

# Risk adjusted premiums for excess of loss reinsurance with reinstatements<sup>\*</sup>

Antonella Campana<sup>1</sup>

Dipartimento di Scienze Economiche, Gestionali e Sociali  
Università degli Studi del Molise  
Via F. De Sanctis s.n.c., 86100 Campobasso, Italy  
campana@unimol.it

**Abstract.** In this paper we considered the initial premium under different risk adjusted premium principles for an excess of loss reinsurance contract with paid reinstatements when the distribution of aggregate claims to the layer is a discrete one or it is approximated by an arithmetic distribution using the method of mass dispersal.

**Keywords.** Excess of loss reinsurance, reinsurance premiums, reinstatements, risk adjusted premium principles, distortion risk measures.

**M.S.C. classification.** 62P05.

**J.E.L. classification.** G22.

## 1 Introduction

The present paper is concerned with the concept of excess of loss reinsurance with reinstatements. The problem was first studied by Simon [9] and Sundt [10]. More recent papers on the subject include Walhin and Paris [12], Mata [8], Walhin [11], Hess and Schmidt [5], Hürlimann [6] and Campana [1].

Simon [9] studied some of the relationships in the catastrophe reinsurance area.

Sundt [10] explained concepts known in reinsurance jargon as reinstatements, aggregate deductible and aggregate limit. He also studied pure premiums and premiums loaded by the standard deviation principle to price a layer with any number of free or paid reinstatements.

Walhin and Paris [12] studied the effect on the probability of ruin for the cedent when it buys excess of loss reinsurance with reinstatements and the reinsurance premiums are calculated under different premium principles.

<sup>\*</sup> This research was partially supported by MIUR.

Mata [8] developed a methodology to calculate the distribution of total aggregate losses for two or more consecutive layers when there is a limited number of reinstatements.

Walhin [11] studied two approximations in order to price reinstatements related to excess of loss reinsurance.

Hess and Schmidt [5] considered optimal premium plans for a reinsurance contract with reinstatements by assuming that constant reinstatement premiums are due when reinsurer's loss exceeds certain bounds. They showed existence and uniqueness of a premium plan which minimizes the expected squared difference between the loss and the total premium income of the reinsurer.

Hürlimann [6] considered the situation of incomplete information, where only a few characteristics of the aggregate claims to an excess of loss layer can be estimated. He showed that the method of stop-loss ordered bounds yields a simple analytical distribution-free approximation to pure premiums of excess of loss reinsurance with reinstatements.

Campana [1] studied premium calculation when the reinstatement percentages are derived to obtain local equilibrium for each reinstatement and the aggregate claims to the layer are generated by a discrete distribution

In this paper, we assume that the distribution aggregate claims to an excess of loss layer is a discrete one or it is approximated by an arithmetic distribution using the method of mass dispersal. Then we study how to calculate the initial premium under different risk adjusted premium principles for an excess of loss reinsurance with paid reinstatements when the reinstatement percentages are given.

This paper is divided as follows. In section 2 we introduce the notation and some basic concepts such as aggregate deductibles, aggregate limits and reinstatements. In section 3 we describe the methodology developed by Sundt [10] to price excess of loss reinsurance with reinstatements for pure premiums and the standard deviation principle. Then we examine risk adjusted premium principles, like the PH-transform premium principle which belong to the class of distortion risk measures defined by Wang [14]. In section 4 we study the case in which the distribution of aggregate claims is a discrete one. In section 5 we show how the distribution of aggregate claim to the layer can be approximated by an arithmetic distribution by using the method of mass dispersal. In section 6 we give numerical examples which illustrate formulae obtained when there are only total losses, i.e. losses completely hitting the layer: the reinsurance companies normally assess treaties on this basis that is very nearly the true situation. Finally, in section 7 we give some concluding remarks.

## 2 Excess of loss reinsurance with reinstatements

Let us recall the main definitions and notations for the non-proportional reinsurance covers described in detail by Sundt [10].

Given an insurance portfolio in one-year period, let  $N$  indicate the number of claims occurring in the portfolio during the year and  $Y_i$  the  $i$ -th claim size ( $i = 1, 2, \dots, N$ ).

An excess of loss reinsurance (in short, XL reinsurance) for the layer  $m$  in excess of  $d$ , written  $m$  xs  $d$ , covers for each claim the amount

$$L_{Y_i}(d, d + m) = \min(\max(0, Y_i - d), m).$$

The aggregate claims to the layer is given by

$$X = \sum_{i=1}^N L_{Y_i}(d, d + m). \quad (1)$$

We make the convention that  $X \equiv 0$  if  $N = 0$ .

In practise the reinsurer does not pay all the claims that hit the layer during the period under consideration because there are often an aggregate deductible  $D$  and an aggregate limit  $M$ .

An XL reinsurance for the layer  $m$  xs  $d$  with aggregate deductible  $D$  and aggregate limit  $M$  covers the aggregate claims to the layer that exceeds  $D$  but with a limited payment  $M$ , that is

$$L_X(D, D + M) = \min(\max(0, X - D), M).$$

This cover is called an XL reinsurance for the layer  $m$  xs  $d$  with aggregate layer  $M$  xs  $D$ .

The aggregate limit  $M$  is usually given as a whole multiple of the limit  $m$ , i.e.  $M = (K + 1)m$ , and one speaks of a limit in the number of the losses covered by the reinsurer, where a loss is defined in the aggregate as a layer of the same size of the maximum amount of an individual claim to the reinsurer.

This reinsurance cover is called an XL reinsurance for the layer  $m$  xs  $d$  with aggregate deductible  $D$  and  $K$  reinstatements and provides total cover for the following amount

$$L_X(D, D + (K + 1)m) = \min(\max(0, X - D), (K + 1)m).$$

If the aggregate payment exceeds a multiple of the limit  $m$ , the layer has to be "reinstated".

There are two kinds of reinstatements: free and paid reinstatements. With paid reinstatements, every time a claim hits the layer the ceding company has to reinstate the layer by paying an extra premium which is charged at a pre-determined rate, pro rata to the claim size.

The premium is in practise expressed as percentage of the premium initially paid for the layer.

The initial premium  $P$  is due at the beginning of the contract and covers the original layer, that is

$$L_X(D, D + m) = \min(\max(0, X - D), m).$$

The premium for the  $k$ -th reinstatement covers the amount

$$L_X(D + km, D + (k + 1)m) = \min(\max(0, X - D - km), m).$$

Since the reinstatement is paid pro rata, this premium is a random variable so defined:

$$\frac{c_k P}{m} L_X(D + (k - 1)m, D + km), \quad (2)$$

where  $c_k \geq 0$  is the  $k$ -th reinstatement percentage. If  $c_k = 0$  the  $k$ -th reinstatement is free.

The total premium income required for this reinsurance contract is given by

$$\delta(P) = P \left( 1 + \frac{1}{m} \sum_{k=0}^{K-1} c_{k+1} L_X(D + km, D + (k + 1)m) \right). \quad (3)$$

The simplest case is when all the reinstatements are free and the ceding company has to pay a fixed premium only at the beginning of the contract.

For simplicity we assume henceforth that there is no aggregate deductible (i.e.  $D = 0$ ) and so we consider an  $XL$  reinsurance for the layer  $m$  xs  $d$  in the aggregate with  $K$  reinstatements. It is not difficult to extend the results for any aggregate deductible.

The aggregate claims  $S = L_X(0, (K + 1)m)$  paid by the reinsurer for this reinsurance cover satisfies the identity

$$S = \sum_{k=0}^K L_X(km, (k + 1)m), \quad (4)$$

where  $L_X(km, (k + 1)m)$  denotes the layer  $m$  xs  $km$  covered by the  $k$ -th reinstatement:

$$L_X(km, (k + 1)m) = \min(\max(0, X - km), m).$$

### 3 Reinsurance premiums

If the reinstatements are paid the total premium income  $\delta(P)$  is a random variable correlated to the aggregate claims  $S$  and it is not obvious how to calculate the initial premium  $P$ .

According to Sundt [10], under the pure premium principle the initial premium  $P$  should be such that the expected value of the aggregate claims  $S$  equals the expected value of the total premium income  $\delta(P)$ :

$$E[S] = E[\delta(P)]. \quad (5)$$

Under the standard deviation principle, Sundt [10] proposed to solve the following equation for  $P$ :

$$E[\delta(P)] = E[S] + \gamma \sqrt{\text{Var}(S + P - \delta(P))} \quad (6)$$

where  $\gamma$  is a positive constant.

Walhin and Paris [12] calculated the initial premium  $P$  under the Proportional Hazard transform premium principle which belongs to the class of distortion risk measures defined by Wang [14] by using the concept of distortion function as introduced in Yaari's dual theory of choice under risk (see [15]).

A distortion function  $g$  is defined as a non-decreasing function  $g : [0, 1] \rightarrow [0, 1]$  such that  $g(0) = 0$  and  $g(1) = 1$ .

The distortion risk measures associated with the distortion function  $g$ , for any non-negative real valued random variable  $Y$  with tail function  $H_Y$ , is defined by

$$W_g(Y) = \int_0^\infty g(H_Y(y)) dy. \quad (7)$$

Common risk measures in actuarial science are premium principles. The PH-transform premium principle  $\Pi_\rho$  corresponds to the distortion function (see [13])

$$g(x) = x^{\frac{1}{\rho}}, \quad 0 \leq x \leq 1, \quad \rho \geq 1, \quad (8)$$

and it is defined such as follows

$$\Pi_\rho(Y) = \int_0^\infty (H_Y(y))^{\frac{1}{\rho}} dy. \quad (9)$$

Walhin and Paris [12] proposed that the initial premium  $P$  should be the solution of the following equation:

$$P = \Pi_\rho(S + P - \delta(P)). \quad (10)$$

They solved equation (10) by using a numerical recursion. Mata [8] showed that it is not necessary to solve (10) numerically.

By applying the same scheme as for pure premiums (5), the initial risk adjusted premium must be such that the following equality holds:

$$W_g(S) = W_g(\delta(P)). \quad (11)$$

The layers  $L_X(km, (k+1)m)$ ,  $k = 1, 2, \dots, K+1$ , are comonotonic risks. Hence, by using the property of additivity for comonotonic risks of  $W_g$  (see [2, ?]), we find

$$W_g(S) = \sum_{k=0}^K W_g(L_X(km, (k+1)m)) \quad (12)$$

and, by the properties of linearity and additivity of  $W_g$ , setting  $D = 0$ , from (3)

$$W_g(\delta(P)) = P \left( 1 + \frac{1}{m} \sum_{k=0}^{K-1} c_{k+1} W_g(L_X(km, (k+1)m)) \right). \quad (13)$$

Therefore, by (11) the initial premium  $P$  must satisfy:

$$P = \frac{\sum_{k=0}^K W_g(L_X(km, (k+1)m))}{1 + \frac{1}{m} \sum_{k=0}^{K-1} c_{k+1} W_g(L_X(km, (k+1)m))}. \quad (14)$$

#### 4 The discrete case

Let  $H_X(x) = Pr\{X > x\}$  be the tail function of the aggregate claims to the layer defined in (1).

For the aggregate claims  $S$  paid by the reinsurer for an XL reinsurance for the layer  $m$  xs  $d$  with aggregate limit  $M = (K+1)m$  we find:

$$H_S(x) = \begin{cases} H_X(x) & 0 \leq x < (K+1)m \\ 0 & x \geq (K+1)m \end{cases} \quad (15)$$

The tail function of the layer  $L_X(km, (k+1)m)$  is given by

$$H_{L_X(km, (k+1)m)}(x) = \begin{cases} H_X(km+x) & 0 \leq x < m \\ 0 & x \geq m \end{cases} \quad (16)$$

Now let us assume that the layer  $L_X(km, (k+1)m)$  satisfies the following equality in distribution:

$$L_X(km, (k+1)m) \stackrel{d}{=} m B_{p_{k+1}}, \quad (17)$$

where  $B_{p_{k+1}}$  denotes a Bernoulli random variable such that

$$Pr[B_{p_{k+1}} = 1] = p_{k+1} = 1 - Pr[B_{p_{k+1}} = 0].$$

Under this assumption, the random variable  $S$  expressed by (4) has the following piecewise constant tail function:

$$H_S(x) = \begin{cases} \sum_{k=0}^K p_{k+1} I(km \leq x < (k+1)m) & 0 \leq x < (K+1)m \\ 0 & x \geq (K+1)m \end{cases} \quad (18)$$

where  $I(km \leq x < (k+1)m)$  is the indicator function which equals 1 if  $km \leq x < (k+1)m$  and 0 otherwise and

$$p_{k+1} = Pr\{S \geq (k+1)m\} = Pr\{S > km\}. \quad (19)$$

We derive that also the premium for the  $k$ -th reinstatement (2) is a two-point random variable distributed as  $c_k P B_{p_k}$ . Therefore the reinsurance premiums due when the reinsurer's loss exceeds certain bounds are constant and the total premium (3) can be written as

$$\delta(P) = P \left( 1 + \sum_{k=0}^{K-1} c_{k+1} B_{p_{k+1}} \right). \quad (20)$$

From (17) it follows that the distortion risk measure associated with the layer  $L_X(km, (k+1)m)$  is given by

$$W_g(L_X(km, (k+1)m)) = m g(p_{k+1}) \quad (21)$$

Then, by (14) we obtain

$$P = \frac{m \sum_{k=0}^K g(p_{k+1})}{1 + \sum_{k=0}^{K-1} c_{k+1} g(p_{k+1})}. \quad (22)$$

We can assume that there are only total losses, i.e. losses hitting the layer completely. While this assumption is not strictly true, the reinsurance companies normally assess treaties on this basis, especially in the catastrophe reinsurance area. It is very nearly the true situation. In order to price reinstatements related to excess of loss reinsurance, the reinsurance companies often use the rate on line method, which assumes that there are only total losses (see [11]). If this assumption causes difficulty, it may be necessary to apply this model to narrow sub-layers of a given treaty (see [9]).

If there are only total losses, the aggregate claims to the layer has a discrete distribution and satisfies the following equality in distribution:

$$S \stackrel{d}{=} m N. \quad (23)$$

Let  $v_k$  denote the probability  $Pr\{N \geq k\}$ ,  $k = 0, 1, \dots$

Now the layer  $L_X(km, (k+1)m)$  follows a discrete law with only two jumps and satisfies this equality in distribution (see (17))

$$L_X(km, (k+1)m) \stackrel{d}{=} m B_{v_{k+1}}. \quad (24)$$

and from (19) it follows that  $v_{k+1} = p_{k+1}$ .

Then, the initial premium is given by

$$P = \frac{m \sum_{k=0}^K g(v_{k+1})}{1 + \sum_{k=0}^{K-1} c_{k+1} g(v_{k+1})}. \quad (25)$$

## 5 Constructing arithmetic distributions

The evaluation of XL premiums often relies on recursive algorithms or Fast Fourier Transform methods. In order to implement recursive methods, the easiest approach is to construct a discrete claim size distribution on multiples of a convenient unit of measurement  $h$ , the *span*. Such distribution is defined on the non-negative integers and it is called *arithmetic* (see [7]).

The method of local moment matching is often used to approximate a given claim size distribution  $F$  by a discrete equidistant one. In this method the probability mass  $F(ih) - F((i-1)h)$  of the interval  $((i-1)h, ih]$  ( $i = 1, 2, \dots$ ) is replaced by point masses at the two end-points  $(i-1)h$  and  $ih$  in order to match the first  $p$  moments of the arithmetic and the true severity distribution. This

method includes the method of mass dispersal as a special case for  $p = 1$  (see e.g. [3, 10]).

Let  $F_S = 1 - H_S$  be the distribution function of the aggregate claims to the layer  $S$ .

When (18) is satisfied, the distribution of  $S$  is arithmetic and the span is  $m$ .

If the distribution of  $S$  is not arithmetic, we can approximate it with an arithmetic distribution. Such approximation can also be performed if  $F_S$  actually is arithmetic but with a different span. Then we approximate it to obtain a span which equals  $m$ , the limit on the payment of each claim.

Now we apply the method of mass dispersal in order to approximate the distribution of  $S$  by a discrete distribution on the  $K + 2$  points  $km$  with  $k = 0, 1, \dots, K + 1$ .

We replace the probability mass  $F_S((k + 1)m) - F_S(km)$  on the interval  $(k, (k + 1)m]$  by point masses at the two end-points  $km$  and  $(k + 1)m$  such that the mean is preserved. For the interval  $[0, m]$  we include the left end-point. As the mean is preserved for each of the discretization intervals, the approximation also preserves the total mean of  $S$ .

Let  $f_k$  denote the probability  $Pr\{S = km\}$ . The method of mass dispersal results in the following formulae (see [7], p.608):

$$f_0 = 1 - \frac{1}{m} E[\min(S, m)],$$

$$f_k = \frac{1}{m} (2 E[\min(S, km)] - E[\min(S, (k - 1)m)] - E[\min(S, (k + 1)m)]),$$

with  $k = 1, 2, \dots, K$ .

We derive the following equalities

$$E[\min(S, km)] - E[\min(S, (k - 1)m)] = E[L_X((k - 1)m, km)],$$

$$E[\min(S, (k + 1)m)] - E[\min(S, km)] = E[L_X(km, (k + 1)m)].$$

Therefore we can write:

$$f_0 = 1 - \frac{1}{m} E[L_X(0, m)],$$

$$f_k = \frac{1}{m} (E[L_X((k - 1)m, km)] - E[L_X(km, (k + 1)m)]),$$

with  $k = 1, 2, \dots, K$ .

The probability defined in (19) can be expressed as

$$p_1 = 1 - f_0,$$

$$p_{k+1} = p_k - f_k, \quad k = 1, 2, \dots, K.$$

Therefore, by induction we derive the following formulae:



$$p_{k+1} = \frac{1}{m} E[L_X(km, (k+1)m)], \quad k = 0, 1, \dots, K. \quad (26)$$

From (26), we have:

$$E[L_X(km, (k+1)m)] = m p_{k+1}, \quad (27)$$

If the random variable  $S$  is discrete with the tail function given by (18), formula (27) clearly follows from (17).

## 6 Numerical examples

In order to give numerical examples, we assume that there are only total losses so that the tail function of the aggregate claims to the layer  $S$  satisfies (23).

The number of claims  $N$  follows a Poisson distribution with parameter  $\lambda$  or alternatively a negative binomial distribution (with the same mean) so defined:

$$Pr\{N = k\} = \binom{r+k-1}{k} p^r q^k, \quad k = 0, 1, \dots \quad (28)$$

As it is known, the parameters of the distribution (28) are  $r$  ( $r > 0$ ),  $p$  and  $q$ , where  $p + q = 1$  and  $0 < p < 1$ ; the mean and the variance are given by:

$$E[N] = \frac{r q}{p}, \quad (29)$$

$$Var(N) = \frac{r q}{p^2}. \quad (30)$$

The premium principles used to calculate initial premiums by formula (25) correspond to the following distortion functions (see [14]), where  $x \in [0, 1]$ :

- a) PH-transforms, see (8);
- b) Logarithmic functions:

$$g(x) = \begin{cases} \frac{\log(1+\alpha x)}{\log(1+\alpha)}, & \alpha > 0 \\ x & \alpha = 0. \end{cases} \quad (31)$$

- c) Exponential functions:

$$g(x) = \begin{cases} \frac{1-e^{-\beta x}}{1-e^{-\beta}}, & \beta > 0 \\ x & \beta = 0. \end{cases} \quad (32)$$

- d) Quadratic functions:

$$g(x) = (1 + \gamma)x - \gamma x^2, \quad 0 \leq \gamma \leq 1. \quad (33)$$

- e) Dual-power transforms:

$$g(x) = 1 - (1 - x)^\delta, \quad \delta \geq 1. \quad (34)$$

We choose the parameters of these distortion functions in order to obtain the same value for the following initial risk adjusted premium

$$P = \frac{m g(0, 25)}{1 + c_1 g(0, 25)} \quad (35)$$

which is derived by (25) when  $N$  is a Bernoulli random variable with  $Pr\{N = 1\} = 1 - Pr\{N = 0\} = 0, 25$ ,  $c_1 = 1$  and  $m = 1000$ .

For simplicity we consider only two cases.

The first one is the practical case of equal reinstatements, i.e.  $c_k = c$  for  $k = 1, 2, \dots, K$ . In particular, we set  $c = 1$ . A common case in reinsurance is for a catastrophe treaty to have only one automatic reinstatement provision (with  $c_1 = 1$ ).

In the second case, we set  $c_1 = 1$ ,  $c_2 = 0, 9$ ,  $c_3 = 0, 8$ ,  $c_4 = 0, 7$  and  $c_5 = 0, 6$ .

As shown in [1], in order to obtain local equilibrium for each reinstatement, the values of  $c_k$  to apply are different (they tend to decrease when  $k$  is increasing) and the corresponding initial premium doesn't depend on the number of reinstatements.

Under these assumptions, we calculate the initial risk adjusted premium.

The following tables show numerical results obtained using formula (25) for different number of reinstatements  $K$ , with varying expected number of claims  $E[N]$  and fixed  $m = 1000$ .

As it was to be expected, the initial premium tends to increase monotonically with the expected number of claims. When the number of reinstatements  $K$  increases and  $E[N]$  is given, the initial premium tends to increase if the rates  $c_k$  are decreasing, while it tends to decrease if all the rates  $c_k$  are equal.

From the tables one can see that the PH-transform assigns the lowest initial premium to the reinsurance treaty, while the highest premium is given by the dual-power transform.

We observe that, for a given expected number of claims  $E[N]$ , the difference between numerical results derived under Poisson and negative binomial distributions depends on the value of  $q$ : the smaller the value of  $q$ , the negative binomial probabilities correspond more closely to the Poisson ones.

For example, the numerical results obtained for the Poisson distribution if  $\lambda$  equals 6 or 9 hold also for the negative binomial distribution if  $q$  equals 0.0001 and  $r$  takes the values 60000 or 90000, respectively.

If we set an higher value of  $q$ , the initial premiums derived under Poisson distribution are always higher than those obtained under negative binomial distribution.

## 7 Concluding remarks

Many problems in actuarial science involve the definition of a mathematical model that can be used to forecast or predict insurance costs in the short-term future. The claim size distribution which is often implemented is a discrete distribution like the observed distribution.

**Table 1.** Initial Risk Adjusted premiums:  $N \sim \text{Poisson}$ ,  $E[N] = \lambda = 6$ ,  $c_k = 1$  ( $k = 1, 2, \dots, 5$ ).

Principle	Parameter	$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$
PH-transform	$\rho = 1.200$	993	983	968	949	930
Logarithmic	$\alpha = 0.880$	994	984	970	953	934
Exponential	$\beta = 0.662$	994	985	972	954	935
Quadratic	$\gamma = 0.347$	994	986	973	956	936
Dual-Power	$\delta = 1.315$	998	991	979	961	939

**Table 2.** Initial Risk Adjusted premiums:  $N \sim \text{Poisson}$ ,  $E[N] = \lambda = 9$ ,  $c_k = 1$  ( $k = 1, 2, \dots, 5$ ).

Principle	Parameter	$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$
PH-transform	$\rho = 1.200$	999	998	996	991	984
Logarithmic	$\alpha = 0.880$	1000	998	996	992	985
Exponential	$\beta = 0.662$	1000	999	996	992	986
Quadratic	$\gamma = 0.347$	1000	999	996	993	987
Dual-Power	$\delta = 1.315$	1000	1000	998	996	990

**Table 3.** Initial Risk Adjusted premiums:  $N \sim \text{Poisson}$ ,  $E[N] = \lambda = 6$ ,  $c_1 = 1$ ,  $c_2 = 0.9$ ,  $c_3 = 0.8$ ,  $c_4 = 0.7$ ,  $c_5 = 0.6$ .

Principle	Parameter	$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$
PH-transform	$\rho = 1.200$	993	1016	1044	1072	1099
Logarithmic	$\alpha = 0.880$	994	1018	1047	1077	1104
Exponential	$\beta = 0.662$	994	1019	1049	1078	1106
Quadratic	$\gamma = 0.347$	994	1020	1050	1080	1108
Dual-Power	$\delta = 1.315$	998	1025	1057	1087	1114

**Table 4.** Initial Risk Adjusted premiums:  $N \sim \text{Poisson}$ ,  $E[N] = \lambda = 9$ ,  $c_1 = 1$ ,  $c_2 = 0.9$ ,  $c_3 = 0.8$ ,  $c_4 = 0.7$ ,  $c_5 = 0.6$ .

Principle	Parameter	$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$
PH-transform	$\rho = 1.200$	999	1033	1076	1125	1177
Logarithmic	$\alpha = 0.880$	1000	1033	1077	1126	1179
Exponential	$\beta = 0.662$	1000	1033	1077	1127	1180
Quadratic	$\gamma = 0.347$	1000	1033	1077	1127	1181
Dual-Power	$\delta = 1.315$	1000	1034	1079	1131	1187

**Table 5.** Initial Risk Adjusted premiums:  $N \sim \text{NB}$ ,  $E[N] = 6$ ,  $r = 4$ ,  $q = 0.6$ ,  $c_k = 1$  ( $k = 1, 2, \dots, 5$ ).

Principle	Parameter	$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$
PH-transform	$\rho = 1.200$	963	948	934	922	912
Logarithmic	$\alpha = 0.880$	967	953	939	927	916
Exponential	$\beta = 0.662$	968	954	940	928	917
Quadratic	$\gamma = 0.347$	970	956	942	930	918
Dual-Power	$\delta = 1.315$	980	965	949	934	921

**Table 6.** Initial Risk Adjusted premiums:  $N \sim \text{NB}$ ,  $E[N] = 9$ ,  $r = 6$ ,  $q = 0.6$ ,  $c_k = 1$  ( $k = 1, 2, \dots, 5$ ).

Principle	Parameter	$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$
PH-transform	$\rho = 1.200$	992	986	979	971	963
Logarithmic	$\alpha = 0.880$	993	987	981	974	966
Exponential	$\beta = 0.662$	993	988	982	975	967
Quadratic	$\gamma = 0.347$	994	989	983	976	968
Dual-Power	$\delta = 1.315$	997	994	988	981	973

**Table 7.** Initial Risk Adjusted premiums:  $N \sim \text{NB}$ ,  $E[N] = 6$ ,  $r = 4$ ,  $q = 0.6$ ,  $c_1 = 1$ ,  $c_2 = 0.9$ ,  $c_3 = 0.8$ ,  $c_4 = 0.7$ ,  $c_5 = 0.6$ .

Principle	Parameter	$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$
PH-transform	$\rho = 1.200$	963	979	1004	1034	1066
Logarithmic	$\alpha = 0.880$	967	984	1010	1040	1073
Exponential	$\beta = 0.662$	968	986	1012	1042	1075
Quadratic	$\gamma = 0.347$	970	988	1014	1044	1077
Dual-Power	$\delta = 1.315$	980	997	1022	1051	1082

**Table 8.** Initial Risk Adjusted premiums:  $N \sim \text{NB}$ ,  $E[N] = 9$ ,  $r = 6$ ,  $q = 0.6$ ,  $c_1 = 1$ ,  $c_2 = 0.9$ ,  $c_3 = 0.8$ ,  $c_4 = 0.7$ ,  $c_5 = 0.6$ .

Principle	Parameter	$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$
PH-transform	$\rho = 1.200$	992	1020	1057	1099	1145
Logarithmic	$\alpha = 0.880$	993	1021	1059	1102	1150
Exponential	$\beta = 0.662$	993	1022	1060	1103	1151
Quadratic	$\gamma = 0.347$	994	1023	1061	1105	1153
Dual-Power	$\delta = 1.315$	997	1028	1067	1112	1161

In this paper we have considered an excess of loss reinsurance with paid reinstatements and we have investigated how to calculate the initial premium under different risk adjusted premium principles when the distribution of aggregate claims to an excess of loss layer is discrete or it is approximated by an arithmetic distribution by using the method of mass dispersal.

By following this approach we have derived formulae for initial risk adjusted premiums that can be easily used for pricing reinsurance treaties with paid reinstatements, like the reinstatements related to catastrophe reinsurance treaties.

The reinsurance companies often assess treaties under the assumption that there are only total losses (i.e. losses hitting the layer completely). This happens, for example, when they use the rate on line method to price catastrophe reinsurance. It is a realistic hypothesis especially in catastrophe reinsurance area.

By assuming that there are only total losses and the number of claims follows a Poisson or a negative binomial distributions, we have derived numerical examples of initial risk adjusted premium under different distortion functions.

## References

1. Campana, A.: Reinstatement premiums for an excess of loss reinsurance contract. *Giornale dell'Istituto Italiano degli Attuari* **LXIX** (2006) 55-68
2. Dhaene, J., Vanduffel, S., Tang, Q.H., Goovaerts, M.J., Kaas, R., Vyncke, D.: Solvency capital, risk measures and comonotonicity: a review. Research Report OR 0416. Department of Applied Economics of the K.U. Leuven, Leuven (2004)
3. Gerber, H.U.: On the numerical evaluation of the distribution of aggregate claims and its stop-loss premiums. *Insurance: Mathematics and Economics* **1** (1982) 13-18
4. Goovaerts, M.J., Dhaene, J.: On the characterization of Wang's class of premium principles. *Transactions of the 26th International Congress of Actuaries* **4** (1998) 121-134
5. Hess, K.Th., Schmidt, K.D.: Optimal premium plans for reinsurance with reinstatements. *ASTIN Bulletin* **34** (2004) 299-313
6. Hürlimann, W.: Excess of loss reinsurance with reinstatements revisited. *ASTIN Bulletin* **35** (2005) 211-238
7. Klugman, S.A., Panjer, H.H., Willmot, G.E.: *Loss models*. Wiley, Hoboken (1998)
8. Mata, A.J.: Pricing excess of loss reinsurance with reinstatements. *ASTIN Bulletin* **30** (2000) 349-368
9. Simon, L.J.: Actuarial applications in catastrophe reinsurance. *Proceedings of the Casualty Actuarial Society LIX*, San Francisco (1972) 196-202
10. Sundt, B.: On excess of loss reinsurance with reinstatements. *Bulletin of the Swiss Association of Actuaries* **1** (1991) 51-65
11. Walhin, J.F.: Some comments on two approximations used for the pricing of reinstatements. *Bulletin of the Swiss Association of Actuaries* **1** (2001) 29-47
12. Walhin, J.F., Paris, J.: Excess of loss reinsurance with reinstatements: premium calculation and ruin probability of the cedent. *Proceedings of the third IME Congress*, London (1999)
13. Wang, S.: Insurance pricing and increased limits ratemaking by proportional hazards transforms. *Insurance: Mathematics and Economics* **17** (1995) 43-54
14. Wang, S.: Premium calculation by transforming the layer premium density. *ASTIN Bulletin* **26** (1996) 71-92
15. Yaari, M.E.: The dual theory of choice under risk. *Econometrica* **55** (1987) 95-115

**Mathematical Methods in Economics and Finance – m<sup>2</sup>ef**

Vol. 5/6, No. 1, 2010/11

ISSN print edition: 1971-6419 – ISSN online edition: 1971-3878

Web page: <http://www.unive.it/m2ef/> – E-mail: [m2ef@unive.it](mailto:m2ef@unive.it)

