A tale of two systems: Winners and Losers when moving from defined benefit to defined contribution pensions

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Abstract. There is a trend among employers to prefer Defined Contribution instead of Defined Benefit pension plans, since the former transfer all risks associated with investment return, longevity, etc from the employer to the employee. However, Defined Contribution plans also allow the individual to enter into positions contingent on the individual situation. This paper investigates the individual welfare consequences of different plans. We used the recent transition from defined benefit to defined contribution for white-collar workers in Sweden as the benchmark for our analysis. The framework for our analysis is a life cycle model of a borrowing-constrained individual’s consumption- and portfolio choices in the presence of uncertain labour income. The main result is that individuals with the characteristic of a low expected pre-retirement income relative to average income and high variance in earnings are winners (men with university degree in the private sector), and that those with the opposite characteristic (women with university degree in the public sector) are losers.

Keywords. Life-cycle, portfolio choice, defined contribution, defined benefit, income process.

M.S.C. classification. 62M45, 65C05, 68T05.

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1 Introduction

Defined Contribution—DC pension plans\(^1\) are now often the preferred pension system among employers. This is not very surprising since the shift from Defined Benefit—DB\(^2\), transfers all the risks associated with investment return and longevity from the employer to the employee. However, there are also several advantages for the individual with a DC plan: it allows the individual to enter into specific positions, which reduces the consequences of forcing all individuals into one-size-fits-all, in terms of risk and return characteristics; it facilitates portability when the agent transfers from one employer to another; and not the least it assures the risks of lower wages in the final years of employment. Which of these systems that are beneficial to the individual is very state dependent and merits this research.

In this paper we have analysed the welfare consequences for the individual when transferring from a DB to a DC system. As a benchmark for this analysis, we have chosen the recently negotiated transfer from a DB to a DC plan between private white-collar workers union and their employers in Sweden, cf. [21]. This analysis is even more pertinent, since this transfer will most likely be the blueprint for a similar future settlement for the public employees.

The main result is that individuals with the characteristic of a low expected pre-retirement income relative to average income and high variance in earnings are winners (men with university degree in the private sector), and that those with the opposite characteristic (women with university degree in the public sector) are losers.

Our analysis draws heavily on the literature highlighting: life-cycle saving and consumption, [20] and [15]; and portfolio-choice, [22] and [19], [11], [6] and [16] created life-cycle models with uncertain wages and borrowing constraints; which showed that market-incompleteness is important when explaining individual choice and welfare effects. [10] and others extended the model with portfolio-choice between a risk-free and a risky asset. [2] added a mandatory pension scheme to the model.

The introduction of non-tradeable human capital into the intertemporal life cycle model with portfolio choice and consumption, creates an asset that will influence—how much the individual saves and the optimal portfolio choice in savings. These choices depend on the expected individual dividend profile from human capital and associated uncertainties, but also on the characteristics of other assets; primarily private savings, pension savings and housing.

Labour generates two types of dividends: wages and pension contributions. In this paper we estimated the income process that should be used as the underlying for calculating the derivatives—net wages and pension contributions; keeping the “dividends” from human capital separate from other types of asset-income.

[7], [16], [10], and other similar earlier studies generally treated returns from human capital as equal to earned income net of return from private savings.

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\(^1\) A DC plan accumulates a proportion of every salary as a contribution.

\(^2\) A DB plan pays a proportion of final salary as a pension.
Such a wide definition lead to some double-counting, for those who retire early or receive pension benefits dependent on contributions during their working life. In our definition of returns from human capital, we only included income that stem from individual productivity and insurances against, e.g. disability, parental leave, unemployment etc.—not from early withdrawals from retirement savings.

We are interested in the expected income profile as the underlying for pension contributions and taxes, which influence the individuals future choices in terms of saving and portfolio allocation. It is therefore natural to model individuals rather than households\(^3\), since pension contributions and taxes are primarily dependent on the individual instead of family incomes.

The remainder of the paper is organised as follows: Section 2 describes the model, while Section 3 describes the optimisation problem, and Section 4 the calibration of the model. Section 5 discusses the results, and the final Section 6 summarizes and draws some conclusions.

2 The Model

2.1 Individual preferences

We assume that an individual maximise the expected utility over their adult life-cycle, which starts at the age of \(\tau_0\), and dies no later than at the age of \(T\). We assume that an individual has constant relative-risk-aversion preferences for a single non-durable consumption good—\(C_{\tau}\).

Individual preferences at time—\(m\) are defined as

\[
\frac{C_{\tau}^{1-\gamma}}{1-\gamma} + E_{m} \sum_{\tau=m+1}^{T} \delta^{\tau-m} \left( \prod_{j=m}^{\tau-1} p_{j} \right) \left\{ p_{\tau-1} \frac{C_{\tau}^{1-\gamma}}{1-\gamma} + b(1-p_{\tau-1}) \frac{D_{\tau}^{1-\gamma}}{1-\gamma} \right\}, \tag{1}
\]

\(\gamma\) is the coefficient of relative risk aversion, \(p_{\tau}\) is the one-year age-contingent survival-probability, \(\delta\) is the discount factor, \(b\) is the bequest parameter and \(D_{\tau}\) is the bequest amount.

2.2 The labour-income process

Following [7], we assume that the individual income process during working life—\(L_{\tau t}\), is exogenously given by

\[
\log(L_{\tau t}) = l_{\tau t} = f(\tau, Z_{\tau t}) + v_{\tau t} + \varepsilon_{\tau t}, \tau \leq K, \tag{2}
\]
where \( f(\tau, Z_i) \) is a deterministic function of individual \( i \)'s age—\( \tau \), and a vector of the individual characteristics—\( Z \), where—\( K \) is the retirement age, and—\( v_{i\tau} \) is given by

\[
v_{i\tau} = v_{i\tau-1} + u_{i\tau},
\]

where the permanent shock—\( u_{i\tau} \sim N(0, \sigma_u^2) \) is independent from the idiosyncratic temporary shock—\( \varepsilon_{i\tau} \sim N(0, \sigma_{\varepsilon_i}^2) \). The permanent shock—\( u_{i\tau} \), consists of a group aggregate component—\( \xi_{k\tau} \sim N(0, \sigma_{\xi_k}^2) \) as well as an idiosyncratic component—\( \omega_{i\tau} \sim N(0, \sigma_{\omega_i}^2) \),

\[
u_i = \xi_{k\tau} + \omega_{i\tau}.
\]

### 2.3 Assets

There are two assets, one risky and one risk-free asset, with after-tax real log-returns equal of \( r_e^r \) and \( r_f^r \) respectively. Excess return is defined as

\[
r_e^r - r_f^r = \mu^e + \eta_r,
\]

where the noise—\( \eta \) is correlated with the group-aggregate innovation in permanent labour-income—\( \xi_k \), which allows for a group specific sensitivity to the risky asset,

\[
\begin{bmatrix}
\xi \\
\eta
\end{bmatrix}
\sim N
\left(
\begin{bmatrix}
-\frac{1}{2}\sigma^2_{\xi_k} & \Sigma \\
\Sigma & \sigma^2_{\eta}
\end{bmatrix}
\right).
\]

### 2.4 Past and present mandatory savings and retirement benefits

In the old system\(^5\), individuals have a defined-benefit and a defined contribution plan. The defined benefit plan has a payout of 10%, 65% and 32.5% of incomes at retirement\(^6\) in the intervals \([0, 320)\), \([320, 850)\), and \([850, 1270)\) respectively.\(^7\)

\[
PODB_{i\tau} = 0.1 \min \left[ L_{64}^P ; 320 \right] + 0.65 \min \left[ \max \left( L_{64}^P - 320 ; 0 \right) ; 850 - 320 \right] + 0.325 \min \left[ \max \left( L_{64}^P - 850 ; 0 \right) ; 1270 - 850 \right].
\]

The defined contribution plan has contributions at 4.5% of annual labour income up to 320.

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\(^4\) i.e. age, marital status, family size, and number and age of children.

\(^5\) Individuals born before 1979.

\(^6\) In reality it depends on the wage during the five years prior to retirement. However, modelling this rule correctly would have necessitated additional state variables. We therefore approximate this by only including the permanent income changes until retirement.

\(^7\) In the following, we express all amounts in thousands of SEK. The present exchange rate is \textit{circa} 6 SEK / USD.
The new system is only based defined contributions (cf. [21]), with contributions set to: 7% for annual incomes up to 320 and 30% for incomes above this limit. Contributions to the defined contribution plans—DC are therefore,

\[ DC_{i\tau} = \begin{cases} 
0.045 \min [L_{i\tau}; 320], & \text{if in old system,} \\
0.07 \min [L_{i\tau}; 320] + 0.3 \max (L_{i\tau} - 320; 0), & \text{if in new system.} 
\end{cases} \]  

(8)

Individuals can choose the fraction, \( \lambda \) of the defined contribution wealth, \( DCW_{i\tau} \) to allocate to the risky asset,

\[ DCW_{i\tau} = \begin{cases} 
e^{r_f} [1 + \lambda_{i\tau-1}(e^{\rho_r+\eta_r} - 1)] DCW_{i\tau-1} + DC_{i\tau}, & \tau < 65, \\
e^{r_f} [1 + \lambda_{i\tau-1}(e^{\rho_r+\eta_r} - 1)] DCW_{i\tau-1} - PODC_{i\tau}, & \tau \geq 65, 
\end{cases} \]  

(9)

where, \( PODC \), is the mortality-adjusted annuity from the defined contribution plan.

Irrespective of system, all individuals also receive social security pension benefits—SS, which depend on the individual’s labour-income trajectory during working life. In [4], we modelled this system as state dependent and from the simulated trajectories we have estimated a piece-wise linear retention-rate,

\[ SS_{i\tau} = 0.4 \min [L_{P64}^{64}; 320] + 0.1 \min [\max (L_{P64}^{64} - 320; 0); 850 - 320], \]  

(10)

dependent on the permanent part of labour income—\( L_{P64}^{64} \),

\[ L_{P64}^{64} = e^{l_k(Z_{64})+v_{64}}. \]  

(11)

All payouts from these pension plans are assumed to be forfeited in the event of death.

### 2.5 Labour income and taxes

Wage and retirement income—\( L \) can now be defined as

\[ L_{i\tau} = \begin{cases} 
e^{l_{i\tau}}, & \tau < 65, \\
PODC_{i\tau} + PODB_{i\tau} + SS_{i\tau}, & \tau \geq 65 \text{ if in old system,} \\
PODC_{i\tau} + SS_{i\tau}, & \tau \geq 65 \text{ if in new system,} 
\end{cases} \]  

(12)

According to Swedish tax rules,\(^8\) labour income and pension benefits are taxed at a common rate, separate from capital income. To calculate net income—\( L_{i\tau}^n \), we first deduct a general allowance of 10; then a municipal tax of 30%; then national tax of 20% on all income above 300; and finally, an additional national tax of 5% on income above 450. Net income is bounded below by the social welfare minimum-benefit and government-guaranteed minimum pension at 60. Therefore

\(^8\) We use the tax rules for incomes earned in 2003.
\[
L_i^\tau = \max[L_i^\tau - 0.3 \max(L_i^\tau - 10; 0) - \\
0.2 \max(L_i^\tau - 300; 0) - 0.05 \max(L_i^\tau - 450; 0); 60].
\] (13)

All the threshold-values that create kinks in tax-rates and benefits\(^9\) are indexed to the expected growth in national labour income—\(\mu^l\), except the social welfare minimum benefit which is kept constant in real terms.

### 2.6 Private savings and consumption

An individual starts her optimisation life with initial wealth set to \(F\). In the following pre-retirement years they receive wages, and in subsequent years retirement benefits. The individual has two control variables: the proportion of cash on hand to consume—\(\theta_i\), and what proportion of savings—\(\alpha_i\), to allocate to the risky asset. The cash on hand—disposable wealth, is therefore,

\[
X_{i\tau} = \begin{cases} 
\exp[\tau (1 + \alpha_i \tau - 1)] [1 - \theta_i X_{i\tau-1} + L_n^\tau, & \tau > \tau_0, \\
F_i + L_n^\tau, & \tau = \tau_0,
\end{cases}
\] (14)

of which consumption is,

\[
C_{i\tau} = \theta_i X_{i\tau}.
\] (15)

There are also constraints on both borrowing and short-sales,

\[
0 \leq \theta_i \leq 1, \\
0 \leq \alpha_i \leq 1.
\] (16)

### 3 Optimisation

To simplify the calculation\(^10\), we introduce a decision rule that defines the asset allocation in the defined contribution account. This rule originates from [19] and states that; in complete markets—the allocation to risky assets—\(\lambda\), is dependent on the relative size of investable assets to total wealth. In our model, total wealth is the sum of: present value of human capital, cash on hand and expected after-tax\(^11\) DC wealth—\(DCW^{af}\). The present value of human capital is the sum of: income plus defined benefits and defined contributions, net of taxes and adjusted for survival probabilities. Prior to retirement, the human capital is discounted with the complete market rate—\(s\),

\(^9\) This is similar to the US since the “bend points” when calculating the primary insurance amounts (PIA) are adjusted by average earnings growth.

\(^10\) The portfolio choice in the DC-account and for private savings is highly interdependent, making a simultaneous choice very complicated numerically.

\(^11\) The after-tax rate is set to the municipal-tax only, since this is typically the only tax that an agent pays when in retirement.
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\[ s = r^f - \frac{\sigma^2}{2} + \beta_k (\mu + \frac{\sigma^2}{2}), \] 

where \( \beta_k = \frac{\text{Cov}(\eta, \xi)}{\text{Var}(\eta)} \), 

(17)

and with the risk-free rate after retirement. Our decision rule is adjusted for the implicit equity exposure through the present value of human capital—\( \beta_k \Delta \),

\[ \lambda = \min \left\{ \frac{\mu^* [DCW_{at} + (1 - \theta)X + PV(HC)]}{\gamma^2 \sigma^2_n [DCW_{at} + (1 - \theta)X]} - \beta_k \Delta PV(HC) \right\}, \] 

(18)

where \( \Delta \) is the change in present value of human capital from a group specific permanent income shock—\( \xi_k \).

The individual’s problem therefore has four state variables (\( \tau, v, X \) and \( DCW \)) and two choice variables (\( \theta \) and \( \alpha \)) as well as four stochastic variables (\( \epsilon, \omega, \xi \) and \( \eta \)). The value function of their intertemporal consumption and investment problem can then be written as

\[ V_{\tau} (\Gamma_{\tau}) = \max_{\theta_{\tau}, \alpha_{\tau}} \left\{ \frac{C_{1\tau}}{1 - \gamma} + \delta E_{\tau} \left[ p_{\tau} V_{\tau+1} (\Gamma_{\tau+1}) + (1 - p_{\tau}) b^{\gamma_{\tau+1}} \right] \right\} \] 

\[ \Gamma_{\tau} = \{X_{\tau}, v_{\tau}, DCW_{\tau}\}, \] 

(19)

The solution to this maximisation problem together with our decision-rule from 18 gives us the state dependent policy rules,

\[ \theta_{\tau} = \theta_{k\tau}(\Gamma_{\tau}), \] 

\[ \alpha_{\tau} = \alpha_{k\tau}(\Gamma_{\tau}), \] 

\[ \lambda_{\tau} = \lambda_{k\tau}(\Gamma_{\tau}). \] 

(20)

We solved the problem numerically by backward recursion from the final year—\( T \), using by-now standard methods, cf. [18] and [10].

4 Calibration of parameters

4.1 Estimation of labour income process

Following [7], we modelled the log of real income as deterministic part with both permanent and temporary shocks. Their description of the income-process has been used in several life cycle models, cf. [2], [10], [4], [9], [5] and [23]. The deterministic part of Equation 2 was estimated (cf. Appendix 6 for details) using a longitudinal panel of data—LINDA, (cf. [12] for details), that covers the Swedish population in the age interval [28,64] for fourteen years during 1992–2005, resulting in more than 1.4 million observations.

The data set augmented with wealth information, has recently received attention in cf. [1], [3] and [14]. The data-set was divided into twelve non-intersecting groups, depending on sex, education and sector (private and public). Using the
methodology of [7], we estimated the variances of the permanent $\sigma_u^2$ and transitory $\sigma_e^2$ components of shocks to income as specified in Equation 2 (cf. Appendix 6).

4.2 Individual parameters
We used a standard set of assumptions with respect to the individual parameters for the reference case. First, we set the coefficient of relative risk aversion $-\gamma$ to 5 and the discount factor $-\delta$ to 0.98. The gender specific survival probabilities $-p$ were taken from the Swedish life-insurers when underwriting new policies, i.e., they are forward looking. The bequest parameter $-b$ was set to 1. Adult life is divided into two intervals: working life $[28, 64]$ and retirement $[65, 100]$. The importance of the risk aversion parameter $-\gamma$ will be elaborated on when we report on the sensitivity analysis in Section 5.4.

4.3 Assets and correlations
In the optimisation, we set the risk-free after-tax rate $-r_f$ to 1.5%, which is consistent with the present gross return of less than 2% for long-dated index-linked bonds. The mean after-tax equity premium $-\mu^e$ was set to 3%, which is lower than the historical average, but corresponds well with forward-looking estimates (cf. [8], [13]). Because of uncertainty about the equity-premium, we analysed its sensitivity in Section 5.4. Volatility $\sigma_\eta$ was set to 17% for the risky asset.

Next, we followed the procedure of [10] to estimate the correlation $-\rho_{\xi_\eta}$ between group specific permanent labour income shocks $-\xi_k$ and lagged equity returns $-\eta_{t-1}$. Table 5, shows the estimated correlation, using the returns on the Swedish equity-index—OMX and on the 12-month Swedish Treasury Bills as proxies for risky returns and the risk-free rate respectively.

We also set the growth in average labour income $-\mu_l$ to 1.8%, which is the estimate used by the National Social Insurance Board. Finally, the initial wealth $-z$ is set to 40, corresponding to the mean wealth for individuals at the age of 28.

5 Results

5.1 Labour income process
For reference, we plotted the average of the simulated income profiles for some$^{12}$ of the groups, cf. Figure 5.1.

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$^{12}$ In order to increase readability, we omitted the groups with similar profiles to the group with the lowest income.
Three findings are notable: First, individuals with a university degree experienced a significantly faster income growth in mid-life than did the other groups, a result which matches stylised facts from the US, cf. [17], [16] and [10]. Secondly, at each level of education, men had higher income than did women, at all stages of the life-cycle. Thirdly, that remunerations in the private sector was typically higher than in the public sector.

Our results also show a strikingly lower permanent variance if the agent is employed by a public vs. a private entity, whereas the temporary variance was similar, except for those with university degrees. After controlling for private vs. public sector, most of the gender differences in variance, that we found in our previous study ([4]) disappeared.

Figure 5.1 shows the large effect that a higher variance in the permanent component for Men in Private sector will have on labour income variation at retirement, when compared to Females in Public sector; albeit both are groups with a University degree.

We also note that the permanent shocks to income has the highest correlation with the equity market for the privately employed with an university degree, and that gender is less of an importance.

5.2 Winners & Losers

We simulated the individual behaviour from age—28 until 100 with 10000 trajectories. Contingent on their random experience, individuals chose responses determined by the policy rules in Equation 20.

Since the change of pension system was negotiated by consenting adults—we would expect that on average the two systems would generate similar benefits. However, under the new system, individuals have a much larger responsibility for the appropriate management of the DC-account, since the outcome rests solely with the employee. In Figure 5.2 we show (for the highest income group—Men with University degree in the private sector), the variation in size of the DC-accounts.

In order to discover to what extent this new pension system generated winners and losers, we evaluated the value function (Equation 19), for the different groups in the first period; using both the old and the new pension system. For each group, we then added an initial amount to the DC-account that was associated with the lowest value of the value function, until the value of the value functions were equal for both pension systems. The results for a subset of the groups are presented in Table 1.

Intuitively, we would expect the group with the highest expected final pre-retirement income relative to average income, to lose from the transition and vice versa. Another factor, is that high uncertainty in final pre-retirement income will decrease the expected utility of a defined benefit pension. Men with an university degree in the private sector has an early earnings career and a more pronounced decline in income prior to retirement. They are therefore the winners from a transition. The gain for this group is increased, as they also have a higher variance in income, which makes their expected final income less certain.
Table 1. Initial amount in Old or New DC-account necessary to equalise the value to the individual of the pension systems

<table>
<thead>
<tr>
<th>Amount in KSEK</th>
<th>Pension System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Old</td>
</tr>
<tr>
<td>Private Men</td>
<td></td>
</tr>
<tr>
<td>High school</td>
<td>19</td>
</tr>
<tr>
<td>University</td>
<td>101</td>
</tr>
<tr>
<td>Women</td>
<td></td>
</tr>
<tr>
<td>High school</td>
<td>32</td>
</tr>
<tr>
<td>University</td>
<td>12</td>
</tr>
<tr>
<td>Public Men</td>
<td></td>
</tr>
<tr>
<td>University</td>
<td>40</td>
</tr>
<tr>
<td>Women</td>
<td></td>
</tr>
<tr>
<td>University</td>
<td>80</td>
</tr>
</tbody>
</table>

The defined-contribution system recently negotiated in the private sector is a likely blueprint for a potential change of system for those in the public sector as well. Our analysis shows (cf. Table 1) that publicly employed would on average lose and that this loss is most pronounced for women. Women typically have their earnings-career later in life (cf. Figure 5.1), and therefore have less benefits from early contributions; and secondly that, the lower variance in earnings among publicly employed and will make the Defined Benefit pension closer to a risk-free asset.

5.3 Effects on portfolio choice

The positive labour income profile and short-sales constraints will typically make younger individuals "more" constrained, i.e. with an equity allocation quite different from the complete market solution. Cash on hand is very small in comparison to the human capital and since their DC-account cannot be used for precaution, we get a maximum allocation to equities in the DC-account. The difference in equity allocation ($\alpha - \lambda$) decrease—as cash on hand accumulates and as the income profile flattens. On average, we therefore expect the allocation to equities in the defined contribution account vs. cash on hand to differ a lot in early life and that the differences will decrease at a later stage.

Figure 5.3, shows the average equity proportion of the DC-account. Irrespective of system, since mandatory savings in pension systems cannot be used for precaution or bequest; there is no reason for young individuals to have anything but equities in the DC-account.

With increasing age, the combined effect of: the DC-account being a much larger proportion of total wealth in the new system and the old Defined Benefit pensions being less risky; will lead to a more conservative behaviour for an agent in the new system. After retirement, when the Defined Benefit benefits become risk-free and hence $\Delta = 0$ in Equation 18, we can identify a large increase in the equity exposure for an individual in the old system.
In Figure 5.3, we show the same profile, but now for cash on hand. There is a large difference between the risky weight in cash on hand vs. DC-account in early life, for precautionary and bequest reasons. After retirement, with decreasing present value of human capital, there is a gradual decline in equity-exposure towards the complete-market solution.

It is important to note that; the profiles reflect the simulated averages for one individual. Figure 5.3 shows some percentiles of equity exposure for an agent in the old pension system. The large variation is solely due to the accumulated effect of individual experiences. If we in addition, also could account for differences among individuals in: e.g. risk aversion, discounting or expected equity premia; then the variation would most likely be even larger.

5.4 Sensitivity analysis

In order to analyse to what extent our results are parameter-dependent, we performed a sensitivity analysis using the group whose benefits are most affected by the change in pension systems—men with a university degree in the private sector. Table 2 shows the initial amounts that the DC-account must be increased with, in order to equalise the value of the two pension systems, with respect to changing risk-aversion and a higher equity-premium.

<table>
<thead>
<tr>
<th></th>
<th>μe</th>
<th>γ</th>
<th>KSEK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>3</td>
<td>5</td>
<td>101</td>
</tr>
<tr>
<td>Low risk aversion</td>
<td>3</td>
<td>2</td>
<td>179</td>
</tr>
<tr>
<td>High risk aversion</td>
<td>3</td>
<td>8</td>
<td>70</td>
</tr>
<tr>
<td>High equity premium</td>
<td>4</td>
<td>5</td>
<td>106</td>
</tr>
</tbody>
</table>

In all cases, it was beneficial for this group to move into the new system. The result show that changes in the equity premium—μe does not have a large impact, whereas the benefit to the less risk-averse was increased substantially.

In Equation 18, we created a decision-rule for the equity share in the DC-account. If this rule is too crude, we would expect individuals to compensate for any such errors in the allocation of their private savings. We therefore "tested" this rule by calculating the difference between the equity share in DC-account and in cash on hand—(λ1τ − α1τ).

A priori, we would expect this difference to be small and show little variance for unconstrained individuals when the precautionary motive is weak, e.g., after retirement. Early in life, however, when individuals are borrowing-constrained, we know that differences between trajectories can be large. Figure 5.4 plots this difference and the variation after retirement is not very large, which indicates that our rule seems to work.
6 Conclusions

This paper aims to contribute to the understanding of the welfare effects of moving from primarily a defined-benefit to a defined-contribution pension system, and the changes in optimal individual behaviour required by such a change. The setting is a borrowing-constrained individual’s consumption- and portfolio-choice in the presence of uncertain labour-income, with group-dependent labour-income processes and realistically-calibrated tax- and pension-systems. We found that those employed in the private sector had higher income-variance than those in the public sector, while gender differences (after controlling for private vs. public employment) were small.

We have used the recently negotiated change from defined benefit to defined contribution pension systems as a benchmark for our analysis. The finding was that agents with low expected final income relative to average income and those with high income variance are set to gain from this transfer. Winners are men with an university degree within in the private sector, and losers would be women in the public sector with an university degree. The value of the different systems to the individual are dependent on the risk preferences, but will not change the preference of system.

Introducing a defined contribution system will necessitate that the individual has to manage the assets differently in private and pension savings; and that the differences in portfolio choice between agents due to individual situations are relatively large, even if we do not account for differences in terms of risk-aversion, etc, between individuals. One-size-fits-all kind of life-cycle funds, where the equity allocation depend on age alone, will therefore not fulfil the individual demand for advice about asset allocation.

Appendix: Data and Estimations

A.1 Estimation of the labour-income process

The data set was divided into twelve non-intersecting groups, depending on sex (Male, Female), education (Compulsory school-, High-school- or University-degree) and employer (Public, Private). The matrix of individual characteristics—Z, includes variables for the number of children in different age-intervals as wells as a dummies for marital status, age. Income was adjusted to real values by deflating with the official consumer price-index. Measured income is an aggregate including gross wages, also all social security benefits (primarily income-compensation for unemployment, disability and childcare) and pension benefits.

To avoid double-counting, we deleted all observations where income included voluntary pension benefits, i.e., individuals above the age of 55 receiving pension pay-outs at their own request. Pension benefits paid prior to age 55 can be considered as insurance payouts and were therefore included. Progressive taxation will induce most agents to make these early withdrawals only if the individual has simultaneously reduced the ordinary wage income. Finally, we exclude an
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observation if income is less than 100,000 SEK. Individuals with income lower than this level are assumed to be voluntarily unemployed.

The following random-effects linear model was used to estimate the deterministic function for each group,

\[ l_{it} = \beta_0 + Z_{it} \beta + \vartheta_i + e_{it}, \]

\[ e_{it} = \rho e_{it-1} + \varepsilon_{it}, \]

\[ \vartheta_i \sim N(0, \sigma_\vartheta^2), \]

\[ \varepsilon_{it} \sim N(0, \sigma_e^2), \]  

(21)

where—\( Z_{it} \) are the nonstochastic regressors and \( \beta \) is the vector of regression coefficients. Estimation results are presented in Table 3.

**Table 3.** Labour Income Process: Coefficients from Regression

<table>
<thead>
<tr>
<th>AR(1) Random effects Regression</th>
<th>#Children at age</th>
<th>Marr-</th>
<th>Std. in</th>
<th>Std. in R^2</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2004 KSEK</td>
<td>1-2</td>
<td>3-5</td>
<td>6-17</td>
</tr>
<tr>
<td>Private Men</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compulsory</td>
<td>-.00599</td>
<td>-.00076</td>
<td>-.00284</td>
<td>-.02846 .5305</td>
<td>.2378</td>
</tr>
<tr>
<td>High school</td>
<td>-.00762</td>
<td>-.00242</td>
<td>-.00027</td>
<td>-.02957 .5356</td>
<td>.2688</td>
</tr>
<tr>
<td>University</td>
<td>-.00786</td>
<td>-.00077</td>
<td>-.00035</td>
<td>-.03185 .5469</td>
<td>.3784</td>
</tr>
<tr>
<td>Women</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compulsory</td>
<td>-.08666</td>
<td>-.04532</td>
<td>-.02430</td>
<td>.03215 .5433</td>
<td>.2223</td>
</tr>
<tr>
<td>High school</td>
<td>-.12572</td>
<td>-.06728</td>
<td>-.03316</td>
<td>.03444 .5116</td>
<td>.2306</td>
</tr>
<tr>
<td>University</td>
<td>-.17466</td>
<td>-.10077</td>
<td>-.05480</td>
<td>.01108 .4645</td>
<td>.3194</td>
</tr>
<tr>
<td>Public Men</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compulsory</td>
<td>-.02575</td>
<td>-.00889</td>
<td>-.00838</td>
<td>-.04473 .4857</td>
<td>.2462</td>
</tr>
<tr>
<td>High school</td>
<td>-.01951</td>
<td>-.01389</td>
<td>-.00617</td>
<td>-.03341 .5076</td>
<td>.2642</td>
</tr>
<tr>
<td>University</td>
<td>-.00655</td>
<td>-.00014</td>
<td>-.00201</td>
<td>-.01905 .5394</td>
<td>.3190</td>
</tr>
<tr>
<td>Women</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compulsory</td>
<td>-.06417</td>
<td>-.03328</td>
<td>-.01257</td>
<td>.02817 .5480</td>
<td>.2018</td>
</tr>
<tr>
<td>High school</td>
<td>-.10207</td>
<td>-.04860</td>
<td>-.02063</td>
<td>.03488 .5274</td>
<td>.1699</td>
</tr>
<tr>
<td>University</td>
<td>-.13186</td>
<td>-.06781</td>
<td>-.02867</td>
<td>.03442 .5038</td>
<td>.2368</td>
</tr>
</tbody>
</table>

We then calculate the deterministic component of labour income—\( \exp \{ f_k(\tau, Z_{it}) \} \), adjusted for age dummies with the averages of the characteristics. This was then used to estimate a third-degree polynomial with respect to age, cf. Equation 22 Table 4, and Figure 5.1.

\[ \exp \{ f_k(\tau, Z_{it}) \} = \sum_{m=0}^{3} a_{km} (AGE_{\tau} - 18)^m. \]  

(22)
Table 4. Coefficients in the age polynomial of the forward-looking income profile

<table>
<thead>
<tr>
<th>Income profile, 2004 KSEK, (AGE-18)</th>
<th>Constant (a_0)</th>
<th>Age (a_1)</th>
<th>Age(^2) (a_2)</th>
<th>Age(^3) (a_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Men</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compulsory</td>
<td>187.4410</td>
<td>3.7553</td>
<td>-0.0149</td>
<td>-0.0009</td>
</tr>
<tr>
<td>High school</td>
<td>192.5214</td>
<td>5.4665</td>
<td>-0.0449</td>
<td>-0.0008</td>
</tr>
<tr>
<td>University</td>
<td>50.6314</td>
<td>26.8242</td>
<td>-0.5615</td>
<td>0.0028</td>
</tr>
<tr>
<td>Women</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compulsory</td>
<td>170.8340</td>
<td>0.0714</td>
<td>0.0986</td>
<td>-0.0020</td>
</tr>
<tr>
<td>High school</td>
<td>200.2780</td>
<td>-2.5127</td>
<td>0.2277</td>
<td>-0.0037</td>
</tr>
<tr>
<td>University</td>
<td>170.6430</td>
<td>5.3380</td>
<td>0.0553</td>
<td>-0.0027</td>
</tr>
<tr>
<td>Public Men</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compulsory</td>
<td>150.9325</td>
<td>3.0467</td>
<td>0.0011</td>
<td>-0.0008</td>
</tr>
<tr>
<td>High school</td>
<td>176.5778</td>
<td>2.5334</td>
<td>0.0325</td>
<td>-0.0013</td>
</tr>
<tr>
<td>University</td>
<td>91.3953</td>
<td>14.9683</td>
<td>-0.2048</td>
<td>0.0000</td>
</tr>
<tr>
<td>Women</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compulsory</td>
<td>154.1382</td>
<td>0.4386</td>
<td>0.0563</td>
<td>-0.0012</td>
</tr>
<tr>
<td>High school</td>
<td>181.6734</td>
<td>-2.1792</td>
<td>0.1781</td>
<td>-0.0027</td>
</tr>
<tr>
<td>University</td>
<td>217.9771</td>
<td>-5.5608</td>
<td>0.4423</td>
<td>-0.0065</td>
</tr>
</tbody>
</table>

A.2 Variance Decomposition

We followed [7] in decomposing permanent and temporary variances. By combining the error terms from Equation 2—\(v_{it} + \varepsilon_{it}\) with the estimated residual—\(\hat{e}_{it}\) from Equation 21, we get:

\[
\Delta e_{it}(d) = e_{it+d} - e_{it} = (v_{it+d} + \varepsilon_{it+d}) - (v_{it} + \varepsilon_{it}) = (u_{it+d} + \ldots + u_{it}) + (\varepsilon_{it+d} - \varepsilon_{it})
\]  

(23)

and consequentially the variance is,

\[
Var(\Delta e_{it}(d)) = d \cdot \sigma_u^2 + 2 \cdot \sigma_{\varepsilon}^2.
\]  

(24)

Following [7], we allowed for serial correlation in the transitory shock of the order \(MA(2)\), and therefore excluded observations with a time distance less than 3. OLS on Equation 24 was then used to estimate \(\sigma_u^2\) and \(\sigma_{\varepsilon}^2\).

A.3 Income correlation with the equities

We followed [10] in estimating the correlation between labour-income shocks and equity–returns. Using Equation 2, the first difference in \(l^*_{ikt} = l_{ikt} - f_k(\tau, \mathbf{Z}_{ikt})\) can be written as

\[
\Delta l^*_{ikt} = \xi_{ikt} + \omega_{ikt} + \Delta \epsilon_{ikt}.
\]  

(25)
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Taking the average over individuals in each group gives us the group-aggregate component,

$$\Delta l_{kt}^* = \xi_{kt}. \quad (26)$$

Finally, we estimated the correlations—$$\varrho_{\xi_{kt}}$$, by applying OLS to,

$$\Delta l_{kt}^* = \beta_k(r_{t-1}^e - r_{t-1}^f) + \phi_t. \quad (27)$$

Table 5 presents the result from this regression using the real return of the Swedish equity index OMX as a proxy for equity-returns—$$r^e$$ and the real return on 12-month Swedish Treasury Bill as the risk-free rate—$$r^f$$.

**Table 5. Variance decomposition and equity correlations**

<table>
<thead>
<tr>
<th>Number of Estimated Variance of the Correla-</th>
<th>Number of Estimated Variance of the Correla-</th>
<th>Number of Estimated Variance of the Correla-</th>
</tr>
</thead>
<tbody>
<tr>
<td>obs</td>
<td>permanent</td>
<td>transitory</td>
</tr>
<tr>
<td>of the</td>
<td>of the</td>
<td>component</td>
</tr>
<tr>
<td>Full sample</td>
<td>1 423 930</td>
<td>0.0211</td>
</tr>
<tr>
<td>Private Men</td>
<td>585 446</td>
<td>0.0142</td>
</tr>
<tr>
<td>Compulsory</td>
<td>140 413</td>
<td>0.0104</td>
</tr>
<tr>
<td>High school</td>
<td>310 835</td>
<td>0.0183</td>
</tr>
<tr>
<td>University</td>
<td>134 198</td>
<td>0.0284</td>
</tr>
<tr>
<td>Women</td>
<td>290 776</td>
<td>0.0048</td>
</tr>
<tr>
<td>Compulsory</td>
<td>67 364</td>
<td>0.0048</td>
</tr>
<tr>
<td>High school</td>
<td>152 254</td>
<td>0.0183</td>
</tr>
<tr>
<td>University</td>
<td>71 158</td>
<td>0.0284</td>
</tr>
<tr>
<td>Public Men</td>
<td>152 243</td>
<td>0.0021</td>
</tr>
<tr>
<td>Compulsory</td>
<td>17 039</td>
<td>0.0029</td>
</tr>
<tr>
<td>High school</td>
<td>47 543</td>
<td>0.0115</td>
</tr>
<tr>
<td>University</td>
<td>87 661</td>
<td>0.0044</td>
</tr>
<tr>
<td>Women</td>
<td>395 465</td>
<td>0.0034</td>
</tr>
<tr>
<td>Compulsory</td>
<td>41 921</td>
<td>0.0030</td>
</tr>
<tr>
<td>High school</td>
<td>176 607</td>
<td>0.0138</td>
</tr>
</tbody>
</table>

**References**

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Fig. 1. Income profiles for different groups. Simulated real gross wages—$L$ without productivity change.
Fig. 2. Income variation. 25\textsuperscript{th} and 75\textsuperscript{th} percentiles for simulated real gross wages—\( L \) without productivity for Men and Females with University degrees employed in the Private and Public sector.
Fig. 3. DC-account variation. 25\textsuperscript{th} and 75\textsuperscript{th} percentiles of the DC-account in the Old and the New pension system for Males with University degree in the Private sector
Fig. 4. Simulated Average Equity Exposure in DC-Account. Simulated average equity share—$\lambda$ in the DC-account for Males with University degree in the Private sector, for the old and new pension system.
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Fig. 5. Simulated Average Equity Exposure in Cash-on-Hand. Simulated average equity share—\( \alpha \) in the cash on hand—X for Males with University degree in the Private sector, for the old and new pension system.
Fig. 6. Variation in equity exposure in DC-Account. Simulated 5th, 25th, 75th and 95th percentiles for the equity share—λ in the DC—account for Males with University degree in the Private sector in the Old pension system.
Fig. 7. Variation in the difference in equity exposure between DC-Account and cash on hand. Simulated $25^{th}$ and $75^{th}$ percentiles for the difference in equity share between DC-account—$DCW$ and cash-on-hand—$X$ and for Males with University degree in the Private sector in the Old pension system—$(\lambda - \alpha)$