<th>Prob &gt; chi2 = 0.0000</th>
<th>Number of obs = 9343</th>
<th>Censored obs = 8080</th>
<th>Uncensored obs = 1263</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex</td>
<td>-.172057</td>
<td>.0720142</td>
<td>-2.39</td>
<td>0.017</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>.024358</td>
<td>.0074358</td>
<td>3.28</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Education</td>
<td>-.0258421</td>
<td>.0098673</td>
<td>-2.62</td>
<td>0.009</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First job seeker at t-1</td>
<td>-.0925774</td>
<td>.0736423</td>
<td>-1.26</td>
<td>0.209</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>.7586267</td>
<td>.2051763</td>
<td>3.70</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Selection's model

| Unemployed at t-1 | Coef. | Std. Err. | t-student | P>|t| |
|-------------------|-------|-----------|-----------|-----|
| Sex               | -.1431562 | .0411798 | -3.48     | 0.001 |
| Age               | -.0805853 | .0022481 | -35.85    | 0.000 |
| Education         | -.0122851 | .0053699 | -2.29     | 0.022 |
| Residence North Italy | -.8817713 | .0452353 | -19.49    | 0.000 |
| Residence Central Italy | -.5050591 | .0537468 | -9.40     | 0.000 |
| Constant          | 2.090866 | .0972613 | 21.50     | 0.000 |

| Athrho            | -.5635978 | .1290556 | -4.37     | 0.000 |
| rho               | -.510642  | .0954037 |           |     |

LR test of indep. eqns. (rho = 0):

chi2(1) = 19.04  Prob > chi2 = 0.0000

2.4 Income at time t for individuals unemployed at time t-1

<table>
<thead>
<tr>
<th></th>
<th>SS</th>
<th>dF</th>
<th>MS</th>
<th>Number of obs = 464</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>50.2</td>
<td>9</td>
<td>5.5824</td>
<td>F( 9, 454) = 9.82</td>
</tr>
<tr>
<td>Residual</td>
<td>258.0</td>
<td>454</td>
<td>.5683</td>
<td>Prob &gt; F = 0.0000</td>
</tr>
<tr>
<td>Total</td>
<td>308.3</td>
<td>463</td>
<td>.6658</td>
<td>R-squared = 0.1630</td>
</tr>
<tr>
<td>Adj R-squared = 0.1464</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Income   | Coef. | Std. Err. | t     | P>|t| |
|----------|-------|-----------|-------|-----|
| Sex      | .4507261 | .0742028 | 6.07  | 0.000 |
| Age      | .1085761 | .0252875 | 4.29  | 0.000 |
| Age²     | -.0015099 | .0003669 | -4.12 | 0.000 |
| Education| .0394855 | .0103662 | 3.81  | 0.000 |
| Residence: North Italy | .1380269 | .0831604 | 1.66  | 0.098 |
| Residence: Central Italy | .1201938 | .1002724 | 1.20  | 0.231 |
| Town size| -.0021112 | .0875808 | -0.02 | 0.981 |
| D4       | -.3355757 | .1073248 | -3.13 | 0.002 |
| D5       | -.4094192 | .1013203 | -4.04 | 0.000 |
| constant | 6.968618  | .4044257 | 17.23 | 0.000 |