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Keywords
Behavioral Finance, Cumulative Prospect Theory, Hedonic Framing, Options Trading Strategies

JEL Codes
C63, D81, G13

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Covered call writing
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Abstract
The covered call writing, which entails selling a call option on one’s underlying stock holdings, is perceived by investors as a strategy with limited risk. It is a very popular strategy used by individual, professional and institutional investors; moreover, the CBOE developed the Buy Write Index which tracks the performance of a synthetic covered call strategy on the S&P 500 Index. Previous studies analyze behavioral aspects of the covered call strategy, indicating that hedonic framing and risk aversion may explain the preference of such a strategy with respect to other designs. In this contribution, following this line of research, we extend the analysis and apply Cumulative Prospect Theory in its continuous version to the evaluation of the covered call strategy and study the effects of alternative framing.

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1 Introduction

The covered call writing (or buy-write) is a popular strategy, used both by experienced investors and non-professional traders who are not so familiar with derivatives, but perceive the strategy as risk-reducing with respect to an investment on a single stock or a stocks’ portfolio. In May 2002 the CBOE released the Buy Write Monthly Index, later called Buy Write Index (BXM), which tracks the performance of a synthetic covered call strategy on the S&P 500 Index (SPX). The methodology and performance of the BXM are described by Whaley in [15]; the BXM is a passive total return index formed with a long position on an S&P 500 portfolio and a sequence of short positions on one-month at-the-money (or just out-of-the-money) covered calls on the S&P 500 Index. The BXM has become a benchmark for measuring the performance of buy-write implementations. Based on past performance, the BXM had volatility which is about 30% lower than the volatility of the underlying SPX Index. Starting from March 2006, the CBOE launched a new OTM Buy Write Index (BXY) that uses the same methodology of the BXM, but calculated considering out-of-the-money calls on the S&P 500 Index.

The seller of the call option owns the underlying asset and her/his risk is limited, but this is not sufficient to explain the success of such a strategy amongst investors, or the preference for covered calls despite several alternative and less known strategies with similar profit profiles, which register significantly lower trading volume. Using modern Prospect Theory arguments, we are able to analyze some aspects that characterize the behavior and choices of the decision makers.

Shefrin and Statman [11] were the first to suggest hedonic framing [12] and risk aversion in the domain of gains as main reasons for departure from standard financial theory\textsuperscript{1}: writers of covered call prefer this strategy to a stock-only position and are loath to repurchase the call when this entails a

\textsuperscript{1} Other studies along this line are [5], [2], [9], and [3].
realization of a loss, out-of-the-money calls are preferred to in-the-money calls in the implementation of the strategy, fully covered positions are preferred to partially covered ones, covered call is preferred to alternative strategies (such as naked puts).

Recently, Hoffmann and Fischer [6] test empirically all these hypothesis: 60.1% of the respondents prefer the stock-only position with respect to the covered call strategy when the profits and losses are described graphically, whereas the covered call preference is 60.6% when the profits and losses are described by text; a fully covered position is preferred to a partially covered call in 69.7% of the cases; out-of-the-money calls are preferred (87.4%) to in-the-money calls when forming the strategy; at-the-money covered calls are preferred (79.3%) to at-the-money naked puts. Such results highlight that decision frames and also the way alternatives are presented to investors do influence actual investment choices.

In this contribution, we extend the analysis of [11] based on a simple one-period binomial model, by evaluating the covered call strategy under Cumulative Prospect Theory [13] in a continuous framework [4], focusing in particular on the effects of alternative framing of the results.

The remainder of the paper is organized as follows: Section 2 summarizes the main concepts of prospect theory; Section 3 introduces the notions of mental accounting and hedonic framing; Sections 4–7 describe the evaluation of the covered call portfolio under different frames; Section 8 compares the covered call writing with alternative trading strategies. Section 9 concludes.

2 Prospect theory

According to Prospect Theory risk attitude, loss aversion and probability perception are described by two functions: a value function \( v \) and a weighting function \( w \). Outcomes are evaluated relative to a certain reference point instead of in terms of final wealth. The shape of the value and weighting
functions describe actual investors’ behaviors. Function \( v \) is typically convex in the range of losses (risk aversion) and concave in the range of gains (risk-seeking), it is steeper for losses (loss aversion). Decision makers have also biased probability estimates: they tend to underweight high probabilities and overweight low probabilities.

Let us denote with \( x_i \), for \(-m \leq i < 0\) negative outcomes and for \(0 < i \leq n\) positive outcomes, with \( x_i \leq x_j \) for \(i < j\). The prospect value is displayed as follows

\[
V = \sum_{i=-m}^{n} \pi_i \cdot v(x_i),
\]

(1)

with decision weights \( \pi_i \) and values \( v(x_i) \) based on relative outcomes.

Specific parametric forms have been suggested in the literature for the value function. A function which is used in many empirical studies is

\[
\begin{cases}
  v^- = -\lambda (-x)^b & x < 0 \\
  v^+ = x^a & x \geq 0,
\end{cases}
\]

(2)

with positive parameters that control risk attitude (\(0 < a \leq 1\) and \(0 < b \leq 1\)) and loss aversion (\(\lambda \geq 1\)); \(v^-\) and \(v^+\) denote the value function for losses and gains, respectively. Function (2) has zero as reference point; it is concave for positive outcomes and convex for negative outcomes, it is steeper for losses. Parameters values equal to one imply risk and loss neutrality.

In Cumulative Prospect Theory \([13]\) decision weights \( \pi_i \) are differences in transformed cumulative probabilities of gains or losses. A probability weighting function \( w \) models probabilistic risk behavior. Formally:

\[
\pi_i = \begin{cases}
  w^-(p_{-m}) & i = -m \\
  w^- \left( \sum_{j=-m}^{i} p_j \right) - w^- \left( \sum_{j=-m}^{i-1} p_j \right) & i = -m + 1, \ldots, -1 \\
  w^+ \left( \sum_{j=-i+1}^{n} p_j \right) - w^+ \left( \sum_{j=-i+1}^{n-1} p_j \right) & i = 0, \ldots, n - 1 \\
  w^+(p_n) & i = n,
\end{cases}
\]

(3)

with \(w^-\) for losses and \(w^+\) for gains, respectively.

In financial applications, in particular in the evaluation of options, prospects may involve a continuum of values; hence, Prospect Theory cannot be ap-
plied directly in its original or cumulative versions. Davies and Satchell in [4] provide the continuous cumulative prospect value:

\[ V = \int_{-\infty}^{0} \psi^{-}(F(x)) f(x) v^{-}(x) \, dx + \int_{0}^{+\infty} \psi^{+}(1 - F(x)) f(x) v^{+}(x) \, dx, \quad (4) \]

where \( \psi = \frac{dw(p)}{dp} \) is the derivative of the weighting function \( w \) with respect to the probability variable, \( F \) is the cumulative distribution function and \( f \) is the probability density function of the outcomes.

The weighting function \( w \) is uniquely determined, it maps the probability interval \([0, 1]\) into \([0, 1]\), and is strictly increasing, with \( w(0) = 0 \) and \( w(1) = 1 \). In this work we will assume continuity of \( w \) on \([0, 1]\), even though in the literature discontinuous weighting functions are also considered. The curvature of the weighting function is related to the risk attitude towards probabilities. Empirical evidence suggests a particular shape of probability weighting functions: small probabilities are overweighted \( w(p) > p \), whereas individuals tend to underestimate large probabilities \( w(p) < p \). This turns out in a typical inverse-S shaped weighting function: the function is initially concave (probabilistic risk-seeking) for probabilities in the interval \((0, p^{*})\), and convex (probabilistic risk aversion) in the interval \((p^{*}, 1)\), for a certain value of \( p^{*} \). A linear weighting function describes probabilistic risk neutrality or objective sensitivity towards probabilities, which characterizes expected utility. Empirical findings indicate that the intersection between the weighting function and the 45° line, \( w(p) = p \), is for \( p^{*} \) in the interval \((0.3, 0.4)\).

Different parametric forms for the weighting function with the above mentioned features have been proposed in the literature, and their parameters have been estimated in many empirical studies. In this work we applied, in particular, the function suggested by Tversky and Kahneman in [13]:

\[ w(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}}, \quad (5) \]

where \( \gamma \) is a positive constant (with some constraint in order to have an increasing function), with \( w(0) = 0 \) and \( w(1) = 1 \). The parameter \( \gamma \) captures
the degree of sensitivity toward changes in probabilities from impossibility (zero probability) to certainty. When $\gamma < 1$, one obtains the typical inverse-S shaped form; the lower the parameter, the higher is the curvature of the function.

3 Hedonic framing

Standard finance and, in particular, option pricing assume frame invariance, where investors are indifferent among frames of cash flows, but many empirical studies highlight that the framing of alternatives exerts a crucial effect on investment choices. Moreover, investors might not be aware of the frames that affect their actual decisions. Prospect theory posits that individuals evaluate outcomes with respect to deviations from a reference point rather than with respect to net final wealth. Decision frames are influenced by the way in which alternatives are presented to investors. People may keep different mental accounts for different types of outcomes. When combining these accounts to obtain overall result, typically they do not simply sum up all monetary amounts, but intentionally use hedonic frame (see Thaler [12]) such that the combination of the outcomes appears more favorable and increases their utility.

Consider the simple case of two sure outcomes $x$ and $y$, the hedonic optimizer would combine the results according to the following rule:

$$V = \max\{v(x + y), v(x) + v(y)\}.$$  \hspace{1cm} (6)

Outcomes are aggregated or segregated depending on what leads to the highest possible prospect value: multiple gains are preferred to be segregated (narrow framing) to enjoy the gains separately, losses are preferred to be integrated with other losses (or large gains) in order to ease the pain of the loss. Considering mixed outcomes, these would be integrated in order to cancel out losses when there is a net gain or a small loss; in case of large losses and a
small gain, they usually are segregated in order to preserve the silver lining. This is due to the shape of the value function in prospect theory, characterized by risk-seeking or risk aversion, diminishing sensitivity and loss aversion. Loss aversion implies also that the impact of losses is more important than that of gains of same amount.

An investment in a portfolio can be isolated (narrow framing) or integrated, which reflects a broader decision frame of the investors (see [8]). Framing can be intertemporal: gains and losses can be time segregated or aggregated, depending on how the perception of the results is affected by the evaluation period. Invertors are influenced by narrow framing and regret when they sell portfolio “winners” and keep portfolio “losers”, in order to avoid the realization of a loss (disposition effect), but they integrate outcomes through simultaneous trades when they sell a loser together with a winner to reduce their regret (see [8]). When we consider a financial option, the premium is cashed in advance with respect to the realization of the payoff, hence these outcomes could be evaluated into separate mental accounts or jointly. The option premium itself might be perceived as an income, cash that can be used immediately for consumption (according to the life cycle theory [11]).

Covered calls are frequently promoted as “an investment strategy that can make you extra money” and proposed to the investors as the sum of three possible sources of profit (three mental accounts): the call premium (considered as an extra dividend), regular dividends, the capital gain on the stock (when the strategy is formed with out-of-the-money calls, and in case of exercise).

In the following sections we will evaluate the results of a covered call portfolio under alternative frames, analyzing the effects of the hedonic and time framing.
4 Covered call writing versus stock only position

In this and the following sections, we consider strategies which include options on non-dividend paying assets for which early exercise is not convenient or possible (American or European calls on single stock or indexes). We retain the usual assumptions of frictionless markets; in particular, we do not consider the effects of taxes and transaction costs, which may lead to different choices. We assume that the dynamics of the underlying price process $S$ is driven by a geometric Brownian motion\(^2\).

Shefrin and Statman [11] evaluate the covered call strategy in a simple one-period binomial model under prospect theory in its original version [7]; they use only a value function, do not consider probability weighting, and assume a zero risk-free interest rate $r$. We extend the analysis and model the problem under continuous cumulative prospect theory.

Let $S_0 > 0$ be the current stock price, it is the amount paid to buy one share of stock at time $t = 0$; consider a time horizon of $T$ years, the buyer of the stock registers a profit if $S_T > S_0 e^{rT}$ (a loss otherwise).

\(^2\)In this work, the dynamics of $S$ is governed by the following stochastic differential equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \quad S_0 > 0,$$

with parameters $\mu \in \mathbb{R}$ and $\sigma > 0$ and $W = (W_t)_{t \in [0,T]}$ is a standard Wiener process, given the usual assumptions.

Then the probability density function of the underlying price at maturity, $S_T$, is

$$f(x) = \frac{1}{x\sigma \sqrt{2\pi T}} \exp \left( -\frac{\ln(x/S_0) - (\mu - \sigma^2/2)T}{2\sigma^2 T} \right),$$

where $\mu$ and $\sigma > 0$ are constants, and the cumulative distribution function is

$$F(x) = \Phi \left( \frac{\ln(x/S_0) - (\mu - \sigma^2/2)T}{\sigma \sqrt{T}} \right),$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard Gaussian random variable.
The prospect value of the stock-only position $V^s$ is displayed as

$$V^s = \int_0^{S_0e^{rT}} \psi^-(F(x)) f(x) v^-(x - S_0e^{rT}) dx + \int_{S_0e^{rT}}^{+\infty} \psi^+(1 - F(x)) f(x) v^+(x - S_0e^{rT}) dx,$$

where $f$ and $F$ are the probability density function and cumulative probability function of $S_T$.

In the definition of the prospect value $V^s$, we have assumed that the price $S_0$ could have been invested at the risk-free interest rate $r$. Under this hypothesis, a risk-neutral investor (setting the parameters $a = b = 1$, $\lambda = 1$, $\gamma^+ = \gamma^- = 1$, and taking risk-neutral dynamics for the process $S$) has a value $V^s$ equal to zero, equivalent to the status quo.

Assume now an out-of-the-money call option written on the same stock, with strike price $X$ (with $X > S_0e^{rT}$) and maturity $T$. Let $c$ be the option premium (computed with the Black-Scholes formula). The prospect value of a covered call position, when the option premium and the covered call result are segregated into two mental accounts, is given by

$$V^{cc} = v^+(c e^{rT}) + \int_0^{S_0e^{rT}} \psi^-(F(x)) f(x) v^-(x - S_0e^{rT}) dx + \int_{S_0e^{rT}}^{X} \psi^+(1 - F(x)) f(x) v^+(x - S_0e^{rT}) dx + \int_{X}^{+\infty} \psi^+(1 - F(x)) f(x) v^+(X - S_0e^{rT}) dx.$$  

(8)

It is worth noting that in the first term in (8), we have assumed that the call premium is not used for consumption, but invested at the risk-free interest rate (otherwise one has to replace the term with $v^+(c)$).

Shefrin and Statman argue that “the prospect theory expected value of the covered call position exceeds the prospect theory value of the stock-only position for investors who are sufficiently risk-averse in the domain of gains”.

---

3Here we take an out-of-the-money forward option; alternatively, the strategy can be built with out-of-the-money options with $X > S_0$. 

9
This hypothesis seems not to be confirmed by [6], whereas the authors find strong evidence for framing effects. [5] argue that a prospect theory investor with above average risk aversion for gains prefers the covered call.

The covered call position (8) is preferred to the stock only position (7) if

$$v^+(c e^{rT}) + \int_X^{+\infty} \psi^+(1 - F(x)) f(x) v^+(X - S_0 e^{rT}) dx > \int_X^{+\infty} \psi^+(1 - F(x)) f(x) v^+(x - S_0 e^{rT}) dx.$$ (9)

Note that inequality (9) depends only on the value function in the domain of gains (when applying function (2), it depends only on the value of the parameter \(a\)), and on the weighting function \(w^+\). Considering a concave value function \(v^+\), one obtains a higher value on the right hand side of (9) when the option premium is segregated from the (positive) result of selling the stock at \(X > S_0 e^{rT}\). Numerical results confirm a framing effect.

## 5 Fully covered versus partially covered call position

The covered call strategy discussed in the previous section is a fully covered position, which entails buying one share and selling one call option. Empirical evidence suggests that most covered call traded strategies are fully covered. Hoffmann and Fischer [6], in their investigation, find significant preference for fully covered over partially covered positions.

Shefrin and Statman [11] compare in their one-period binomial model a fully covered position with a partially covered one composed by 1.25 shares and 2.5 calls. Both positions have the same profit and loss profiles. The authors examine the prospect values of two possible frames with identical cash flows, consisting of:

1. a fully covered position of 1.25 shares and a short position of 1.25 (naked) calls;
(ii) a stock only position of 1.25 shares and a short position of 2.5 (naked) calls.

They conclude that the prospect value in the first frame exceeds the one in the second frame for investors who are highly risk-averse in the domain of gains and highly risk-seeking in the domain of losses. Moreover, the fully covered position is preferred to the partially covered one when the concavity and convexity of the value function for gains and losses, respectively, are sufficiently pronounced.

We can generalize the analysis of [11] in a continuous setting, comparing the following two frames:

(i) a fully covered position of $\alpha \beta$ shares (with $0 < \alpha < 1$) and a short position of $(1 - \alpha)\beta$ (naked) calls ($\beta > 0$);

(ii) a stock only position of $\alpha \beta$ shares and a short position of $\beta$ (naked) calls.

As limit cases, when $\alpha = 0$ we have a short position of $\beta$ (naked) calls, and when $\alpha = 1$ the short position of $\beta$ calls is fully covered.

The prospect value in the first frame, when the option premium and the covered call net result are segregated into separate mental accounts, is given by

$$V_p = V^c(\alpha \beta) + V^sc((1 - \alpha)\beta)$$

$$= \left[ \psi^+(F(x)) f(x) \right]_{\alpha \beta} + \int_0^{S_0 e^{rT}} \psi^-(F(x)) f(x) \left( \alpha \beta (x - S_0 e^{rT}) \right) dx +$$

$$\int_{S_0 e^{rT}}^{\infty} \psi^+(1 - F(x)) f(x) \left( \alpha \beta (x - S_0 e^{rT}) \right) dx +$$

$$\int_{X}^{\infty} \psi^+(1 - F(x)) f(x) \left( \alpha \beta (X - S_0 e^{rT}) \right) dx +$$

$$\int_{X}^{\infty} \psi^-(1 - F(x)) f(x) \left( (1 - \alpha)\beta (X - x) \right) dx .$$

(10)
The last two terms in equation (10) represent the prospect value of the writer's position of $(1-\alpha)\beta$ calls in the time segregated frame $V^{sc}$ (see the Appendix), when the option premium received at time $t=0$ is evaluated in a separate mental account. We assume that the premium is reinvested at the risk free interest rate $r$.

Alternatively, the naked position could be evaluated in a time aggregated mental account $V^{ac}$ (see the Appendix) as follows:

\[ V^{pcc} = V^{cc}(\alpha\beta) + V^{ac}((1-\alpha)\beta), \]

where

\[ V^{ac}((1-\alpha)\beta) = \int_{S_0}^{X+ce^{rT}} \psi^+(F(x)) f(x) v^+ \left((1-\alpha)\beta \left(ce^{rT} - (x - X)\right)\right) dx + \]

\[ + \int_{X+ce^{rT}}^{+\infty} \psi^- (1-F(x)) f(x) v^- \left((1-\alpha)\beta \left(ce^{rT} - (x - X)\right)\right) dx. \]

Note also that the covered call strategy in (10) is built with out-of-the-money (forward) calls; other assumptions are also possible and will be discussed in the next section.

The prospect value in frame (ii) is the sum of a stock only position of $\alpha\beta$ shares and a short position of $\beta$ naked calls; when the option premium and the option payoff are segregated into separate mental accounts, one obtains

\[ V^{pcc} = V^{sc}(\alpha\beta) + V^{sc}((1-\alpha)\beta), \]

\[ = \int_{0}^{S_0 e^{rT}} \psi^-(F(x)) f(x) v^- \left(\alpha\beta(x-S_0 e^{rT})\right) dx + \]

\[ + \int_{S_0 e^{rT}}^{+\infty} \psi^+(1-F(x)) f(x) v^+ \left(\alpha\beta(x-S_0 e^{rT})\right) dx + \]

\[ + v^+ (\beta ce^{rT}) + \]

\[ + \int_{X}^{+\infty} \psi^- (1-F(x)) f(x) v^- (\beta(X-x)) dx. \]

The last two terms in equation (12) represent the prospect value of the writer’s position in the time segregated frame; as an alternative, such a position can be evaluated in the time aggregated case.
6 Covered call writing with out-of-the-money versus in-the-money options

In the previous sections, covered call strategy was formed with out-of-the-money options. This is the way in which more often the strategy has been proposed to the investors, in order to obtain a profit in case of exercise. Due to the shape of the value function and hedonic framing, prospect investors will tend to segregate such a gain.

We consider a call option which is in-the-money forward with \( X < S_0 e^{rT} \) (the case \( X < S_0 \) can be treated analogously). When \( S_T \geq X \) the option is exercised and the profit and loss profile for the covered call investor is:

\[
ce^{rT} - (S_T - X) + (S_T - S_0 e^{rT}),
\]

assuming that both the call premium and the amount \( S_0 \) could be invested at the risk free rate\(^4\). By construction of the strategy, the difference \( X - S_0 e^{rT} \) is negative. On the other hand, when \( S_T < X \) the option is not exercised and the result of the long position on the underlying, \( S_T - S_0 e^{rT} \), is negative as well. The covered call strategy gives rise to a profit only if the call premium is sufficiently higher than the loss on the combined positions on the call and the underlying.

An investor who applies hedonic framing will segregate the call premium from other losses in order to obtain a better prospect value and, when such a value is negative, to preserve the silver lining. Formally we have:

\[
V^{\text{ce-utm}} = v^+(ce^{rT}) + \int_0^X \psi^-(F(x)) f(x)v^-(x - S_0 e^{rT}) dx + \\
+ \int_X^{\infty} \psi^-(1 - F(x)) f(x)v^-(X - S_0 e^{rT}) dx.
\]

Shefrin and Statman [11] compare two portfolios formed with stocks, short positions in different proportions of out-of-the-money or in-the-money

\(^4\)This hypothesis can be easily relaxed, considering as in [11] that the cashed premium could be used for consumption or disregarding the effects of a non zero interest rate.
call options and borrowed cash (when the strategy is formed with in-the-money options), in a one-period binomial framework. In their particular example, both portfolios entail the same initial cash outflow and the same gains or losses in the up and down states. When comparing the prospect value of the two strategies, the authors conclude that covered call with out-of-the-money options is preferred when investors are sufficiently risk-averse in the domain of gains and risk-seeking in the domain of losses.

In our analysis, we consider continuous variables, and the comparison of portfolios with identical cash outflows and results is no longer possible. Nevertheless, we observe that both the integrals in (13) result in a negative value which has to be compensated by the positive value of the call premium in order to obtain a positive prospect value. This requires that the option is over-evaluated by the prospect investors, in order to prefer the strategy to the status quo. It is difficult to separate the different effects of framing, subjective evaluation of gains and losses, and the over- and under-weighting of probabilities, which requires a large numerical analysis which is not included in this contribution. Numerical experiments based on Tversky and Kahneman [13] sentiment parameters yield negative prospect values. In particular, when the parameter $\lambda$, which models loss aversion in (2), departs from the value 1, one obtains negative prospect values (excluding the effects of the probability weighting function). When we include in the analysis the weighting function, low probabilities of extreme events are overweighted and this may lead to different decisions about the preferred frame.

7 Selling the underlying asset versus buying back the option

Shefrin and Statman [11] argue that a prospect theory investor will not repurchase the call option when it is likely to be exercised, because this would imply the realization of a loss. They also explain that this happens when
investors are sufficiently risk-averse in the domain of gains. Hoffmann and Fischer [6] tested this hypothesis, finding no statistical evidence for the preference of repurchasing the call or selling the stock.

We compare two different frames: first we consider an investor who segregates the call premium from the combined result of the call and stock positions; second, the investor separates the result of the call position from the gains or losses on the stock. In both cases let us assume \( X > S_0e^{rT} \), hence the covered call strategy is formed with out-of-the-money calls.

In the first frame the prospect value \( V^{cc} \) is given as in (8), whereas the prospect value of the covered call strategy when the call is repurchased is:

\[
V^{ccr} = w^+(F(X))v^+(ce^{rT}) + \int_{X}^{X+ce^{rT}} \psi^+(F(x))f(x)v^+(ce^{rT} - (x - X)) \, dx + \\
+ \int_{X+ce^{rT}}^{+\infty} \psi^-(1 - F(x))f(x)v^- (ce^{rT} - (x - X)) \, dx + \\
+ \int_{0}^{S_0e^{rT}} \psi^-(F(x))f(x)v^- (x - S_0e^{rT}) \, dx + \\
+ \int_{S_0e^{rT}}^{+\infty} \psi^+(1 - F(x))f(x)v^+ (x - S_0e^{rT}) \, dx.
\]

(14)

In this case, the prospect value of the call position is given by the first three terms, and the gains and losses on the stock are evaluated through the last two integrals.

If \( V^{ccr} \) is lower than \( V^{cc} \) as defined in (8), the investor would prefer selling the stock (hence realizing a gain), rather than repurchasing the option which entails the realization of a loss. It is worth noting that the preference depends not only on the shape of the value function in the region of gains, but also on the risk-seeking behavior of the investor in the domain of losses, and his or her loss aversion.

In some numerical experiments, first neglecting the effects of the probability weighting function (taking \( \gamma^+ = \gamma^- = 1 \)), we found that due to the shape of the value function the frame defined by (8) is preferred, giving as a result
a higher prospect value, even with values of the parameters $a$ and $b$ close to unit. As in the comparisons with other frames in the previous sections, the loss aversion parameter plays an important role, because it enhances the negative valuation of losses. When also the probability perception through the weighting function is considered, this can lead to different results, due to the fact that low probabilities of extreme events are overweighted.

8 A comparison of covered call writing with alternative trading strategies

The covered call profit and loss profile can be compared with that of a naked put. [6] in their empirical study find that investors have a significant preference for at-the-money covered calls over at-the-money naked puts and explain such a result in terms of perceived higher riskiness of the long position in a put option. Another possible reason for this is the framing effect.

Remember that the prospect value of the position in the covered call, when the option premium and the covered call result are evaluated into separate mental accounts is provided by equation (8). Consider now a covered call strategy formed with an at-the-money call option with $X = S_0$.

\begin{equation}
V_{cc-atm} = v^+(e^{rT}) + \int_0^X \psi^-(F(x)) f(x) v^- (x - S_0) \, dx .
\end{equation}

(15)

Let $p$ be the put option premium at time $t = 0$, the prospect value of a naked put position is

\begin{equation}
V_{np} = v^- (-pe^{rT}) + \int_0^X \psi^+(F(x)) f(x) v^+ (X - x) \, dx .
\end{equation}

(16)

Note that this is the prospect value considered from the viewpoint of the holder, when the option premium and the final payoff are segregated. The current underlying price does not appear in (16), but influences directly the option premium and the probability of exercising the option.
Assume that also the put option is at-the-money with $X = S_0$, and for simplicity consider the same parameters for $w^+$ and $w^-$. Then the covered call strategy is preferred if $V_{cc-atm} > V_{np}$ and the choice is influenced by the shape of the value function in the domain of gains and losses. Formally, one has to check for the following condition

$$v^+(c e^{rT}) + \int_0^X \psi^-(F(x)) f(x) v^-(x - S_0) \, dx > v^-(p e^{rT}) + \int_0^X \psi^+(F(x)) f(x) v^+(X - x) \, dx.$$  \hspace{1cm} (17)

Substituting the value function (2), one obtains

$$(c e^{rT})^a + \int_0^X \psi^-(F(x)) f(x) \left(-\lambda (X - x)^b\right) \, dx >\hspace{1cm} (18)$$

$$> -\lambda (p e^{rT})^b + \int_0^X \psi^+(F(x)) f(x) (X - x)^a \, dx.$$  

Assuming $\gamma = 1$ in the probability weighting function, the previous inequality simplifies into

$$(c e^{rT})^a - \lambda \int_0^X (X - x)^b f(x) dx >\hspace{1cm} (19)$$

$$> -\lambda (p e^{rT})^b + \int_0^X (X - x)^a f(x) dx ,$$

hence it depends directly on the value of the parameters $a$, $b$ and $\lambda$ which model risk attitude and loss aversion.

9 Concluding remarks

In this paper we have applied hedonic framing and prospect theory on the evaluation of one of the most popular trading strategies. The covered call writing is perceived as a strategy with limited risk, but this is not sufficient to explain its success amongst investors, or the preference for covered call with respect to alternative strategies with similar profit profiles, which register significantly lower trading volume. Preliminary numerical results suggest
that different frames may lead to different preference for alternative designs of the covered call strategy.

The analysis presented in this contribution can be extended to the valuation of other structured financial products can be evaluated (see [3]), with important insights and implications for the marketing and investment decisions.

Appendix: European options valuation

[14] provide the prospect value of writing call options, considering different intertemporal frames. The option premium represents a sure gain for the writer, whereas the negative payoff is a potential loss; the writer may aggregate or segregate such results in different ways. [10] extend the model of [14] to the case of put options, considering the problem both from the writer’s and holder’s perspectives, and use alternative weighting functions.

Let $S_t$ be the price at time $t \in [0,T]$ of the underlying asset of a European option with maturity $T$. Let $c$ be the call option premium with strike price $X$. At time $t = 0$, the option’s writer receives $c$ and can invest the premium at the risk-free rate $r$, obtaining $ce^{rT}$. At maturity, the amount $S_T - X$ is paid to the holder if the option expires in-the-money.

In the time segregated case the option premium is evaluated separately (through the value function) from the option payoff. Considering zero as a reference point (status quo), the prospect value of the writer’s position is

$$V^{sc} = v^+ (ce^{rT}) + \int_{X}^{+\infty} \psi^- (1 - F(x)) f(x) v^- (X - x) \, dx, \quad (20)$$

with $f$ and $F$ being, respectively, the probability density function and the cumulative distribution function of the future underlying price $S_T$, and $v$ is defined as in (2). When one equates $V^{sc}$ at zero and solve for the price $c$ finds

$$c = e^{-rT} \left( \lambda \int_{X}^{+\infty} \psi^- (1 - F(x)) f(x) (x - X)^b \, dx \right)^{1/a}, \quad (21)$$
which requires numerical approximation of the integral.

In the time aggregated frame, gains and losses are integrated in a unique mental account, then one obtains the prospect value

\[ V_{ac} = w^+ (F(X)) v^+ (ce^{rT}) + \int_{X+ce^{rT}}^X \psi^+ (F(x)) f(x) v^+ (ce^{rT} - (x - X)) \, dx + \int_{X+ce^{rT}}^{+\infty} \psi^- (1 - F(x)) f(x) v^- (ce^{rT} - (x - X)) \, dx. \]  

(22)

In this case, the option price evaluated by a prospect theory investor is implicitly defined by the equation \( V_a = 0 \) and has to be determined numerically.

In order to obtain the value of a European put option, we can no longer use put-call parity arguments. Let \( p \) be the put option premium at time \( t = 0 \); the prospect value of the writer’s position in the time segregated case is

\[ V_{sp} = v^+ (p e^{rT}) + \int_0^X \psi^- (F(x)) f(x) v^- (x - X) \, dx, \]  

(23)

and one obtains

\[ p = e^{-rT} \left( \lambda \int_0^X \psi^- (F(x)) f(x) (X - x)^b \, dx \right)^{1/a}. \]  

(24)

In the time aggregated case the put option value is implicitly defined equating at zero the following expression

\[ V_{ap} = \int_0^{X-p e^{rT}} \psi^- (F(x)) f(x) v^- (p e^{rT} - (X - x)) \, dx + \int_{X-p e^{rT}}^X \psi^+ (1 - F(x)) f(x) v^+ (p e^{rT} - (X - x)) \, dx + \int_{X-p e^{rT}}^{+\infty} \psi^- (1 - F(X)) f(x) v^+ (p e^{rT}) \, dx. \]  

(25)

and numerical techniques are required to find the price of the put option \( p \).

References


