DEA models
for ethical and non ethical mutual funds*

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Abstract. In this paper we propose some models for the evaluation of the performance of mutual funds within a DEA approach that are able to tackle the problem of the presence of negative average rates of returns. The three models presented adjust some models previously proposed in the literature and regard a model that can be used for investments in mutual funds which have profitability as main objective and two models that are specifically formulated for ethical mutual funds. These two latter models include also the ethical objective among the outputs and differ in the way the ethical goal is pursued by investors.
The models proposed are applied to the European market of ethical mutual funds. In order to do so, a measure of the ethical level which takes into account the main socially responsible features of each fund is built.

Keywords. Data envelopment analysis, Performance evaluation, Mutual funds, ethical investment.

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1 Introduction

In this paper we present some models for the evaluation of the performance of mutual funds within a data envelopment analysis (DEA) approach that are able to tackle a problem which is often present in financial data, namely the presence of negative average rates of returns.

DEA models have been proposed in the literature in order to compare the performance of mutual funds by taking into consideration different aspects of the investment process: first of all, profitability and riskiness, but also initial and exit fees, and possible further objectives such as those which drive socially responsible investments. Along this line, we find the models proposed in Murthi,

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Choi and Desai [11], Morey and Morey [10], Basso and Funari [3], [5] and some generalizations to the measurement of the performance of ethical mutual funds proposed in Basso and Funari [4].

However, a problem may arise with these models, due to the assumption, implicitly made in DEA approaches, that all the input and output values are non negative. As a matter of fact, in slump periods of the business cycle the average rate of return of most stocks is negative, and that of many mutual funds as well, so that one of the output variables may well take negative values.

In this contribution we present some extensions of the DEA models for the evaluation of the performance of mutual funds that enable to compute the performance measure also in the presence of negative rates of returns. These extensions regard a model that can be used for investments in mutual funds which have profitability as main objective and two models, specifically formulated for ethical mutual funds, that include the ethical objective among the outputs.

The two models for ethical funds differ in the way the ethical goal is assumed to be pursued by investors: the first model is appropriate in the case in which investors try to maximize both the return and the ethical level of the investment at the same time, whereas the second one is more appropriate when investors choose the ethical level a priori and try to maximize the return of their investment while satisfying the desired ethical level.

The models proposed are applied to the European market of ethical mutual funds. In order to do so, we have built a measure of the ethical level which takes into account the main socially responsible features of each fund. The analysis carried out concerns the main ethical equity funds of the Western European market, as well as a set of non ethical equity funds included for comparison. On average, in the period considered in the analysis the ethical funds turn out to perform somewhat better than the non ethical funds if a model that takes the ethical level into account is used, while they are overcome by the non ethical funds if the ethical goal is not explicitly considered.

The paper is organized as follows. Section 2 tackles the problem of the presence of negative mean returns in computing a performance indicator for mutual funds with a DEA model and presents an adjusted model that ensures the positivity of the output values. Section 3 presents a method to build an ethical measure for mutual funds starting from available information on the ethical features of the funds. Sections 4 and 5 propose two generalizations of the DEA models for ethical funds while Section 6 discusses the connections among the DEA performance measures obtained. In Section 7 we present the results of the analysis carried out on the European market. Finally, Section 8 gives some concluding remarks.

2 DEA performance evaluation of mutual funds in the presence of negative rates of returns

In order to measure the performance of mutual funds Murthi, Choi and Desai [11] and Basso and Funari [3] propose some models which apply a DEA approach.

Moreover, special DEA models have been proposed in Basso and Funari [4] to evaluate the performance of ethical mutual funds.

Actually, it can be shown that the DEA technique can be used to define mutual fund performance indexes that take into account several inputs, such as different risk measures and the initial and exit fees of the investment, as well as several outputs, such as a return indicator and an ethical measure ([3]).

Let us consider a set of n mutual funds $j = 1, 2, \ldots, n$ with risky rates of return R_j and assume to have to compare their performances. We denote by $E(R_j)$ the expected rate of return of fund j and by $\sqrt{Var(R_j)}$ the standard deviation of the rate of return, often used as a risk indicator for a fund investment.

It is usual to evaluate the performance of mutual funds over past periods and use this performance measure in order to assess the ability of the fund managers. This is often done by substituting the average rates of return

$$\overline{R}_j = \frac{1}{T} \sum_{t=1}^{T} r_{jt} \qquad j = 1, 2, \ldots, n \tag{1}$$

obtained by the funds in the period considered and the historical volatilities of the returns

$$\sigma_j = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (r_{jt} - \overline{R}_j)^2} \qquad j = 1, 2, \ldots, n \tag{2}$$

for the expected rates of return $E(R_j)$ and the standard deviations $\sqrt{Var(R_j)}$, respectively, where $r_{j1}, r_{j2}, \ldots, r_{jT}$ denote the rates of return obtained by fund j in the periods $1, 2, \ldots, T$.

Moreover, let us also take into account the initial and exit fees usually required for an investment in a fund, f_j^I and f_j^E, respectively.

It is common in classical DEA models to assume that all the input and output values are non negative (see for example [7]). This is indeed a crucial assumption in the measurement of performance with the DEA technique. On the other hand, when some output variables may take negative values, the DEA performance measure may give non satisfactory results; for some examples on this subject see [6].

Actually if, as suggested in [11] and [3], we use in the DEA analysis as return indicator the average excess return $\overline{R}_j - r$, where r is the riskless rate of return, its value is negative in the period considered for all the funds which obtain a rate of return lower than the riskless interest rate. On the other hand, if instead we use as return indicator the average rate of return $\overline{R}_j$, as also suggested in [3], this often turns out to be negative for many mutual funds in the slump periods of the business cycle.

In order to ensure the positivity of all data, one might choose a different return indicator, defined in such a way as it is positive under all circumstances and thus it can be directly used as an output variable in a DEA model.

To this purpose, it would be sufficient to use a suitable DEA model which is translation invariant. A model is said translation invariant if the optimal value of the objective function, which represents the DEA efficiency measure, is invariant for translations of the original input and output values consequent to an addition of a constant to the original data.

A DEA model which has such a property is the additive model (on additive DEA models see e.g. [7], Section 4.3), and actually this model is often used in order to tackle the problem of negative data in DEA analysis. In particular, it can be proved (see [1] and [9]) that the additive model is indeed translation invariant, while the basic CCR DEA model is not.

However, an additive DEA model discriminates between efficient and inefficient DMUs, but it cannot gauge the depth of eventual inefficiencies: indeed, the efficiency measure given by an additive model does not provide a radial efficiency measure such as that given by the basic CCR model.

Another approach, proposed in [12], treats the problem of negative data in DEA models by modifying the efficiency measure used, but neither this approach is directly connected to radial efficiency.

For this reason we prefer to take into consideration a return indicator which is financially meaningful and cannot take negative values: this can be found in the capitalization factor $\overline{U}_j = 1 + \overline{R}_j$, which gives the final value of a unit initial investment at the end of a unit period. This quantity cannot become negative, since in the worst case we may at most lose all the capital invested in a mutual fund.

At a first sight, this modification in the output variables seems a detail of minor importance, one that does not substantially change the evaluation results when applied to decision making units with positive data, i.e. to mutual funds with positive mean returns. However, a test carried out on a set of European mutual funds shows that this is not the case; on the contrary, it turns out that the results do change greatly.

More precisely, let us consider as risk measure the historical volatility σ_j and let us take into consideration among the inputs the initial and exit fees f_j^I and f_j^E, respectively. We have first computed the efficiency score obtained using the I_{DEA-1} index proposed in [3] with these inputs and the mean return $\overline{R}_j$ as output. Then we have computed the efficiency scores obtained with the newly defined *DEA-cf* model which has the same input variables but the capitalization factor $\overline{U}_j = 1+\overline{R}_j$ as output. The set of mutual funds used for such a test includes all the mutual funds that will be used in the empirical analysis presented in Section 7 and have a positive mean return (56 funds out of 269, only 21 % of the funds analyzed).

The results are summarized in the first three columns of table 1 and show that in our test not only the ranking obtained changes but also does the efficiency set, so much that the efficiency set obtained with the new output is completely different than the one obtained with the original *DEA-1* model. Such an outcome does not seem much desirable and has pushed us to try to build a slightly different model which at least preserves the efficient units.

To this aim, since the output $\overline{U}_j$ represents the final value of the investment, let us include among the inputs also the initial capital invested in the mutual

Table 1. Comparison of the efficient set and the minimum and average efficiency scores obtained with *DEA-1*, *DEA-cf* and *DEA-S* models for the subset of funds with positive mean returns.

	DEA-1	*DEA-cf*	*DEA-S*
Efficient set			
Fund-22		•	•
Fund-27		•	•
Fund-39	•		•
Fund-55	•		•
Fund-56	•		•
Minimum score	0.016	0.360	0.855
Average score	0.368	0.602	0.941

fund; in the comparison analysis, the same initial capital $C_0 = 1$ is assumed to be invested in all the funds under examination. With these choices, we obtain the following DEA model

$$\max_{\{u,v_i\}} \quad \frac{u\overline{U}_{j_0}}{v_1 C_0 + v_2\sigma_{j_0} + v_3 f^I_{j_0} + v_4 f^E_{j_0}} \tag{3}$$

subject to

$$\frac{u_1\overline{U}_j}{v_1 C_0 + v_2\sigma_j + v_3 f^I_j + v_4 f^E_j} \leq 1 \qquad j = 1, 2, \ldots, n \tag{4}$$

$$u \geq \varepsilon, \tag{5}$$

$$v_i \geq \varepsilon \qquad i = 1, 2, 3, 4, \tag{6}$$

with $j_0 = 1, 2, \ldots, n$.

The DEA performance measure for fund j_0, $I_{j_0,DEA-S}$, is the optimal value of the objective function (3)

$$I_{j_0,DEA-S} = \frac{u^*\overline{U}_{j_0}}{v_1^* C_0 + v_2^*\sigma_{j_0} + v_3^* f^I_{j_0} + v_4^* f^E_{j_0}} \tag{7}$$

and lies in the interval $[0, 1]$.

We have compared the results obtained with such a model with those obtained with the I_{DEA-1} and I_{DEA-cf} models; as can be seen in the last column of table 1, the efficiency set obtained with the I_{DEA-S} model turns out to include the efficiency sets obtained with both I_{DEA-1} and I_{DEA-cf} models, so that the efficient funds keep being efficient. From the mean and average efficiency scores reported in table 1 it is also apparent that the I_{DEA-S} model gives much higher efficiency scores with respect to the other two models.

3 How to define an ethical measure for mutual funds

Let us now turn our attention to the evaluation of the performance of ethical mutual funds.

First of all, in order to evaluate the performance of ethical mutual funds we need to build an ethical measure which can be used as an output variable to be taken into account together with the return indicator.

Various consultant agencies and research institutes analyze the ethical nature of mutual funds. For example, in the 'SRI Funds Service' the European Social Investment Forum (EUROSIF) together with Avanzi rating agency and Morningstar, give some basic information regarding the socially responsible profile of European ethical mutual funds. Such information is organized in various sections; in particular, the funds are analyzed on the basis of the most important questions taken into consideration in order to define negative and positive ethical screening.

Actually, one of the most important strategies applied by socially responsible mutual funds is ethical screening. According to such a strategy, the assets included in the mutual fund portfolios are selected on the basis of social and environmental grounds. The selection can be carried out either with a negative screening, by excluding from the portfolios the assets of the companies with a profile that is bad according to a socially responsible criterion, or with a positive screening, by including in the fund portfolio investments in companies which are selected on the ground of their ethically and socially responsible behaviour.

The most important information on the ethical screening used by the SRI Funds Service takes into consideration a set of features which can be either present or absent in the ethical profile of each fund:

a. **Negative screening issues**: 1. firearms; 2. weapons and military contracting; 3. nuclear energy; 4. tobacco; 5. gambling; 6. human rights and ELO fundamental conventions violations; 7. child labour; 8. oppressive regimes; 9. pornography; 10. alcohol; 11. animal testing; 12. factory farming; 13. furs; 14. excessive environmental impact and natural resources consumption; 15. GMO; 16. products dangerous to health/environment; 17. others.
b. **Positive screening issues**: 1. products beneficial for the environment and quality of life; 2. customers, product safety, advertisement competition; 3. environmental services and technologies; 4. environmental policies, reports, management systems; 5. environmental performances; 6. employees policies, reports, management systems; 7. employees performances; 8. suppliers and measures to avoid human rights violations; 9. communities and bribery; 10. corporate governance; 11. others.

Another important information on the ethical behaviour of mutual funds is the presence or absence of an ethical committee which has the function of defining the guidelines of the socially responsible investments and controlling the actions of the fund management in this respect.

We have used such information in order to define an ethical measure by assigning each ethical feature a weight and then computing their weighted sum.

More precisely, let us consider n mutual funds and let s^N and s^P be the number of negative and positive screening issues taken into account, respectively. Moreover, let s_j^N and s_j^P be the number of negative and positive screening features presented by fund j, with $j = 1, 2, \ldots, n$. Then

$$N_j = \frac{s_j^N}{s^N} \qquad \text{and} \qquad P_j = \frac{s_j^P}{s^P} \tag{8}$$

represent the quota of the positive and negative screening issues which are present in the ethical profile of fund j, respectively. Moreover, let

$$C_j = \begin{cases} 1 & \text{if fund } j \text{ has an ethical committee with full powers} \\ 1/2 & \text{if fund } j \text{ has an ethical committee with partial powers} \\ 0 & \text{if fund } j \text{ does not have an ethical committee.} \end{cases} \tag{9}$$

An ethical measure defined in the real interval $[0, L]$ can be computed as follows:

$$e_j = \omega^N N_j + \omega^P P_j + \omega^C C_j \tag{10}$$

where ω^N, ω^P and ω^C are positive weights assigned to the negative and positive screening and to the ethical committee, respectively, and $L = \omega^N + \omega^P + \omega^C$.

By construction, fund j has a zero ethical measure if and only if it has no ethical profile, so that $e_j = 0$ for non ethical funds.

4 A DEA model for ethical funds with non negative outputs

In Section 3 we have defined a real measure of the ethical level for mutual funds; this measure can be used as an additional output variable in a DEA model. The manner in which the *DEA-S* model (3)–(6) can be extended to handle an ethical objective depends on the actual ethical goal pursuits by investors.

If investors choose the mutual fund in which to invest their money by trying to maximize both the return and the ethical level of the investment simultaneously, then we can resort to the following two-output *DEA-SE* model

$$\max_{\{u_r, v_i\}} \quad \frac{u_1 \overline{U}_{j_0} + u_2 e_{j_0}}{v_1 C_0 + v_2 \sigma_{j_0} + v_3 f_{j_0}^I + v_4 f_{j_0}^E} \tag{11}$$

subject to

$$\frac{u_1 \overline{U}_j + u_2 e_j}{v_1 C_0 + v_2 \sigma_j + v_3 f_j^I + v_4 f_j^E} \leq 1 \qquad j = 1, 2, \ldots, n \tag{12}$$

$$u_r \geq \varepsilon \qquad r = 1, 2 \tag{13}$$

$$v_i \geq \varepsilon \qquad i = 1, 2, 3, 4, \tag{14}$$

which is a direct extension of model (3)–(6).

According to this model, the DEA performance measure for fund j_0, $I_{j_0,DEA-SE}$, is the optimal value of the objective function (11)

$$I_{j_0,DEA-SE} = \frac{u_1^*\overline{U}_{j_0} + u_2^* e_{j_0}}{v_1^* C_0 + v_2^*\sigma_{j_0} + v_3^* f_{j_0}^I + v_4^* f_{j_0}^E}. \tag{15}$$

However, if investors choose the ethical level they desire a priori and then try to maximize the return of their investment by choosing the best mutual fund among all the funds that satisfy the required ethical level, model (11)–(14) is not appropriate. Actually, in this case the output e_j has to be considered as exogenously fixed, beyond the discretionary control of the managers of fund j.

On the other hand, it is known that the presence of an exogenously fixed output has a major consequence in the formulation of a DEA model, as pointed out in [2] and, as concerns the performance of ethical mutual funds, in [4]. In next section we derive a more appropriate model for the case of non negative outputs when the ethical level is exogenously chosen by investors.

5 A DEA model for ethical funds with non negative outputs and exogenously fixed ethical levels

In order to see how the basic DEA model (11)–(14) has to be modified to take into account the presence of an exogenously fixed output, let us observe that this model is equivalent to the following linear programming problem (for the derivation see for example [7]) in output-oriented form

$$\min_{\{u_r, v_i\}} \quad v_1 C_0 + v_2\sigma_{j_0} + v_3 f_{j_0}^I + v_4 f_{j_0}^E \tag{16}$$

subject to

$$u_1\overline{U}_{j_0} + u_2 e_{j_0} = 1 \tag{17}$$

$$-u_1\overline{U}_j - u_2 e_j + v_1 C_0 + v_2\sigma_j + v_3 f_j^I + v_4 f_j^E \geq 0 \quad j = 1, \ldots, n \tag{18}$$

$$u_r \geq \varepsilon \qquad r = 1, 2 \tag{19}$$

$$v_i \geq \varepsilon \qquad i = 1, 2, 3, 4. \tag{20}$$

The DEA performance measure for the ethical fund j_0, $I_{j_0,DEA-SE}$, coincides with the reciprocal of the optimal value of the linear objective function (16).

The dual of this linear problem can be written as

$$\max \quad z_0 + \varepsilon s_1^+ + \varepsilon s_2^+ + \varepsilon \sum_{i=1}^{4} s_i^- \tag{21}$$

subject to

$$\overline{U}_{j_0} z_0 - \sum_{j=1}^{n} \overline{U}_j \lambda_j + s_1^+ = 0 \tag{22}$$

$$e_{j_0} z_0 - \sum_{j=1}^{n} e_j \lambda_j + s_2^+ = 0 \tag{23}$$

$$\sum_{j=1}^{n} C_0 \lambda_j + s_1^- = C_0 \tag{24}$$

$$\sum_{j=1}^{n} \sigma_j \lambda_j + s_2^- = \sigma_{j_0} \tag{25}$$

$$\sum_{j=1}^{n} f_j^I \lambda_j + s_3^- = f_{j_0}^I \tag{26}$$

$$\sum_{j=1}^{n} f_j^E \lambda_j + s_4^- = f_{j_0}^E \tag{27}$$

$$\lambda_j \geq 0 \qquad j = 1, 2, \ldots, n \tag{28}$$

$$s_r^+ \geq 0 \qquad r = 1, 2 \tag{29}$$

$$s_i^- \geq 0 \qquad i = 1, 2, 3, 4 \tag{30}$$

$$z_0 \qquad \text{unconstrained,} \tag{31}$$

where z_0 is the dual variable associated with the equality constraint (17), λ_j (with $j = 1, 2, \ldots, n$) are the dual variables associated with the mutual funds constraints (18) and s_r^+ (with $r = 1, 2$) and s_i^- (with $r = 1, 2, 3, 4$) are the dual variables connected with the output and input weight constraints (19) and (20), respectively.

It is known that the optimal solution of this dual problem enables to identify for each inefficient fund a composite unit made up of a linear combination of the efficient funds, i.e. the funds for which constraint (18) is satisfied as equality and which therefore get an efficiency value equal to 1. Actually, from the complementary conditions of duality in linear programming we have that only the optimal value of the dual variables λ_j associated to these efficient funds can have a strictly positive optimal value, the others being null.

A composite unit is defined as a linear combination of the set of funds $\{ F_1, F_2, \ldots, F_n \}$ considered in the analysis with coefficients given by the optimal values of the dual variables λ_j, namely $\sum_{j=1}^{n} \lambda_j^* F_j$. In a composite unit only the efficient funds can have a strictly positive coefficient, while the coefficients of the other funds are bound to be null.

If we analyze the constraints (22)–(23) of the dual problem when the variables take their optimal value, we can see that the composite units use a level of inputs which is not greater than that employed by fund j_0 and obtains a level of outputs that is not lower than that obtained by fund j_0. In particular, as concerns the output levels we have that

$$\sum_{j=1}^{n} \overline{U}_j \lambda_j^* \geq \overline{U}_{j_0} z_0^* \tag{32}$$

$$\sum_{j=1}^{n} e_j \lambda_j^* \geq e_{j_0} z_0^*, \tag{33}$$

so that the composite units have a level of both the capitalization factor and the ethical indicator that is not lower than that of fund j_0 multiplied by the optimal value z_0^* of the dual variable z_0.

If fund j_0 is radially efficient, then no other fund or combination of funds can increase both outputs without augmenting the value of the inputs and $z_0^* = 1$ (and all the more so for the Pareto-Koopmans efficiency). On the contrary, if fund j_0 is not radially efficient then the optimal value of the objective function of both primal and dual linear problems is greater than 1 and, given the non-Archimedean nature of ε (which is positive and smaller than any positive valued real number), z_0^* will be greater than 1, too. In such a case the composite units give for both the capitalization factor and the ethical indicator a value higher than that of fund j_0.

However, when investors choose the ethical level they desire a priori, we have that a constraint is actually imposed on the fund chosen; indeed, in this case investors choose the fund that maximizes the return of their investment among all the funds that satisfy the required ethical level. Formally, this entails that the ethical level has to be considered as an exogenously fixed output, so that a composite unit is required to have an ethical level not lower than that of fund j_0

$$\sum_{j=1}^{n} e_j \lambda_j^* \geq e_{j_0} \tag{34}$$

and constraint (23) of the dual problem has to be substituted by the following constraint

$$\sum_{j=1}^{n} e_j \lambda_j - s_2^+ = e_{j_0}. \tag{35}$$

Moreover, following the suggestion of Banker and Morey [2], we relax the constraint on the weight u_2 in the primal problem to a pure non negativity constraint; this entails that the coefficient of the slack variable s_2^+ in the objective function of the dual problem vanishes.

The dual problem in the case of an exogenously fixed ethical level can therefore be written as follows

$$\max \quad z_0 + \varepsilon s_1^+ + \varepsilon \sum_{i=1}^{4} s_i^- \tag{36}$$

subject to

$$\overline{U}_{j_0} z_0 - \sum_{j=1}^{n} \overline{U}_j \lambda_j + s_1^+ = 0 \tag{37}$$

$$\sum_{j=1}^{n} e_j \lambda_j - s_2^+ = e_{j_0} \tag{38}$$

$$\sum_{j=1}^{n} C_0\lambda_j + s_1^- = C_0 \quad (39)$$

$$\sum_{j=1}^{n} \sigma_j\lambda_j + s_2^- = \sigma_{j_0} \quad (40)$$

$$\sum_{j=1}^{n} f_j^I\lambda_j + s_3^- = f_{j_0}^I \quad (41)$$

$$\sum_{j=1}^{n} f_j^E\lambda_j + s_4^- = f_{j_0}^E \quad (42)$$

$$\lambda_j \geq 0 \qquad j = 1, 2, \ldots, n \quad (43)$$

$$s_r^+ \geq 0 \qquad r = 1, 2 \quad (44)$$

$$s_i^- \geq 0 \qquad i = 1, 2, 3, 4 \quad (45)$$

$$z_0 \quad \text{unconstrained.} \quad (46)$$

Let us observe that from the dual of the dual problem (36)–(46) we can reconstruct the equivalent fractional programming problem *DEA-SEef*

$$\max_{\{u_r, v_i\}} \quad \frac{u_1\overline{U}_{j_0}}{v_1C_0 + v_2\sigma_{j_0} + v_3f_{j_0}^I + v_4f_{j_0}^E - u_2e_{j_0}} \quad (47)$$

subject to

$$\frac{u_1\overline{U}_j}{v_1C_0 + v_2\sigma_j + v_3f_j^I + v_4f_j^E - u_2e_j} \leq 1 \qquad j = 1, 2, \ldots, n \quad (48)$$

$$u_1 \geq \varepsilon, \quad u_2 \geq 0 \quad (49)$$

$$v_i \geq \varepsilon \qquad i = 1, 2, 3, 4. \quad (50)$$

The DEA performance measure for fund j_0, $I_{j_0,DEA-SEef}$, is the optimal value of the objective function (47) and coincides with the reciprocal of the optimal value of the objective function (36) of the linear dual problem (36)–(46).

If we compare the DEA model for ethical funds (36)–(46) with the exogenously fixed DEA model proposed in Basso and Funari [4], it is apparent that the differences between the two models lie in the expedient used to tackle the case of negative average rates of returns, which has lead to a special choice for the return indicator and to the use of the initial capital as an additional input.

6 Connections among the efficiency measures

We may wonder which relation exists between the DEA scores obtained with the two models for ethical mutual funds (11)–(14) and (47)–(50), and the one obtained with the DEA model (3)–(6) which ignores the ethical objective. The following theorems 1 and 2 answer this question.

Theorem 1. *Let $I_{j_0,DEA-S}$, $I_{j_0,DEA-SEef}$ and $I_{j_0,DEA-SE}$ be the DEA performance measures for fund j_0 obtained by solving the DEA problems (3)–(6), (47)–(50) and (11)–(14), respectively. The following inequalities hold:*

$$I_{j_0,DEA-S} \leq I_{j_0,DEA-SEef} \leq I_{j_0,DEA-SE}. \tag{51}$$

Proof. Let us first prove the inequality $I_{j_0,DEA-S} \leq I_{j_0,DEA-SEef}$. If we compare the DEA fractional programming problems (3)–(6) and (47)–(50), we can observe that problem (3)–(6) can be obtained as a restriction of problem (47)–(50), since it can be obtained from the latter by considering the further constraint $u_2 = 0$. Hence, $I_{j_0,DEA-S}$, which is the optimal solution of the constrained problem (3)–(6), cannot be greater than $I_{j_0,DEA-SEef}$, that is the optimal solution of problem (47)–(50).

Let us now demonstrate the inequality $I_{j_0,DEA-SEef} \leq I_{j_0,DEA-SE}$. Let us consider the dual problem (21)–(31), which is the dual of the linear programming problem (16)–(20) equivalent to problem (11)–(14). Since in both problems (21)–(31) and (36)–(46) the optimal value of the dual variable z_0 is not lower than 1, if we cut the feasible regions of both problems by introducing the additional constraint $z_0 \geq 1$ we do not cut off the optimal solution. In the remaining part of the feasible region, we have $\sum_{j=1}^{n} e_j \lambda_j \geq e_{j_0} z_0 \geq e_{j_0}$ and therefore constraint (23) is more restrictive than constraint (38). As all the other constraints of the feasible regions of problems (21)–(31) and (36)–(46) are equal, we conclude that the feasible region of problem (21)–(31) is a subset of that of problem (36)–(46). Since the difference between the objective functions of the two dual problems is given by εs_2^+ which is lower than any positive real number, given the nature of non-Archimedean infinitesimal of ε, the optimal solution of problem (21)–(31) is lower than or equal to the optimal solution of problem (36)–(46). Hence, for their reciprocal values, which give the DEA performance measures $I_{j_0,DEA-SE}$ and $I_{j_0,DEA-SEef}$, respectively, the reverse inequality holds. □

Another main issue concerns the efficiency measure of the non ethical mutual funds obtained with the models for ethical funds (11)–(14) and (47)–(50). Theorem 2 shows that if the ethical indicator of a fund is equal to 0 the use of these two models does not improve the fund efficiency score.

Theorem 2. *Let j_0 be a mutual fund with ethical measure $e_{j_0} = 0$ and let $I_{j_0,DEA-S}$, $I_{j_0,DEA-SEef}$ and $I_{j_0,DEA-SE}$ be the DEA performance measures obtained by solving the DEA problems (3)–(6), (47)–(50) and (11)–(14), respectively. The following equalities hold:*

$$I_{j_0,DEA-S} = I_{j_0,DEA-SEef} = I_{j_0,DEA-SE}. \tag{52}$$

Proof. The equality $I_{j_0,DEA-S} = I_{j_0,DEA-SEef}$ follows from the observation that when $e_{j_0} = 0$ the DEA fractional programming problem (47)–(50) coincides with problem (3)–(6).

The equality $I_{j_0,DEA-SEef} = I_{j_0,DEA-SE}$ can be proved by observing that when $e_{j_0} = 0$ the feasible regions of problems (21)–(31) and (36)–(46) coincide while their objective functions differ by the quantity εs_2^+ which is the product of a non-Archimedean infinitesimal and a real number and is therefore smaller than any positive real number. The DEA performance measures $I_{j_0,DEA-SE}$ and $I_{j_0,DEA-SEef}$, therefore, coincide. □

7 An analysis of the European market of ethical mutual funds

We have used the DEA models proposed in the previous sections for the evaluation of the performance of ethical mutual funds with non negative outputs in order to analyze the European market of ethical mutual funds.

The analysis refers to the three-year period 31/01/2002 to 31/01/2005 and takes into consideration a large number of ethical funds from western European countries in which the phenomenon of ethical investing is significant.

We have used the 'SRI Funds Service' data base and have included in the analysis all the ethical equity funds for which the data on input and output variables were available during the period investigated. In such a way, a total of 159 ethical equity funds were obtained, domiciled in 11 different countries. The number of ethical mutual funds of equity typology comprised in the study is considerable for Sweden (38 funds), United Kingdom (32 funds), France (27 funds) and Luxembourg (26 funds), while it is less substantial for the other European countries: Belgium (12 funds), The Netherlands (8 funds), Austria (6 funds), Swiss (4 funds), Germany (2 funds), Italy (2 funds) and Dublin (2 funds).

In addition, in order to compare the performance obtained by ethical and non ethical funds, we have included in the set of funds analyzed also a non ethical fund with analogous features for each ethical fund considered, each time one such non ethical fund was offered by the same fund company (source: Morningstar Europe). On the whole, the set consists of 269 equity funds, 159 ethical and 110 non ethical funds.

The input variables considered are the volatility of the fund returns, computed as per cent values on an annual base, the initial and exit fees and the initial capital invested, set equal to 1 for each fund. The output variables are the capitalization factor and the ethical measure (10), computed by using the weights $\omega^N = \omega^P = 2$ and $\omega^C = 1$ (notice that this choice of the weights stresses the screening activity of the ethical funds). Figures 1 and 2 compare the overall behavior of the input and output variables for the sets of ethical and non ethical funds. From these figures we can see that the ethical funds tend to have a slightly higher standard deviation and a slightly lower rate of return (and capitalization factor) than the non ethical funds.

Moreover, as many as 79 % of the funds analyzed exhibit a negative average rate of return in the period considered in our analysis, and for no less that 86 % of the funds the observed excess return is negative. This clearly shows that it is necessary to use a DEA model which is able to cope with such cases.

On the other hand, the negativity of the observed excess return entails the negativity of the widely used Sharpe ratio ([13]) and this can be misleading.

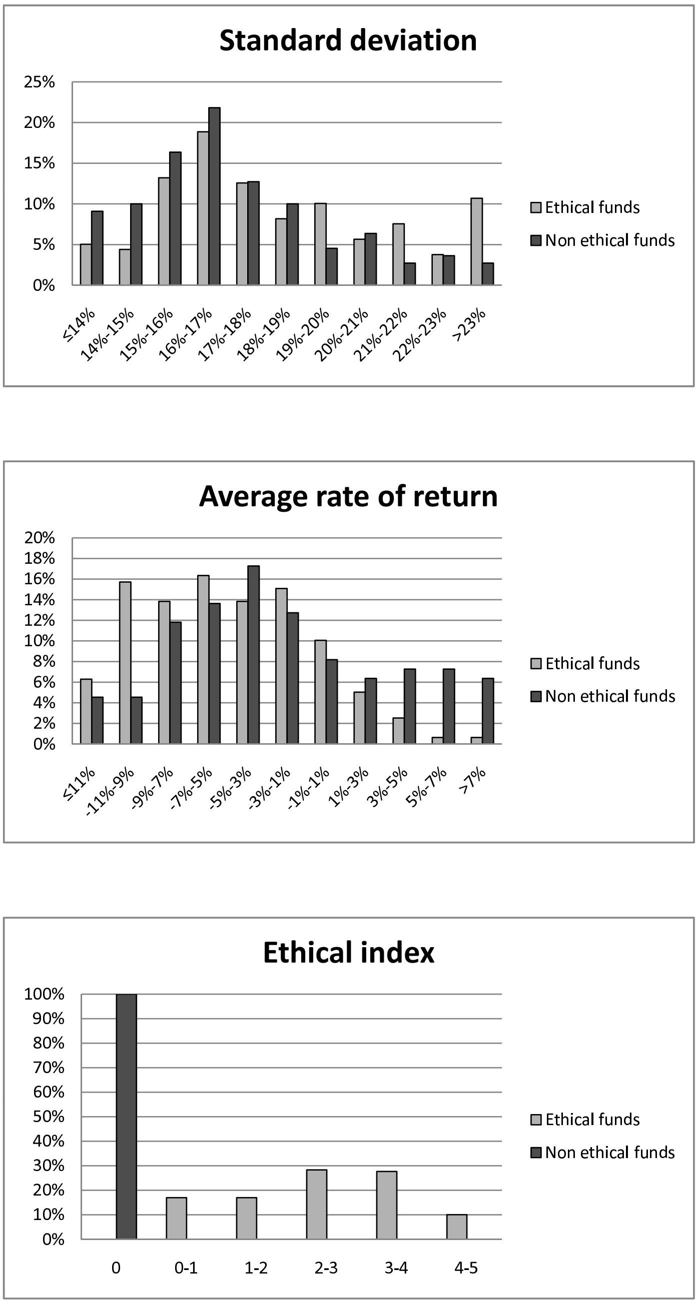

Fig. 1. Histograms of the per cent frequency distributions of the standard deviation, average rate of return and ethical index for the ethical and non ethical European mutual funds.

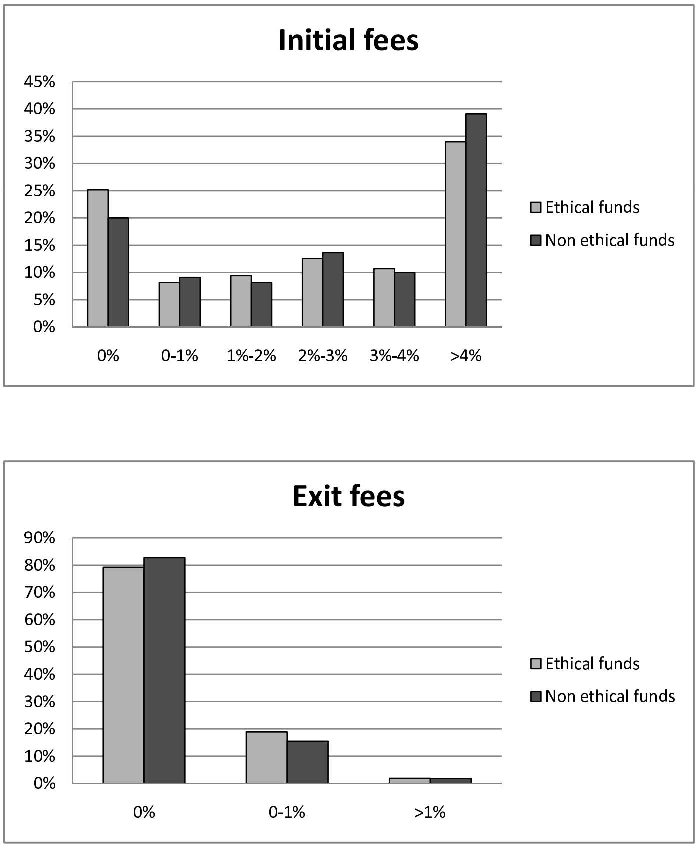

Fig. 2. Histograms of the per cent frequency distributions of the initial and exit fees for the ethical and non ethical European mutual funds.

Indeed, only when the excess return is positive, the (positive) value of the Sharpe ratio decreases with the risk indicator σ_j, as we would expect for a performance indicator; on the contrary, when the excess return is negative, the (negative) value of the Sharpe ratio increases with the value of the standard deviation.

For each fund in the set of funds analyzed we have computed the DEA performance measures I_{DEA-S}, I_{DEA-SE} and $I_{DEA-SEef}$.

The efficient funds are 5 for the *DEA-S* model, 4 non ethical funds and only 1 ethical fund (Ohman Etisk Index Pacific). The number of efficient ethical funds increases considerably when the ethical objective is taken into account; actually, in the ethical models *DEA-SEef* and *DEA-SE* a good 13 ethical funds turn out to be efficient. Of course, as stated by theorem 2, the non ethical funds do not change their efficiency score when the ethical models are used and neither does the number of efficient funds.

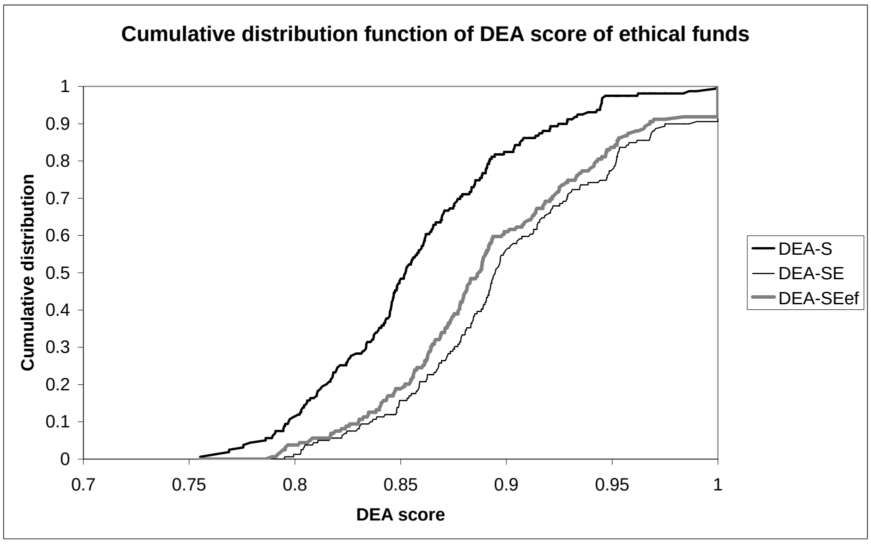

Fig. 3. Empirical cumulative distribution function of the DEA efficiency measures I_{DEA-S}, I_{DEA-SE} and $I_{DEA-SEef}$ for the European ethical funds analyzed.

These funds are efficient in the sense of Pareto-Koopmans, namely they obtain a DEA score equal to 1 and for them it is not possible to reduce any input or increase any output without worsening the value of some other inputs or outputs; they can be identified by solving a convenient two-phase linear programming problem equivalent to the original DEA problem (see for example [8]). For the *DEA-SE* model, we also have 2 funds which have a score equal to 1 but are not Pareto-Koopmans efficient, since not all of their slack variables are equal to 0.

In order to analyze how the efficiency measure changes with the DEA model used, figure 3 compares the empirical cumulative distribution function of the DEA efficiency measures I_{DEA-S}, I_{DEA-SE} and $I_{DEA-SEef}$ obtained for the ethical funds analyzed with the three models considered. By theorem 1, the cumulative distribution function of model *DEA-S* lies above that of model *DEA-SEef*, and the latter lies above that of model *DEA-SE*. Figure 3 shows that the more considerable increase in the DEA score, which corresponds to the more notable shift rightwards in the cumulative distribution function, takes place when the ethical objective is taken into account, while the difference is much slighter between the two ethical models. Actually, the average DEA score of the ethical funds is equal to 0.857 for the *DEA-S* model, 0.894 for the *DEA-SEef* model and 0.903 for the *DEA-SE* one. As regards the non ethical funds, their average DEA score is equal to 0.884, which is higher than the average score of the ethical funds when the ethical measure is not considered, while it is lower than that of

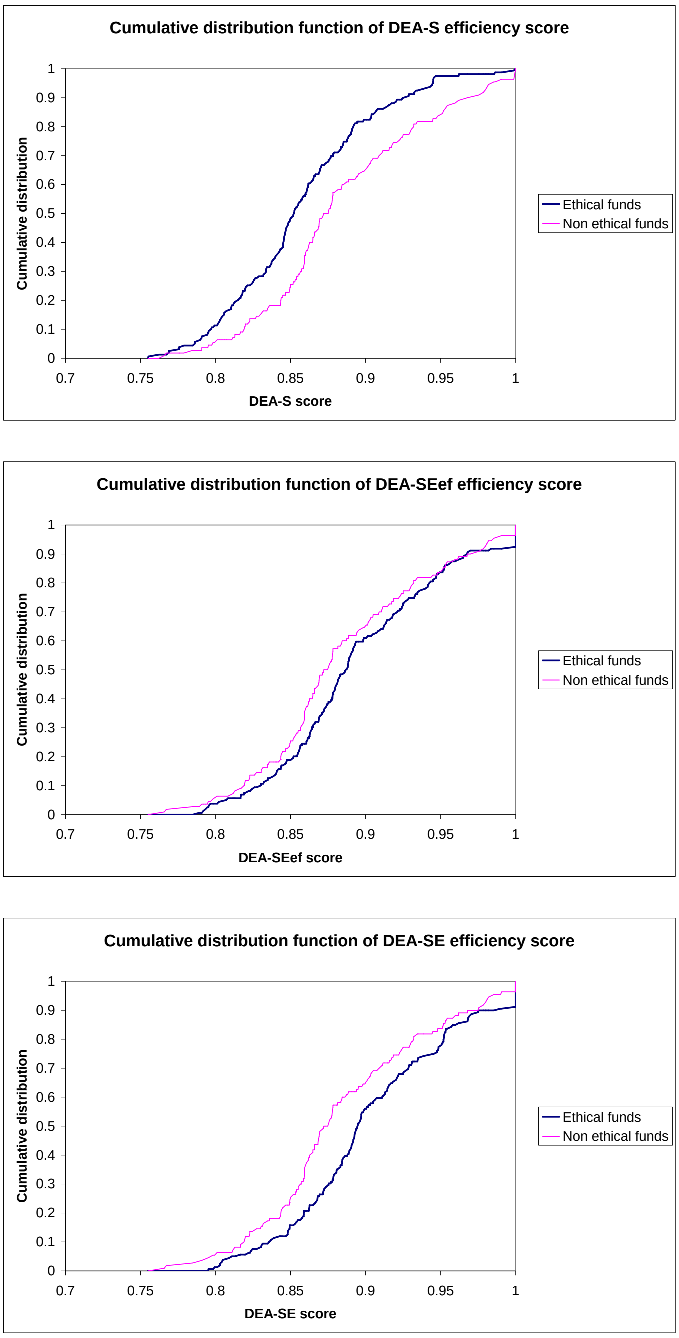

Fig. 4. Comparison of the empirical cumulative distribution functions of the DEA efficiency measures I_{DEA-S}, $I_{DEA-SEef}$ and I_{DEA-SE} for the ethical and non ethical European mutual funds.

the ethical funds in the models that take the ethical measure into account. The comparison of the DEA efficiency measures between ethical and non ethical funds is highlighted in figure 4, which compares the empirical cumulative distribution functions of the DEA efficiency measures I_{DEA-S}, I_{DEA-SE} and $I_{DEA-SEef}$ for the ethical and non ethical mutual funds. It can be noticed that the non ethical funds obtain a sensibly higher efficiency score than the ethical funds when the only output variable taken into consideration is the capitalization factor, i.e. in the *DEA-S* model. The improvement in the efficiency score obtained by the ethical funds when the ethical indicator is included in the set of outputs causes the dominance to reverse in the ethical models, so that with these models the highest efficiency scores are obtained by the ethical funds.

Table 2 shows the efficient sets obtained with the three DEA models applied; for each efficient fund the table also reports the number of times the fund appears in the reference set of the inefficient funds. As a consequence of theorem 1, the efficient set of the *DEA-S* model is included in the efficient set of the *DEA-SEef* model and this is included in the efficient set of the *DEA-SE* model, in turn; actually, from table 2 we may notice that in our empirical study the efficient sets of the two models that take the ethical measure into account coincide. We may also observe that for these two models the frequencies in the reference sets are quite similar.

Table 2. Frequency of the efficient funds in the reference set of the inefficient funds for the *DEA-S*, *DEA-SE* and *DEA-SEef* models; a dash indicates that the fund is not efficient; SE=Sweden, UK=United Kingdom, LU=Luxembourg, BE=Belgium, NL=The Netherlands, CH=Switzerland.

Funds in the efficient set	Ethical	Country	*DEA-S*	*DEA-SE*	*DEA-SEef*
Ing (L) Invest Sustainable Growth P	•	LU	–	0	0
Pioneer Funds Global Ethical Equity F	•	LU	–	4	4
ING Duurzaame Rendement Fonds	•	NL	–	0	0
Postbank Duurzaame Aandelenfonds	•	NL	–	0	0
Triodos Meerwaarde Aandelenfonds	•	NL	–	4	4
F&C Stewardship Income F.Sh.Cl.1	•	UK	–	103	105
Henderson Global Care Income Fund	•	UK	–	25	10
Aktie-Ansvar Sverige	•	SE	–	18	19
KPA Etisk Aktiefond	•	SE	–	30	26
Skandia Cancerfonden	•	SE	–	15	15
Ohman Etisk Index Pacific	•	SE	54	55	57
Raiffeisen-Fonds Futura Global Stock A	•	CH	–	0	0
Raiffeisen-Fonds Futura Swiss Stock A	•	CH	–	48	39
Athena Global Opportunities C		BE	164	80	79
Jupiter UK Smaller Companies Fund		UK	178	110	125
Skandia Smabolag Sverige		SE	174	133	146
Robur Smabolagsfond Norden		SE	51	46	48

8 Conclusions

In this paper we have proposed an extension to some DEA models for the evaluation of mutual fund performance which enables to tackle the problem of negative data that often occurs with the DEA models suggested by the literature in slump periods of the business cycle.

The first extension leads to a basic model for the measurement of the performance of general mutual funds with a pure investment goal. The second extension considers a model, applicable to ethical investments, which includes also an ethical objective among the outputs of the mutual funds. The third extension is applicable to ethical mutual funds in the case in which investors fix a priori the ethical level desired and maximize the investment return by considering an ethical level as an exogenously fixed output.

The investigation carried out on data from the European market of ethical mutual funds shows that the highest values of the performance measure are generally obtained by the ethical funds if we use a model that takes the ethical level into account. On the contrary, the performance score obtained by the ethical funds is generally lower than that of the non ethical funds when the only output variable taken into consideration is the capitalization factor.

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