

Optimal reinsurance programs bearing demographic and financial risks

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Abstract. The objective of this paper is to analyze – in a Risk-Based Capital framework – the equilibrium conditions between the Insurer (Cedant) and the Reinsurer in respect to appropriate combinations of linear and non-linear reinsurance strategies, dealing with longevity and financial risks. The analysis is conducted through a stochastic simulation of the management model of a life insurance company.

Keywords. Reinsurance programs, Solvency Requirements, Probability of Ruin, Expected ROE.

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1 Introduction

To choose a reinsurance program is a typical economic problem of decision under uncertainty. In literature this problem has been analyzed mainly by the point of view of the cedant ([7], [2], [4]), sometimes by the point of view of the reinsurer and rarely simultaneously analyzing both sides of the reinsurance strategies based on the risk theory and expected utility theory.

Some reinsurance programs based on combination of traditional reinsurance treaties have been studied ([5], [15], [10]). This paper aims to analyze the point of view of both sides of the reinsurance market by means of a risk model based on a financial analysis of surplus dynamics ([11], [6]).

Effectiveness of some reinsurance programs on the control of the demographic and financial risks of a life insurance company are measured by means of a criterion based on the return on equity under the ruin probability constraints. Points of mutual advantage at the same level of confidence are identified for both reinsurance market agents; the analysis is conducted through a stochastic simulation of the management model of a life insurance company.

2 Managing a portfolio of life annuities

2.1 An internal actuarial model

Let us consider a portfolio of non deferred time-discrete life annuities, with length n , sold to a cohort of males N_0 aged x at time $t = 0$. In order to ([8]) evaluate risk capital requirements, a discrete time actuarial risk model is adopted, based on a compact formulation of the company's financial surplus i.e. the difference between accrued assets and the present value of relevant liabilities:

$$A_t = \frac{A_{t-1}}{v(t-1, t)} - \sum_{j \in \Pi_t} b_t^{(j)}, \quad (1)$$

where $b_t^{(j)}$ is the annual benefit at time t to the j -th insured, $\Pi_t = \{j : T_x^{(j)} > t, j = 1, \dots, N_0\}$ and $T_x^{(j)}$ is the random residual lifetime; while $v(t-1, t)$ is the one-period random discount factor.

Let us denote with 0^+ the time after the payment of the single premium, so:

$$A_{0^+} = (1 + \delta) V_{0^+, n}^{[I_0]} + M_{0^+}, \quad (2)$$

where:

- A_{0^+} : is the value of assets allocated to the portfolio;
- $V_{0^+, n}^{[I_0]}$: is the value of portfolio reserve at time $t = 0$;
- $M_{0^+} = pV_{0^+, n}^{[I_0]}$: is the value of shareholders' capital set up by the insurer in reference to a solvency ratio p ;
- δ : is the safety loading on single premium.

The value of assets at time t is random subjected to two risk drivers: systematic and non-systematic variations in mortality rate (involving the random number N_t) and the volatility of interest rate acting in the stochastic discount factor $v(t-1, t) = 1/(1 + j_t)$.

For $t > 0$ the value of shareholders' capital is defined by means of the equation:

$$M_t = A_t - V_{t, n}^{[I_t]}, \quad (3)$$

where $V_{t, n}^{[I_t]}$ is the value of portfolio reserve at time t .

In order to investigate effectiveness of some traditional reinsurance treaties or

their combinations in controlling the demographic and financial risks; the reinsurance arrangements efficiency is analyzed in terms of risk and performance, introducing a criterion based on the return on equity under the ruin probability constraints; in particular we have selected the following two indices:

- the Finite-Time Ruin probability over the horizon $(0, t)$, as a risk measure:

$$\varphi(M; T) = Pr \{M_t < 0 \text{ for a least on } t = 1, 2, \dots, T | M_{0+} = M\};$$

- the annual Rate of Expected Return on equity, as a portfolio performance index:

$$i(0, t) = (1 + ROE(0, t))^{\frac{1}{t}} - 1,$$

where $ROE(0, t) = E\left(\frac{M_t - M_{0+}^*}{M_{0+}^*}\right)$ measures the Expected Return on Equity for the horizon $(0, t)$ and $M_{0+}^* = \inf\left\{M_{0+} \geq 0 \mid Pr\left\{\bigcap_{s=0}^T M_s \geq 0\right\} \geq 1 - \varepsilon\right\}$ is the target level of capital, as a risk measure for a given time horizon T and a given ruin probability ε (as in [11]).

2.2 Portfolio characteristics

The model has been investigated on a portfolio of non-deferred time-discrete life annuities (with single premium), with length $n=35$, referred to a cohort of males currently aged $x = 65$. The number of insured at time 0 is $N_0 = 2.500$; the maximum age is assumed to be: $\omega = 100$ years.

We suppose that the distribution of insurance annuities is not uniform, that is the amount of annual benefit is not the same for all insured:

Table 1. Initial amount.

Annuity	5	10	15	20	25	Currency unit
Frequency	35%	25%	20%	15%	5%	100%

Table 2. Statistics.

Average insured amount	11,5
Total insured amount	28.750
s.d.	6
Skewness	0,55

2.3 Demographic hypothesis

For computational aims, the insurance premiums has been calculated by means of the RG48 ¹ mortality table $\{q_x^{RG48}, x = 0, 1, \dots, \omega - 1\}$ as a first order mortality basis.

In order to introduce the systematic and non systematic demographic risk component. The model is a simple way to explain the mortality rate dynamic ([3]) and surely it is not the most robust model to describe the longevity risk, but in our work it gave the possibility to make many simulation in a reasonable time. The model can be presented simply as:

$$\{\widehat{q}_{x,t} = q_{x,t}^{RG48} C_t + \varepsilon_{x,t}, t = 0, 1, \dots, \omega - x - 1\}, \quad (4)$$

where for age x at time t : $q_{x,t}^{RG48}$ is the expected mortality, $\widehat{q}_{x,t}$ is the observed mortality (restricted to $[0, 1]$), C_t is the stochastic process, and $\varepsilon_{x,t}$ is the random noise. The diversifiable risk, i.e. the fluctuation risk, is represented by the additive component ($\varepsilon_{x,t}$).

In its original formulation the model has a multiplicative form, where the potential future observed mortality is defined as the expected mortality times a stochastic process and which is depending on time only, i.e. independent of age. In particular the non-systematic risk is described by means of the $\varepsilon_{x,t}$, additive component with normal distribution and the systematic one by means of a stochastic process lognormal distributed:

$$C_t = e^{X_t} C_{t-1}, \quad \forall t > 0. \quad (5)$$

To amplify the systematic risk component, we have derived from the original form a three parameter model introducing in the equation (5) a parameter d which allows to assets the longevity risk:

$$C_t = [d + e^{X_t}] C_{t-1}, \quad \forall t > 0, \quad (6)$$

where:

- $C_0 = 1$;
- $X_t \sim N(\mu, \sigma_X^2) \quad \forall t$;
- $E[d + e^{X_t}] = 1 + d$;
- $\varepsilon_{x,t} \sim N(0, \sigma_\varepsilon^2), \quad \forall t$;
- $Cov(C_t, \varepsilon) = 0, \quad \forall t$.

The d parameter is the initial point of the range of variable C_t and represent a shift parameter of the bundle of survival's paths. In particular it holds $\{ {}_t \widehat{p}_x^{RG48} = E[{}_t \widehat{p}_x], t = 0, 1, \dots, \omega - x - 1 \}$ for $d = 0$, with $E[{}_t \widehat{p}_x]$ and ${}_t \widehat{p}_x^{RG48}$ respectively the expected survival index derived from a simulation procedure of the equation (4) and the survival index derived from the first order mortality basis. Table 3 shows parameters used in our calculus to simulate demographic scenarios.

¹ The RG48 projected mortality tables are cohort mortality tables elaborated by the State General Accounting Department and refer to the cohort born in 1948

Table 3. Demographic technical basis parameters.

Parameter	First-order	Second-order
d	-0,020	-0,030
σ_X	0,030	0,030
σ_ε	0,001	0,001

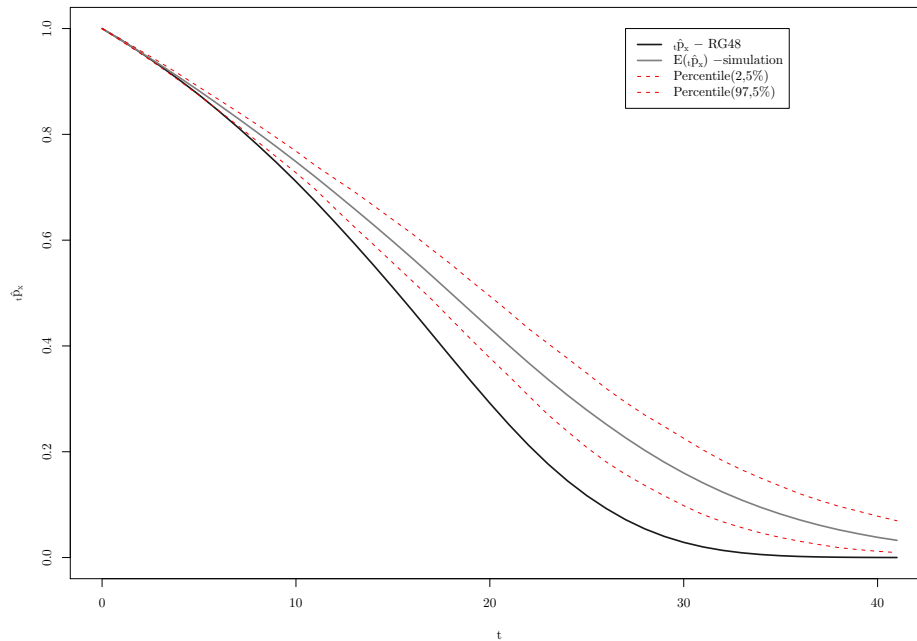
Fig. 1 shows a comparison between the survival index derived from the first order mortality basis ${}_t\hat{p}_x^{RG48}$ and the expected value and the percentiles of the stochastic process ${}_t\hat{p}_x$.

2.4 Financial hypothesis

In order to model the financial risk, we have adopted an autoregressive process (see [16], [13], [14]) for the interest rate dynamic, dependent on the inflation rate:

$$j_{t+1} = j_m + a_1(j_t - j_m) + a_2i_{t-1} + a_3i_{t-2} - i_m + c_1\xi_{t+1}, \quad (7)$$

where:

**Fig. 1.** Comparison between survival index.

- j_m : is the average rate of interest;
- i_m : is the average rate of inflation;
- a_1 : is the coefficient of the process deviation;
- a_2, a_3 : are the inflation rate weights;
- c_1 : is the coefficient of the diffusion factor;
- $\xi_t + 2 \sim \text{Gamma}(4, 2) \quad \forall t$;

and the inflation rate is a Markov process described by means of:

$$i_{t+1} = i_t + a(i_t - i_m) + c\xi_{t+1}, \quad (8)$$

where:

- i_m : is the average inflation rate;
- a : is the coefficient of the process deviation;
- c : is the coefficient of the diffusion factor.

For computational aims, the starting values of j_0 and i_0 have been calibrated in correspondence to a value of 2,5%; in order to assess the influence of different volatility values, two different volatility scenarios have been adopted: a *low* volatility scenario (1% for year) and a *high* volatility scenario (10% for year); the average rate of interest has been set equal to 4%. Table 4 shows parameters used in our calculus to simulate financial scenarios.

Table 4. Parameters.

Parameter	First-order	Second-order (<i>low</i>)	Second-order (<i>high</i>)
j	2,5%		
j_m		4%	4%
c_1		1%	10%
a_1		0,1	0,1
a_2, a_3		0,5	0,5
j_0		2,5%	2,5%
i_m		2,5%	2,5%
c		1%	1%
a		0,25	0,25
i_0		2,5%	2,5%

3 Reinsurance programs and risk analysis

In this paragraph we present result on the risk model discussed above, tested on reinsurance programs made up a combination of linear and non linear strategy. The effectiveness of the programs is measured in terms of ability to produce a positive expected return not less than that achieved without reinsurance for

the cedant, and higher than the risk-free rate for the reinsurer, providing the parties a level of solvency ratio with an assigned probability, over the horizon of reference.

For each reinsurance program, the reinsurance premium and the asset and capital dynamics of the reinsurance programs analyzed is made up as a combination of the quota share arrangement (QS), the excess of loss arrangement (XLT) and the stop-loss arrangement on cash flows (SLCF).

Some preliminary definitions:

QS arrangement: referring to an annuitant alive at time t , the individual random present value of benefit for the cedant and for the reinsurer are respectively defined as follows:

$$Y_t^{QS(j)} = \alpha \sum_{s=1}^{T_{x+t}^{(j)}} b_{t+s}^{(j)} v(t, t+s) = \alpha Y_t^{(j)}, \quad (9)$$

$$X_t^{QS(j)} = (1 - \alpha) Y_t^{(j)}, \quad (10)$$

where $\alpha \in [0, 1]$ is the percentage of the risk retained by the cedant and $Y_t^{(j)}$ the random present value at time t of future benefits to annuitant j , assumed to be alive at time t .

For the reinsurance premium (to be paid at time 0), we assume that the reinsurer adopts the following pricing principle:

$$R^{QS}(0, n) = (1 - \alpha)(1 + \eta) V_{0,n}^{\Pi_0}, \quad (11)$$

where η is security rate on single reinsurance premium.

XLT arrangement: the following notation is adopted:

- T the maximum period of annuity payment such that the cedant does not receive any benefit from the reinsurer (i.e. the retention period);
- $Y_t^{XL(j)}, Y_t^{XL(\Pi_t)}$: individual and portfolio random present value of (net) future benefit for the cedant, at time t ;
- $X_t^{XL(j)}, X_t^{XL(\Pi_t)}$: individual and portfolio random present value of future benefit for the reinsurer, at time t ;
- R_0^{XL} : reinsurance premium to be paid at time 0.

Referring to an annuitant alive at time t , the individual random present value of benefits for the cedant and for the reinsurer are respectively defined as follows:

$$Y_t^{XL(j)} = \begin{cases} \sum_{s=0}^{\min\{T_{x+t}^{(j)}, T-t\}} b_{t+s}^{(j)} v(t, t+s) & t < T \\ 0 & t \geq T \end{cases} \quad (12)$$

$$X_t^{XL(j)} = \begin{cases} v(t, T-t) \sum_{s=0}^{\max\{T_{x+t}^{(j)}-T, 0\}} b_{t+s}^{(j)} v(T-t, T-t+s) & t < T \\ \sum_{s=0}^{T_{x+t}^{(j)}} b_{t+s}^{(j)} v(t, t+s) & t \geq T \end{cases} \quad (13)$$

At the portfolio level it holds:

$$Y_t^{XL(\Pi_t)} = \sum_{j \in \Pi_t} Y_t^{XL(j)}, \quad (14)$$

$$X_t^{XL(\Pi_t)} = \sum_{j \in \Pi_t} X_t^{XL(j)}. \quad (15)$$

Obviously, the random present value of portfolio future benefits at time t is:

$$Y_t^{(\Pi_t)} = Y_t^{XL(\Pi_t)} + X_t^{XL(\Pi_t)}. \quad (16)$$

As far as the reinsurance premium is concerned, we assume that the reinsurer adopts the percentile principle for pricing; hence at time 0:

$$R_0^{XL} = \inf \left\{ u \geq 0 \mid Pr \left\{ X_t^{XL(\Pi_t)} > u \right\} \leq \gamma \right\}, \quad (17)$$

where γ represent the accepted probability of loss. We also disregard a spreading out of the single premium. Hence, $R_t^{XL} = 0, t > 0$.

SLCF arrangement: reinsurance conditions should concern the following items:

- Stop Loss Priority: the minimum amount X_t of benefits (at time t), below which there is no payment by the reinsurer, is equal to the expected value of annuities to be paid increased, by a security rate (for the reinsurer) denoted by ρ . The formula is $L_t = E \left(\sum_{j \in \Pi_t} b_t^{(j)} \right) (1 + \rho)$, with $\rho \geq 0$;
- Reinsurance premium: let us assume that the reinsurance treaty is issued at time 0 and the single premium is based on the percentile principle.

According to the reinsurance conditions, the outflows of the cedant and the reinsurer at time t are:

$$CF_t^Y = \min \left\{ \sum_{j \in \Pi_t} b_t^{(j)}, L_t \right\}, \quad (18)$$

trivially:

$$CF_t^X = \max \left\{ \sum_{j \in \Pi_t} b_t^{(j)} - L_t, 0 \right\}. \quad (19)$$

As far as the reinsurance premium is concerned, we assume that the reinsurer adopts the percentile principle for pricing. Hence at time 0:

$$R_0^{SLCF} = \sum_{t=1}^{\infty} \inf \left\{ u_t \geq 0 \mid Pr \left\{ CF_t^X > u_t \right\} \leq \gamma \right\} (1 + j)^{-t}, \quad (20)$$

where γ represent the accepted probability of loss and j is the discount rate in the premium (first order basis).

3.1 The Quota-share - Excess of Loss (QS-XLT) program and Excess of Loss - Quota-share (XLT-QS) program

The Quota-share - Excess of Loss (QS-XLT) is a program given by the combination of the following two treaties:

1. a proportional coverage (the quota-share, QS treaty) for the entire duration of the contract, with relative reinsurance premium dependent on the retention rate (share of reinsurance);
2. a non proportional coverage (the excess of loss, XLT treaty) for the range $(0, T)$ with relative reinsurance premium dependent on the percentile and the duration of the contract.

The reinsurance premium (to be paid at time 0) is a sum of two terms: a QS reinsurance premium and an α -proportion XLT reinsurance premium:

$$R^{QS-XLT} = R^{QS} + \alpha R^{XLT}. \quad (21)$$

Then, the Excess of Loss - Quota-share (XLT-QS) is a program given by the combination of the following treaties:

1. a non proportional coverage (the excess of loss) with relative reinsurance premium dependent on the percentile and the duration of the contract;
2. a proportional coverage (the quota share treaty) for charges of the cedant for its coverage period specified by the XLT treaty, with the relative reinsurance premium dependent on the retention rate (share of reinsurance).

In this case, the reinsurance premium (to be paid at time 0) is a sum of two terms: XLT reinsurance premium and a QS reinsurance premium:

$$R^{XLT-QS} = R^{XLT} + \alpha R^{QS}(0, T). \quad (22)$$

For both reinsurance programs, recursive formulas of assets and shareholder's capital dynamics are shown, respectively for each of reinsurance market's agents. Shareholder's capital and assets dynamics for the insurer:

$$A_t^C = \begin{cases} M_{0^+}^C + (1 + \delta) V_{0^+,n}^{[I_0]} - R^{(\cdot)} & t = 0^+ \\ A_{t-1}^C (1 + j_t) - \alpha C F_t^{[I_t]} & 0 < t \leq T \\ A_{t-1}^C (1 + j_t) & t > T \end{cases}$$

$$M_t^C = \begin{cases} p V_{0^+,T}^{[I_0]} & t = 0^+ \\ A_t^C - \alpha V_{t,T-t}^{[I_t]} & 0 < t \leq T \\ A_t^C & t > T \end{cases}$$

Shareholder's capital and assets dynamics for the reinsurer:

$$A_t^R = \begin{cases} M_{0^+}^R + R^{(\cdot)} & t = 0^+ \\ A_{t-1}^R (1 + j_t) - (1 - \alpha) C F_t^{[I_t]} & 0 < t \leq T \\ A_{t-1}^R (1 + j_t) - C F_t^{[I_t]} & t > T \end{cases}$$

$$M_t^R = \begin{cases} p(1-\alpha)V_{0^+,T}^{[I_0]} + p_T/V_{0^+,n-T}^{[I_0]} & t = 0^+ \\ A_t^R - \left[(1-\alpha)V_{t,T-t}^{[I_t]} + T-t/V_{t,n-T}^{[I_t]} \right] & 0 < t \leq T \\ A_t^R - V_{t,n-t}^{[I_t]} & t > T \end{cases}$$

Where $R^{(\cdot)} = R^{QS-XLT}$ for the QS-XLT program and $R^{(\cdot)} = R^{XLT-QS}$ for the XLT-QS program.

We remark that for the cedant, the liability is described by means of an α -proportion of the technical provision of a temporary annuity with maturity T , while for the reinsurer the liability is described by means of a sum of two reserve components respectively referred to an $(1-\alpha)$ proportion of a temporary annuity and a deferred annuity with starting point in time T .

For the QS-XLT and XLT-QS program, the Fig. 2 represent both the cedant and the reinsurer performance indices in the case of a reinsurance premium calibrated in correspondence to $\alpha = 60\%$, $\gamma = 95\%$ and $\phi(M;T) = 2,5\%$, for the two volatility scenarios.

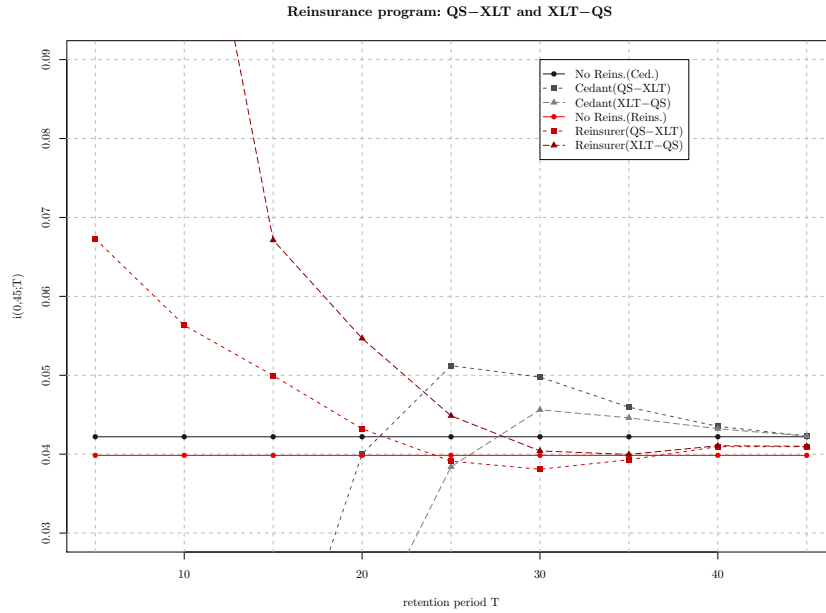


Fig. 2. Index $i(0, t)$ low volatility scenario.

From the Fig. 2 and Fig. 3, we can observe that:

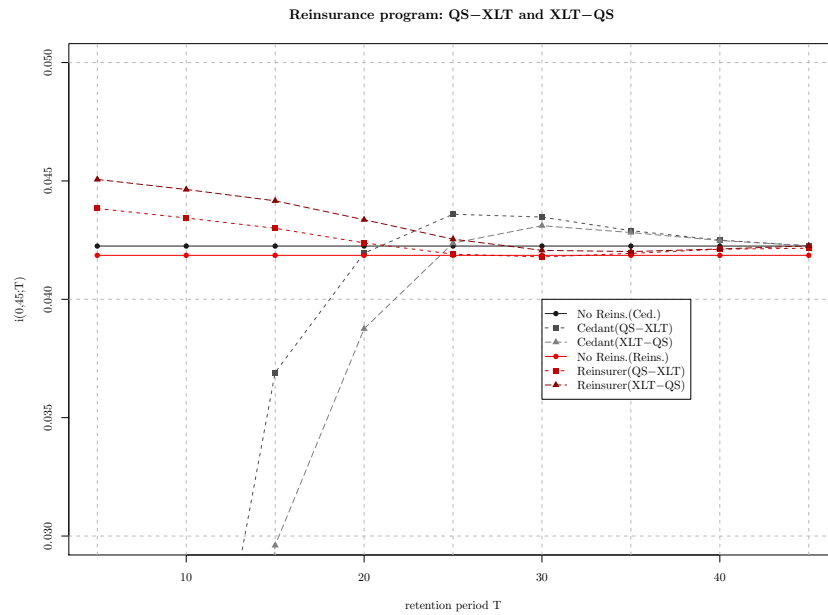


Fig. 3. Index $i(0, t)$ high volatility scenario.

- from the cedant’s viewpoint: to enter the reinsurance treaty incurs an additional charge i.e. the price of reinsurance; the performance index increases with increasing T and a maximum point is reached in reference to a retention period ranging from 26 to 31 years, respectively in the (QS-XLT) and the (XLT-QS) strategy (optimal unilateral solution);
- obviously, from the reinsurer’s point view the expected return on equity decrease rapidly with increasing of T ;
- the optimum level of the return on equity can be achieved in correspondence of a retention period equals to 20 years and 25 years respectively in the (QS-XLT) and the (XLT-QS) strategy, since none of the other possible solution at the same time helps to improve the position of both agents;
- an higher level of volatility on the interest rate does not modify the optimum retention period in the decision problem, while reduces the entire structure of performance index because of greater request of solvency capital.

3.2 Stop loss on cash flow - Excess of Loss (SLCF-XLT) program and Excess of Loss - Stop loss on cash flows (XLT-SLCF) program

The SLCF-XLT program is given by the combination of the following two treaties:

1. a non proportional coverage with a fixed priority level that limits the amount of annual benefits to be paid by the cedant;

2. a second treaty which reduces the period covered by the cedant.

In this way the cedant reduces the annual benefits and its coverage time horizon. The reinsurance premium (to be paid at time 0) is:

$$R^{SLCF-XLT} = R^{SLCF} + R^{XLT|SLCF}. \quad (23)$$

The XLT-SLCF program is a combination of the following two treaties:

1. a treaty that reduces the period covered by the cedant;
2. a non proportional treaty with a fixed period level.

Essentially, the cedant reduces its coverage period and its annual benefits. The reinsurance premium (to be paid at time 0) is a sum of two terms: a XLT reinsurance premium and a SLCF reinsurance premium, both calculated in reference to a percentile approach:

$$R^{XLT-SLCF} = R^{XLT} + R^{SLCF|XLT}. \quad (24)$$

The SLCF-XLT and XLT-SLCF reinsurance programs achieve the same configuration of risk transfer. But the composition of the treaties leads to a different global reinsurance premium. For both reinsurance programs and two reinsurance market's agents, recursive formulas of assets and shareholder's capital dynamic hold:

Shareholder's capital and assets dynamics for the insurer:

$$A_t^C = \begin{cases} M_{0^+} + (1 + \delta) V_{0^+,n}^{II_0} - R^{(\cdot)} & t = 0^+ \\ A_{t-1}^C (1 + j_t) - CF_t^Y & 0 < t \leq T \\ A_{t-1}^C (1 + j_t) & t > T \end{cases} \quad (25)$$

$$M_t^C = \begin{cases} pV_{0^+,T}^{II_0} & t = 0^+ \\ A_t^C - V_{t,T-t}^Y & 0 < t \leq T \\ A_t^C & t > T \end{cases} \quad (26)$$

where $V_{t,T-t}^Y$ is the value of portfolio reserve for the cedant. Shareholder's capital and assets dynamics for the reinsurer:

$$A_t^R = \begin{cases} M_{0^+} + R^{(\cdot)} & t = 0^+ \\ A_{t-1}^R (1 + j_t) - CF_t^X & 0 < t \leq T \\ A_{t-1}^R (1 + j_t) - CF_t^{II_t} & t > T \end{cases} \quad (27)$$

$$M_t^R = \begin{cases} p\beta V_{0^+,T}^{II_0} + p_T/V_{0^+,n-T}^{II_0} & t = 0^+ \\ A_t^R - [V_{t,T-t}^X + T-t/V_{t,n-T}^{II_t}] & 0 < t \leq T \\ A_t^R - V_{t,n-t}^{II_t} & t > T \end{cases} \quad (28)$$

where $V_{t,T-t}^X$ is the value of portfolio reserve for the reinsure, $R^{(\cdot)} = R^{SLCF-XLT}$ for the SLCF-XLT program and $R^{(\cdot)} = R^{XLT-SLCF}$ for the XLT-SLCF program and β is a percentage of portfolio reserve.

For the SLCF-XLT program, the Fig. 4 and Fig. 5 represent respectively the cedant and the reinsurer indexes in the case of a reinsurance premium calibrated in correspondence to the percentile SLCF and XLT $\gamma = 95\%$, $\rho\%$ and $\phi(M;T) = 2, 5\%$.

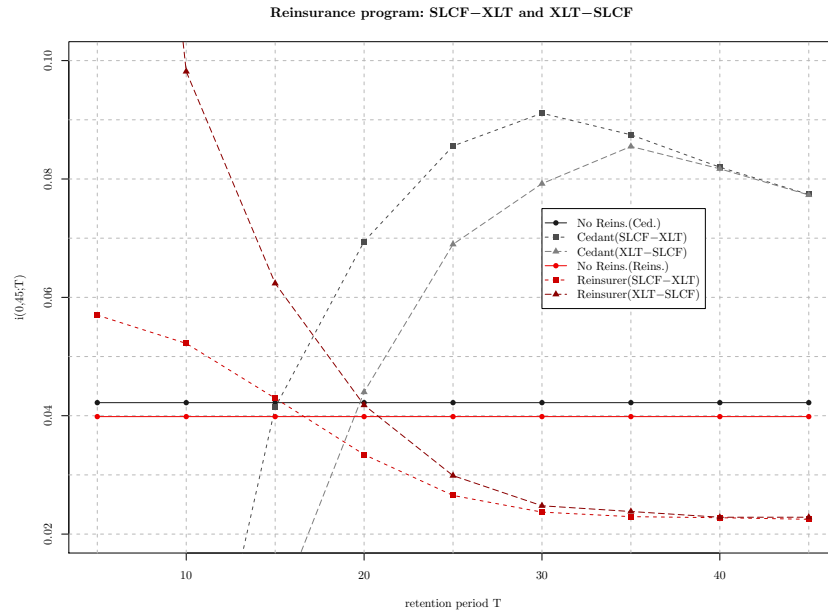


Fig. 4. Index $i(0, t)$ low volatility scenario.

From the Fig. 4 and Fig. 5, we can observe that:

- from the cedant's side, the analyzed reinsurance programs allow to achieve a performance index greater than that one in no reinsurance case, in correspondence to a retention period T not less than 15 and 20 years, respectively, in the (SLCF-XLT) and the (XLT-SLCF) strategies. In reference to the maximum level of the performance index, the two strategies are preferred to both the (QS-XLT) and (XLT-QS) reinsurance arrangements i.e. the QS component is too costly than the request for solvency capital;
- from the reinsurer's side, the two strategies permit to maintain the return on equity above the level achievable in the case of no reinsurance, correspondence to a retention period T not greater than 17 years and 21 years, respectively, in the (SLCF-XLT) and the (XLT-SLCF) strategy;
- the bilateral optimum level of the return on equity can be achieved in correspondence of a retention time equals to 15 years and 20 years respectively

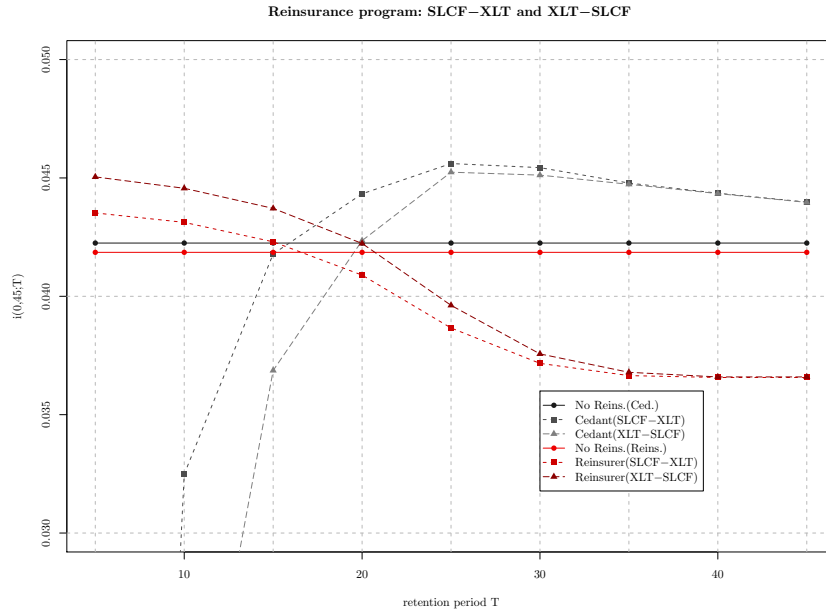


Fig. 5. Index $i(0, t)$ low volatility scenario.

in the (SLCF-XLT) and the (XLT-SLCF) strategy, since none of the other possible solution at the same time helps to improve the position of both agents;

- as expected, an higher level of volatility on the interest rate does not modify the bilateral optimum retention time in the decision problem, while it reduces the entire structure of performance index because of greater request of solvency capital for both market agents. Obviously, the performance index is strongly affected by the interest rate dynamics: Fig 6 and Fig 7 show that a reduction of j_m produces a dramatic decrease on the adjusted return on equity index.

4 Conclusion

It is evident that the risk of default for a portfolio of life annuities is heavily affected by the demographic and financial risk. A natural choice to reduce risk and to get an efficient capital allocation is to transfer a portion of the risk to reinsurers, possibly with a favorable pricing.

In this paper, it is proposed a criterion based on the return on equity the ruin probability constraints as a particular performance index to select the reinsurance strategy, suitable for both sides of the reinsurance markets.

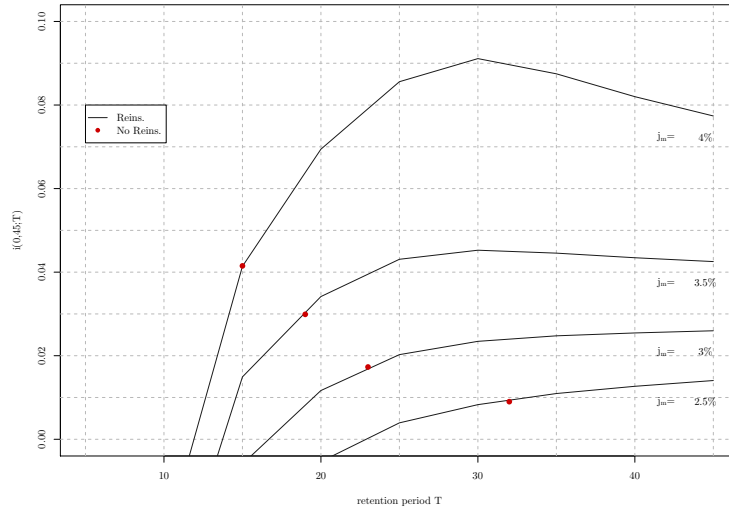


Fig. 6. Cedant's side Index $i(0,t)$ low volatility scenario.

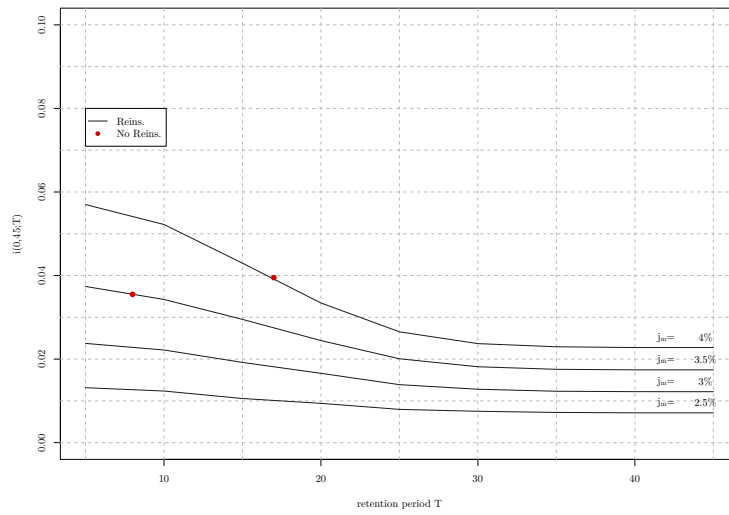


Fig. 7. Reinsurer's side Index $i(0,t)$ low volatility scenario.

With reference to the risk model adopted and the criteria analyzed, we can remark that programs that combine non-proportional reinsurance strategies provide excellent bilateral performance levels, preferable to those reached matching non-proportional and proportional ones.

It is possible to define an efficient frontier based on the trade-off insolvency risk/shareholders return on equity, according to different reinsurance arrangements and different retentions, taking in account the demographic and financial risk.

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