A reserve risk model for a non-life insurance company

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Abstract. The aim of the study is the assessment of the reserve risk for a non-life insurance company that writes two different lines of business; the focus is on the determination of the current estimates incorporated in the measurement of the liabilities (in some jurisdictions referred to as technical provisions or actuarial reserves) of insurance contracts (without risk margins) through a Bayesian stochastic methodology and on the possible methods for the determination of risk margins above current estimates appropriate for the measurement of the liabilities for insurance contracts for regulatory and general purpose financial reports. A new formula for the calculation of the latter is proposed, analyzing its potential advantages in comparison with the existing ones. In order to determine the risk margin the reserve risk is calibrated on a one year time horizon considering both the case of independence between lines of business and the case of dependence.

Keywords. Current estimate, risk margin, Bayesian stochastic methods, Markov Chain Monte Carlo methods.

M.S.C. classification. 62P05, 65C05, 91B30.
J.E.L. classification. C11, C15, G22.

1 Introduction

The aim of the study is the assessment of the reserve risk for a non-life insurance company that writes two different lines of business (LoBs). First of all the stochastic methodology for the assessment of the reserve for a single LoB is described: among the existing solutions the choice is a Bayesian model which represents an improvement of one of the most known deterministic models, the Chain-Ladder method. The implementation of the Bayesian model is possible by using Markov Chain Monte Carlo (MCMC) techniques, specifically the Gibbs algorithm, as described in Gilks, [5], and Scollnik, [13]. Considering a multiLoB insurance company the problem is the determination of a single distribution of the reserve referred to the LoBs jointly, i.e. the aggregate reserve. For this scope the methodology followed requires the use of copula functions. In the specific case the marginal distributions are the probability distribution functions of the reserve of each single LoB. The statistic used to determine the correlation between LoBs is the average cost of LoB. The average cost of LoB is one of the possible alternatives to determine the correlation between LoBs. The ideal approach should consider the two historical series of the amounts booked in the past for the reserves of the two LoBs, adjusting the data for the past inflation and other external effects, for example the settlement policy. Though this approach is not always applicable, especially when the Insurance Company has not sufficient historical data. In this work the choice is to estimate the dependence on the average costs: this choice is due to the fact that the claim reserves are amounts that the Insurance Companies book for the eventual payment of incurred claims. To this end the claim frequency is not so important since the number of claims is known (with the exception of IBNR), while the average costs have a relevant economic role. In order to determine the parameter that represents the dependence structure of each copula the canonical maximum likelihood method is followed. The point estimate is used to determine a preference on the goodness of the fit. In order to evaluate the distribution of the dependence parameter for each copula a simulation algorithm is used, specifically the Metropolis-Hastings algorithm in case of independent sampling. Having determined the distribution of dependence parameter the aggregate reserve is assessed varying the dependence structure and the corresponding dependence parameter. In the last part of the work, using the results obtained, the economic impact of the reserve risk is assessed. The reserve risk is part of the underwriting risk, that is intended as the risk of loss, or of adverse change, in the value of insurance liabilities due to inadequate pricing and provisioning. It should therefore capture the risk arising over the occurrence period (a period of one year over which an adverse event occurs) and their financial consequences over the whole run-off of the liabilities. Two different economic measures are used, the solvency capital requirement (SCR) and the risk margin. The solvency capital requirement corresponds to the economic capital a (re)insurance undertaking needs to hold in order to limit the probability of ruin to 0.5%; it is calculated using Value-at-Risk techniques: all potential losses, including adverse revaluation of assets and liabilities over the next 12 months are to be assessed. The SCR reflects the true risk profile of the undertaking, taking account of all quantifiable risks. The risk margin is the financial cost of uncertainty of liabilities over the whole run-off. The risk margin ensures that the overall value of technical provisions is equivalent to the amount a (re)insurance undertaking would expect to have to pay today if it transferred its contractual rights and obligations immediately to another undertaking. The cost of capital (CoC) method is used to asses the risk margin. The CoC relies on a projection of the Solvency Capital Required to face potential adverse events until the last payment of liabilities over the whole run-off of the reserves. A new proxy for the evaluation of the risk margin is presented that captures the impact of the correlations between LoBs. The paper is consistent with the most recent existing literature that treats these themes, both the assessment of the reserving risk with a one year horizon approach (Merz, [10], [11]) and the assessment of the parameter variability in dependency structures using MCMC techniques (Borowicz, [2]).

2 The current estimate in a Bayesian Framework

In the framework of the Solvency II Project, the European Commission requested the Committee of European Insurance and Occupational Pensions Supervisors to establish well defined solvency and supervisory standards in order to allow a convergent and harmonized application across EU of the general prudential principles in the determination of the insurance technical provisions and the required solvency capitals. In the Solvency II draft Directive framework, [3], the technical provisions have the following definition: “The value of technical provisions shall be equal to the sum of a best estimate and a risk margin; the best estimate shall be equal to the probability-weighted average of future cash flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure; the risk margin shall be such as to ensure that the value of the technical provisions is equivalent to the amount insurance and reinsurance undertakings would be expected to require in order to take over and meet the insurance and reinsurance obligations”. Current estimates have sometimes been referred to as “best estimates”, although the latter term has sometimes also been used to represent the estimate of the most likely possible (modal) outcome rather than the estimate of the probability-weighted expected (mean) value that will be discussed here and that most faithfully represents the current assessment of the relevant cash flows. Such estimates reflect unbiased expectations of the obligation at the report date and are determined on a prospective basis. A current estimate represents the expected present value of the relevant cash flows. In the case where the present value is based on a range of discount rates, it is appropriate to estimate the probability-weighted expected present value of these cash flows. The assumptions used to derive a current estimate reflect the current expectation based on all currently available information about the relevant cash flows associated with the financial item being measured. These expectations involve expected probabilities and conditions (scenarios) during the period in which the cash flows are expected to occur. An assessment of expected future conditions is made rather than blindly applying recent historical or current experience. Although historical or current experience is often the best source from which current expectations of future experience can be derived for a particular portfolio, current estimates of cash flows should not automatically consist of a reproduction of recent experience. In addition, although the observed experience might be relevant to the portfolio as it existed during the observation period, the current portfolio for which estimates are being made may differ in several respects – in many cases, it could be argued that the current portfolio is usually different than the observed portfolio.

Probabilities specify the degree of our belief in some proposition(s) under the assumption that some other propositions are true. The conditioning propositions have to include, at least implicitly, the information used to determine the probability of the conditioned proposition(s). Probability is a relation between conditioned hypothesis and conditioning information - it is meaningless to talk about the probability of a hypothesis without also giving the evidence on which that probability value is based. Bayes' Theorem uses conditional probabilities to reflect a degree of learning. It is central to model empirical learning both because it simplifies the calculation of conditional probabilities and because it clarifies significant features of the subjectivist position. Learning is a process of belief revision in which a "prior" subjective probability P is replaced by a "posterior" probability Q that incorporates newly acquired information. This process proceeds in two stages: first, some of the subject's probabilities are directly altered by experience, intuition, memory, or some other non-inferential learning process; second, the subject "updates" the rest of his/her opinions to bring them into line with his/her newly acquired knowledge. Let $Y[i,j]$ denote the claim amounts paid by the insurance company with a delay of $j-1$ years for accidents reported in the year i , with $i,j=1,...,n$ (where n represents the number of different generations). The value of j is commonly known as the development period. Let $Z[i,j]$ denote the cumulative claim amount for accidents reported in the year i with a delay of $j-1$ years or less. For convenience, it is assumed that the observed data is in the traditional upper triangular form such that $Y[i,j]$ and $Z[i,j]$ are observed for $i=1,...,n$ and $j=1,...,n-i+1$, and unobserved elsewhere. Define the single cell development factor $DF[i,j]$ as

$$DF[i,j] = \frac{Z[i,j+1]}{Z[i,j]}, \tag{1}$$

for $i=1,...,n$ and $j=1,...,n-1$. At the end of reporting year n , these factors are only observed for $i=1,...,n-1$, with $j=1,...,n-i$. Then these estimated development factors are used, in conjunction with (1), to develop estimates of the cumulative claim amounts in the lower triangle and, hence, of the missing incremental claim amounts and the loss reserve. There are many possible ways in which to construct estimates of the missing single cell development factors in each column. One of the most popular set of estimates, known as the volume weighted development factors, is given by:

$$WDF[j] = \frac{\sum_{i=1}^{n-j} Z[i,j+1]}{\sum_{i=1}^{n-j} Z[i,j]}. \tag{2}$$

Observe that the volume weighted development factors are weighted averages of the single cell development factors, with the cumulative claim amounts appearing in the denominator of the latter used as the weights involved in the calculation of the former. The single cell development factors tend to be similarly valued, given the development year j. Thus, moving to a Bayesian framework means specifying stochastic models with equal means for single cell development factors sharing a common development year. Normal models are assumed, although others could be entertained (e.g., gamma or lognormal). Thus,

$$DF[i,j] \sim N\left(\theta_j, \frac{1}{\tau_{i,j}}\right). \tag{3}$$

for $i = 1, ..., n$ and $j = 1, ..., n-1$. Observe that negative incremental claims are permitted by this model, since $DF[i,j]$ may be less than 1. The second parameter appearing in this normal distribution is a precision, or inverse variance, parameter. The precision parameters $\tau_{i,j}$ may be modelled in a variety of ways (for example they can be set equal to a common value for all values i and j, i.e. τ). Possibly, the parameters may be scaled by relevant weights, where the weights could be related to the written premium associated with the different years of origination, or perhaps to the number of claims associated with the different years of origination that settled in the latest development period (for example). Next, it is supposed that the underlying parameters $\theta_{i,j}$ are drawn from a common normal distribution, i.e. :

$$\theta_j \sim N\left(\mu_\theta, \frac{1}{\tau_\theta}\right), \tag{4}$$

for $j = 1, ..., n-1$. The remaining parameters μ_θ, τ_θ, and τ must be assigned prior density specifications in order to complete the definition of a full (i.e., fully specified) probability model. In particular:

$$\tau \sim \Gamma\left(a, b\right), \tag{5}$$

$$\mu_\theta \sim N\left(c, d\right), \tag{6}$$

$$\tau_\theta \sim \Gamma\left(e, f\right). \tag{7}$$

The parameters a, b, c, d, e, f are estimated on the lob historic experience. The Bayesian analysis of this model yields the posterior distribution (i.e., not just point estimates) for all unknown model parameters. This includes the posterior predictive distribution of the unobserved claim and cumulative claim amounts (i.e., the reserves). The implementation of the Bayesian model is possible by using Markov Chain Monte Carlo (MCMC) techniques, specifically the Gibbs algorithm. (Gilks, [5], Scollnik, [13]).

3 The aggregate current estimate

Considering a multiLoB insurance company the problem is the determination of a single distribution of the reserve referred to the LoBs jointly, i.e. the aggregate reserve. For this scope the methodology followed requires the use of copula functions. Copulas have become a popular multivariate modeling tool in many fields where the multivariate dependence is of great interest and the usual multivariate normality is in question. A copula is a multivariate distribution whose marginals are all uniform over $(0,1)$. For a p-dimensional vector U on the unit cube, a copula C is

$$C(u_1,...,u_p) = Pr\left[U_1 \le u_1,...,U_p \le u_p\right], \tag{8}$$

Combined with the fact that any continuous random variable can be transformed to be uniform over $(0,1)$ by its probability integral transformation, copulas can be used to provide multivariate dependence structure separately from the marginal distributions. Copulas first appeared in the probability metrics literature. Let F be a p-dimensional distribution function with margins $F_1,...,F_p$. Sklar, [14], first showed that there exists a p-dimensional copula C such that for all x in the domain of F,

$$F(x_1,...,x_p) = C\left\{F_1(x_1),...,F_p(x_p)\right\}. \tag{9}$$

The last two decades witnessed the spread of copulas in statistical modelling. Joe, [9], and Nelsen, [12], are the two comprehensive treatments on this topic. A frequently cited and widely accessible reference is Genest and MacKay, [4], titled “The Joy of Copulas”, which gives properties of an important family of copulas, Archimedean copulas. Given a dataset, choosing a copula to fit the data is an important but difficult problem. The true data generation mechanism is unknown, for a given amount of data, it is possible that several candidate copulas fit the data reasonably well or that none of the candidate fits the data well. When maximum likelihood method is used, the general practice is to fit the data with all the candidate copulas and choose the ones with the highest likelihood. Suppose that we observe n independent realizations from a multivariate distribution, $\left\{(X_{i1},...,X_{ik})^T : i = 1,...,n\right\}$. Suppose that the multivariate distribution is specified by k margins with cdf F_i and PDF f_i, $i = 1,...,k$ and a copula with density c. Let λ be the vector of marginal parameters and α be the vector of copula parameters. The parameter vector to be estimated is $\theta = \left(\lambda^T, \alpha^T\right)^T$. The loglikelihood function is:

$$l(\theta) = \sum_{i=1}^{n} \log c\left\{F_1(X_{i1};\lambda),...,F_{1k}(X_{ik};\alpha)\right\} + \sum_{i=1}^{n}\sum_{j=1}^{k} \log f_i(X_{ij};\lambda). \tag{10}$$

The ML estimator of θ is $\hat{\theta}_{MLE} = \arg\max_{\theta\in\Theta} l(\theta)$, where Θ represents the parameter space. Actuaries face a difficult task when estimating the parameter values for their chosen copula. In insurance, historical data is often limited. The parameter uncertainty can be significant for small data sets. Using fixed parameters in the copula for claims could lead to misestimation of risk, as there may not be enough historical data for the effect of dependence to become apparent. Common methods of assessing parameter uncertainty include “classical” statistical methods, such as the asymptotic normality of maximum likelihood estimates, and empirical approaches, such as non-parametric bootstrapping. However, these methods have drawbacks when applied to small samples, as is typical in insurance. An alternative general framework for the analysis of parameter uncertainty which does not have such problems is the use of Bayesian methods, in conjunction with Markov Chain Monte Carlo (MCMC) techniques, [5]. This is the followed approach.

4 The risk margin

Insurance obligations are, by their very nature, uncertain. The insurance industry exists to purchase uncertainty from policyholders by transferring at least part of this uncertainty for a price. Measurement of liabilities for insurance contracts is currently under discussion by the International Accounting Standards Board (IASB). The current likely measurement direction of the IASB is based on an exit value, i.e. the amount an insurer would expect to pay or receive at the current date if it transferred its outstanding rights and obligations under a contract to another entity. When deep liquid observable markets exist for financial instruments (such as for many financial assets), the observed exit price already provides an investor with an expected return sufficient for compensation for the risks in that investment relative to alternative investments. In this paper the market price includes both a current estimate of expected cash flows and a risk margin in excess of that amount. If there were a deep liquid market for insurance obligations, the observed market price for an insurance obligation would constitute the exit price. However, as no deep liquid market currently exists for insurance obligations, a model must be constructed that can produce exit values. In putting this methodology into practice, it is assumed that a rational transferee would require something above the current estimate (even if transferor and transferee were to agree perfectly on the level of the current estimate). Otherwise, the transferee would expect to receive nothing for taking on the risk if everything does not work out as expected. The amount, the margin over current estimate, can therefore be regarded as an additional amount "for uncertainty". It therefore can also be regarded as a compensation for the transferee for the risk of taking on an obligation to pay uncertain cash flows. In addition to serving as an element of the exit price, a risk margin makes it possible to absorb reasonable volatility in experience. If experience is more favorable than that assumed in the current estimate, without risk margins, the release of the excess risk margin creates a "profit" that serves as a reward for the investor that has taken the risk; if experience is worse than expected, the risk margin covers some part of the expected losses, also considering that is also a chance of achieving profits. Normally, a purchaser will not be willing to assume a risky obligation unless its expected reward for doing so not only covers the expected costs, but a margin for risk has been provided as well. Three basic approaches (sometimes referred to as methods), or more appropriately, families of approaches, (see IAA, [6], [7]) of determining risk margins have been used in the past:

1. Explicit assumption approaches. These risk margin methods use "appropriate" margins for adverse deviation on top of realistic "current estimate" assumptions.

2. Quantile methods. These risk margin methods express uncertainty in terms of the excess of a percentile (quantile) for a given confidence level above the expected value for a given period, such as the lifetime of the coverage.
3. Cost of capital methods. These risk margin methods are determined based on the cost of holding the capital needed to support the obligation.
   The cost of capital method is based on the explicit assumption that, at each point in time, the risk margin must be sufficient to finance the (solvency) capital otherwise a transferee will be unwilling to pay less than an amount that would fund future capital requirements. Reflection of the estimated current and future economic capital needs of a potential transferee ensures that the amount paid for the transferee for risk provides for the entire risk that will affect the purchaser. In contrast, the quantile and explicit assumption methods do not explicitly reflect current or future required capital. Having fixed the cost of holding the capital (eg. $CoC = 6\%$), the risk margin at the valuation date ($t = 0$) is determined as follows:

$$RM_0 = \sum_{t=1}^{n-1} \frac{CoC \cdot SCR_{t-1}}{(1 + i(0,t))^t}, \tag{11}$$

   where SCR is the solvency capital requirement, $i(0, t)$ is the interest rate.

5 The reserve risk

In the Solvency II draft Directive framework the Solvency Capital Requirement has the following definition: "The SCR corresponds to the economic capital a (re)insurance undertaking needs to hold in order to limit the probability of ruin to 0.5%, i.e. ruin would occur once every 200 years. The SCR is calculated using Value-at-Risk techniques, either in accordance with the standard formula, or using an internal model: all potential losses, including adverse revaluation of assets and liabilities over the next 12 months are to be assessed. The SCR reflects the true risk profile of the undertaking, taking account of all quantifiable risks, as well as the net impact of risk mitigation techniques." Within this framework, the reserve risk is defined as a part of the underwriting risk, as follows: "Underwriting risk means the risk of loss, or of adverse change in the value of insurance liabilities, due to inadequate pricing and provisioning". If we apply this framework to the reserve risk (see IAIS, [8]), the concept of time horizon should distinguish between a period of one year over which an adverse event occurs, i.e. "shock period", and a period over which the adverse event will impact the liabilities, i.e. the "effect period". In any case the reserve risk should capture the risks arising over the occurrence period and their financial consequences over the whole run-off of liabilities (for example, a court judgement or judicial opinion in one year – the shock period – may have permanent consequences for the value of claims and hence will change the projected cash flows to be considered over the full run-off of liabilities – the effect period). To illustrate the concept of a one year horizon year, let's consider the following example. The goal is to assess the reserve risk at $31.12.N$ over a one year horizon, from the triangulation of losses over 12 underwriting years $[Uw\,(N-11)\,;Uw\,(N)]$. Figure 1 is divided into 4 areas (A, B, C, D):

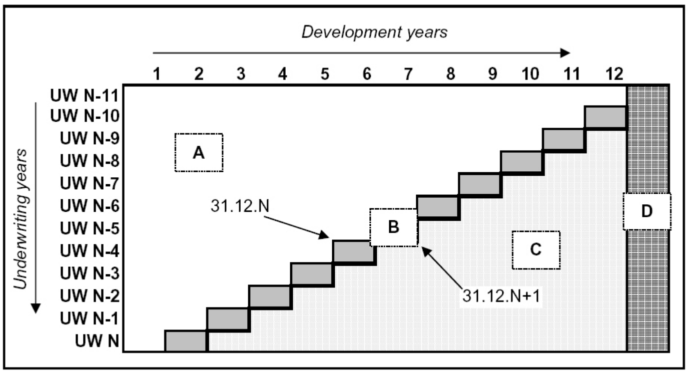

Fig. 1. Graphical representation of a non-life technical liability

A: This area contains the available data/information at $31.12.N$ to assess the reserves at $31.12.N$ (Noted R_n).

B: This area (soft grey) corresponds to a one year period beyond $31.12.N$. This area represents the "shock period". At the end of the shock period (i.e. at $31.12.N+1$), it will be possible to revise R_n a posteriori considering: - the real payments of losses (noted P_{n+1}) over the period $[01.01.N+1; 31.12.N+1]$, - the valuation of reserves at $31.12.N+1$ (noted R_{n+1}) regarding the available information until $31.12.N+1$ for the underwriting years $[Uw\,(N-11)\,;Uw\,(N)]$. The reserve risk at $31.12.N$ measures the uncertainty of the valuation of reserves calculated at $31.12.N$ regarding the additional information over the period $[01.01.N+1; 31.12.N+1]$ that could change this valuation at $31.12.N+1$ (The reserves at $31.12.N+1$ do not include the liabilities related to the underwriting year $N+1$. Indeed the risk associated with this underwriting year is captured in the premium risk). The reserve risk captures the difference between $[P_{n+1}+R_{n+1}]$ and R_n .

C: Under the Solvency II framework and to calculate the reserve risk, this area represents the effect period beyond the shock period. This area contains additional information that could lead to revision of the reserves beyond $31.12.N+1$. This additional information should not be taken into account. The use of the area C should be limited to the assessment of the financial consequences of the adverse events arising during the shock period.

D: This area contains the ultimate costs. These costs are used to assess the risk capital with a VaR methodology. The most usual actuarial methodologies are not consistent with the Solvency II framework since they capture all the adverse events arising beyond the one year horizon. Within the Solvency II framework, it should not be a surprise that some long tail business where adverse movements in claims provisions emerge slowly over many years require less solvency capital than some short tail business exposed to catastrophe risks (for instance).

The uncertainty measurement of reserves in the balance sheet (called risk margin in the Solvency II framework) and the reserve risk do not have the same time horizon. It seems important to underline this point because it may be a source of confusion when the calibration is discussed. The risk margin captures uncertainty over the whole run-off of liabilities. The Solvency II draft Directive framework provides a definition of the risk margin: "The risk margin ensures that the overall value of the technical provisions is equivalent to the amount (re)insurance undertakings would expect to have to pay today if it transferred its contractual rights and obligations immediately to another undertakings; or alternatively, the additional cost, above the best estimate of providing capital to support the (re)insurance obligations over the lifetime of the portfolio" For non-life liabilities (which are non-hedgeable in general) the risk margin is the financial cost of uncertainty of liabilities over the whole run-off giving that this uncertainty is calibrated through the solvency filter: "Where insurance and reinsurance undertakings value the best estimate and the risk margin separately, the risk margin shall be calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the lifetime thereof." Suppose the risk margin is assessed with the cost of capital (CoC) methodology. The level of the CoC relies essentially on the reserve risk calibration. If the reserve risk is over calibrated (i.e. for instance a calibration over the whole run-off of the reserves), the CoC methodology multiplies the level of prudence.

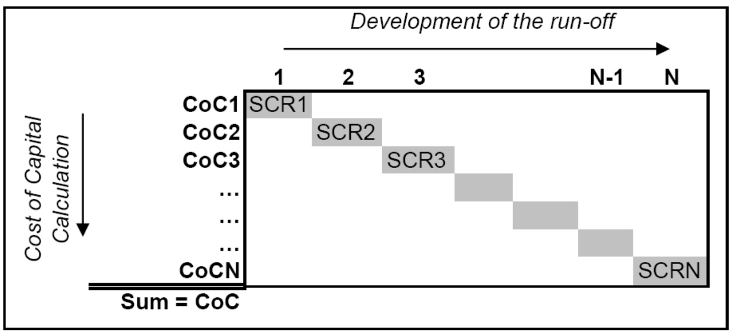

Fig. 2. Cost of capital calculation if the reserve risk is assessed over a one year horizon

For each year horizon, the CoC captures the cost of providing own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the run-off. If the duration of the run-off is N years, the CoC embeds N SCR valuations.

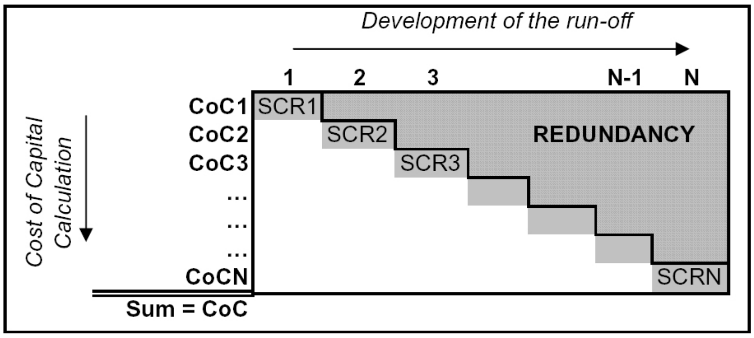

Fig. 3. Cost of Capital calculation if the reserve risk is assessed over the whole run-off

If the reserve risk is calibrated over the whole run-off, or, broadly speaking, if the reserve risk is over calibrated, the CoC creates undue layers of prudence with a leverage effect (see the N "clusters of risks" in Figure 2 versus $N(N+1)/2$ "clusters of risks" in Figure 3).

6 A possible alternative

The Cost of Capital method for the assessment of the risk margin relies on a projection of the Solvency Capital Required to face potential adverse events until the last payment of liabilities, i.e. over the whole run-off of the reserves. Among the problems that can arise in the assessment two have to be considered inevitably: the projection of the capital requirement in future years and the double counting of the risk margin in the approach chosen. One of the possibilities for the calculation is based on the following formula:

$$RM_0 = \sum_{t=1}^{n-1} CoC \cdot SCR_0 \cdot \frac{CE_{t-1}}{CE_0} \cdot \max\left(1, \ln\left(1+\gamma_{t-1}\right)\right) \cdot \frac{1}{\left(1+i\left(0,t\right)\right)^t}, \quad (12)$$

where RM represents the risk margin, CoC the cost of capital, SCR the solvency capital requirement, CE the current estimate, $i(0,t)$ the interest rate, $\gamma_t = \frac{CV(Res_t)}{CV(Res_0)}$ the ratio between coefficients of variation of the random variable Res, which represents the outstanding claim reserve. The capital requirement is determined as follows:

$$SCR_0 = VaR^{99.5\%}\left(Res_0\right) - RM_0 - CE_0, \quad (13)$$

where $VaR^{99.5\%}(Res_0)$ represents the Value at Risk at the valuation date of the outstanding claim reserve at a 99.5% confidence level over a one-year time horizon. Substituting the (13) in (12) the risk margin becomes :

$$RM_0 = \frac{CoC \cdot \left(VaR^{99{,}5\%}(Res_0) - CE_0\right) \cdot ProFact}{1 + CoC \cdot ProFact}. \tag{14}$$

where

$$ProFact = \sum_{t=1}^{n-1} \frac{CE_{t-1}}{CE_0} \cdot \max\left(1, \ln\left(1+\gamma_{t-1}\right)\right) \cdot \frac{1}{\left(1+i\left(0,t\right)\right)^t}. \tag{15}$$

The assessment of the risk margin through (14) has some advantages:

- the solvency capital requirement follows the underlying driver, i.e. the current estimate;
- the formula considers that the variance increases as the time passes and consequently the SCR should increase as well, as the variance is a risk measure;
- the future variance of the current estimate is over estimated at the valuation date: this is due to the lack of information on the development factors for the extreme development years. The increase is mitigated through the use of the function;
- the double counting of risk margin both in the fair value and in the capital requirement is eliminated;
- the formula considers the real variance and the real Value-at-Risk of the current estimate instead of approximations and simplifications. When evaluating the aggregate current estimate, the coefficient of variation is determined considering the correlation between LoBs.

7 Numerical results

This paragraphs shows the results obtained through the methods described in the paper. The initial data set is represented by the run-off triangles of incremental payments of two distinct lines of business: Motor, other classes (LoB 3, Table 1) and Motor, third party liability (LoB 10, Table 2). The risk free interest rates adopted in the discounting[1] are reported in Table 3. Table 4 and Table 5 report the values of the current estimate, the risk margin, the risk capital (the risk covered is the reserve risk, the Solvency Capital Requirement covers also other risks), comparing different possibilities described in literature. The different columns of the tables represent different way of calculations:

- I (method) : the risk margin is given by the 75% percentile;
- II (method) : the risk margin is obtained as indicated by the IAA (see [7]) ;
- III (method) : the risk margin is obtained as the difference between undiscounted reserve and the discounted one;
- IV (method) : the risk margin is obtained through the formula (14).

Table 1. Incremental Payments Triangle LoB 3: Motor, other classes, Values in Euro thousands

Y[i,j]	Development year							
Underwriting year	1	2	3	4	5	6	7	8
1999	15,762	9,348	733	96	25	18	23	10
2000	16,648	9,627	734	20	32	19	13	
2001	18,333	10,346	735	126	50	22		
2002	21,999	14,878	855	61	46			
2003	25,271	13,505	856	190				
2004	23,574	16,385	360					
2005	26,833	14,373						
2006	28,613							

Table 2. Incremental Payments Triangle LoB 10: Motor, third party liability, Values in Euro thousands

Y[i,j]	Development year							
Underwriting year	1	2	3	4	5	6	7	8
1999	22,144	25,601	13,042	2,282	2,239	1,751	1,093	890
2000	25,875	30,065	12,111	5,221	1,993	1,730	6,148	
2001	29,655	36,258	13,132	10,980	4,486	2,758		
2002	31,031	42,815	11,631	9,840	5,957			
2003	42,197	48,728	22,347	11,030				
2004	44,481	60,337	26,212					
2005	49,964	51,001						
2006	49,848							

Table 3. Risk free interest rates

t(years)	1	2	3	4	5	6	7
$i(0,t)$	4.07%	4.12%	4.12%	4.12%	4.11%	4.12%	4.13%

Table 4. LoB 3 Values (Euro thousands)

Values/Methods	I	II	III	IV
Current Estimate	19,274	19,274	19,274	19,274
Risk Margin	3,986	35	2,267	362
Fair Value	23,260	19,309	21,541	19,636
Risk Capital	13,382	17,333	15,101	17,006

Table 5. LoB 10 Values (Euro thousands)

Values/Methods	I	II	III	IV
Current Estimate	210,860	210,860	210,860	210,860
Risk Margin	32,730	5,029	36,249	9,561
Fair Value	243,590	215,889	247,109	220,421
Risk Capital	114,310	142,011	110,791	137,479

The results obtained show both for LoB 3 and LoB 10 that the assessment of the risk margin is sensitive to the approach followed. In particular with the 75% percentile approach (I) the risk margin is close to the undiscounted approach (III), which is in use in the Italian market, and the values are much higher than the ones obtained with the cost of capital (II and IV). Comparing the values obtained with the approaches II and IV it can be noted that the consideration of the future variability (through the formula (14)) considerably increases the value of the risk margin (i.e. for LoB 10 the RM value passes from 5 mil to 10 mil nearly). Table 6 (aggregation without considering dependence LoBs) and Table 7 (aggregation considering dependence between LoBs) show the values of the current estimate, the risk margin, the risk capital when the two lines of business are considered jointly. For the dependence case the values are referred to the Gumbel Copula (which is the copula with the highest likelihood among the ones considered). The value of the dependence parameter is for prudence chosen as the 97.5% percentile of its distribution.

Table 6. Aggregate Values - LoB3 + LoB10, Independence (Euro thousands)

Values/Methods	I	II	III	IV
Current Estimate	230,134	230,134	230,134	230,134
Risk Margin	34,785	5,005	38,515	8,041
Fair Value	264,919	235,139	268,649	238,175
Risk Capital	98,058	127,837	94,328	124,802

The results show that considering the aggregation between LoBs leads to a gain in comparison to the actual approach (that could be summarized in taking the values of each LoB calculated separately). The diversification impact is much higher if the risk capital is considered: if the approach IV is observed, the risk capital to be held is equal to Euro 135,1 mil. that is 12.5% lower compared to sum the two values calculated separately (Euro 154, 6 mil.). It is to be noted

[1] The risk free term structure is the one reported in the QIS3 Solvency II Technical Specifications, derived using swap rates rather than government bonds (see [1]).

Table 7. Aggregate Values - LoB3 + LoB10, Copula Gumbel (Euro thousands)

Values/Methods	I	II	III	IV
Current Estimate	230,134	230,134	230,134	230,134
Risk Margin	36,598	5,031	38,515	8,707
Fair Value	266,732	235,165	268,649	238,841
Risk Capital	107,249	138,816	105,332	135,140

also that the value of the risk capital could be underestimated in case the risks are considered independent (approach IV a risk capital 8% lower).

8 Conclusions

This work describes a possible solution for the calculation of the risk margin and compares the new formula proposed with the approaches already proposed in the literature. The methodology adopted follows the indications outlined in the Solvency II framework that encourages the development of stochastic actuarial models for the assessment of the technical provisions, though the analysis is limited to two lines of business. The numerical results of the case study proposed outline that the Cost of Capital approach (methods II and IV) is less prudential than the method that is actually adopted on the Italian market since the results are significantly lower than the ones obtained determining the Risk Margin through the ultimate cost method. The dependence between different LoBs has a significant impact on the estimate of the Reserve Risk Capital: therefore in the Solvency II framework a key issue will be the definition of the statistic to be used to determine this dependency and the estimation of the parameter, taking into account its variability, as shown in the applications presented in this work. If more LoBs were considered the effects of the dependence could be higher and the Capital requirement could be reduced even further. The intention is to continue the study of the possible solutions extending the analysis to the insurance company as a whole.

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