The volume-volatility relationship: A fractal analysis for a stock index

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Abstract. This paper investigates the contemporaneous as well as the causal relationship between trading volume and a new measure of volatility based on the pointwise Hölder regularity of price process. Using daily data of Nikkei 225 index, evidence of contemporaneous correlation is found. A vector autoregressive (VAR) analysis is employed to test the dynamic relationship and a bidirectional causality is shown.

Keywords. Volatility, Pointwise Hölder regularity, Stock Markets.

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J.E.L. classification. C01, G00.

1 Introduction

One of the most debated topics in financial research is represented by the volume-volatility relation. A large body of literature has investigated their association from both an empirical and theoretical point of view. In fact, according to [30], the volume-volatility relationship may help to better understand some mechanisms of financial markets including, for example, the prediction of market movements for technicians as well as the the appropriate model for the distribution of stock prices. Two main information theories have been developed: the *Mixture of Distributions Hypothesis* (MDH) and the *Sequential Arrival of Information Hypothesis* (SAIH). The former – introduced by [12] in 1973 – suggests that the volatility is positively related to trading volume because of its dependence of a common latent mixing variable represented by the rate of information arrival (see, e.g., [17], [42] and [24]). In particular, the MDH posits that both volume and volatility are driven by the same underlying information flow and, for this reason, they do change contemporaneously as soon as information is processed by

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market participants. Despite many empirical contributions have confirmed the validity of MDH in several markets with different frequency of data (see, e.g., [31], [4], [9], [32], [10], [11] and [29] among others), some authors either have not found any relation ([23] and [15]) or have shown a negative relation when the jump component of volatility was considered or turbulent markets were examined ([21], [2], [22] and [43]). In fact, as pointed out by [8], the MDH is based on a possible statistical explanation for the positive volume–volatility relationship but it remains silent about the underlying economic mechanisms that governs the link between trades and price adjustments to the news. An extension of MDH (the modified MDH model) which includes informational asymmetries is due to Andersen [1]. Under the modified MDH assumptions, the positive relationship between volume and volatility is mainly driven by informed traders, while liquidity volume is unrelated to return volatility. On the other hand, Copeland in 1976 ([14]) introduced the sequential arrival of information hypothesis to explain the evidence of a lead-lag relationship between return and volumes. The model assumes that the information is asymmetrically distributed and spreads sequentially from one trader to another. Later, Copeland's analysis was extended by [37], [27] and [28]. Since the 1990s, the financial researches have moved their interest to causal relation between price changes and trading volume. Typically, unidirectional and/or bidirectional causality between these variables has been addressed by the vector autoregressive (VAR) analysis and/or Granger causality tests. Analyzing stock indexes from emerging markets, [41] finds that volume seems to lead volatility but not viceversa as well as [10] for S&P500 index. Studying Asian markets, [11] reveals that, for a sample of ten indexes, volatility implies volumes while only the indexes of Japan and Taiwan show evidence of volume leading volatility. Examining stock and foreign markets, the results of Chen and Daigler [9] show a one-way Granger causation from volatility to volume. A significant bidirectional causality between price variability and volume was found by [25] for DJIA index as well as [32] for S&P500, TOPIXS and FTSE indexes. Same results are obtained by [33] and [34] for 1-minute intraday data from Taiwan stock exchange.

This study differs from other studies on the volume-volatility relationship because the measure of volatility is calculated exploiting the pointwise regularity of the price process. We examine both the contemporaneous and dynamic relation for Nikkei 225 index from July 2002 to November 2017. Our results confirm the validity of MDH which predicts a simultaneous positive association. As to the the causal relation, a bidirectional causality is found but to different extent.

2 The model and volatility estimation

The empirical applications of this work are based on the assumption that the dynamic of a stock index does follow a particular stochastic process – referred to as *multifractional Brownian motion* (mBm) – which has been proved to size many empirical features displayed by actual time series, i.e. the so-called "stylized facts" (see [13] for an exhaustive literature). The mBm represents an extension of the well-known *fractional Brownian motion* (fBm) introduced by the seminal paper of [35] as a moving average of the *ordinary Brownian motion* (oBm). Briefly, the fBm displays a slowly decaying autocorrelation function which depends on the parameter H (named Hurst or Hölder parameter) belonging to the interval $(0,1]$ (see [36] for a survey bibliography). The increments of fBm are the only zero-mean Gaussian, stationary, self-similar[1] sequence with an autocovariance function $\varrho(h) = \frac{K^2}{2}\{(h+1)^{2H} - 2h^H + |h-1|^{2H}\}$, where $h \geq 0$ represents the lag and K is a scale factor. It is well known that: (1) if $0 < H < 1/2$ the motion displays antipersistence (positive increments tend to be followed by negative increments and viceversa); (2) if $1/2 < H < 1$ fBm shows persistence (increments tend to be followed by increments of the same sign). Finally, when $H = 1/2$ fBm reduces to the ordinary Brownian motion (see [19] for details). The generalization of fBm leads to mBm under the assumption that the Hurst parameter does vary over time becoming a Hölder function[2] $H : (0,\infty) \mapsto (0,1]$. The mBm – introduced independently by [40] and [3] – is a Gaussian process that admits the following non anticipative moving average representation

$$M_{H(t),K(t)}(t) = K(t) \cdot V_{H(t)}^{1/2} \int_{\Re} f_t(s)dB(s) \tag{1}$$

with $f_t(s) = \frac{1}{\Gamma(H(t)+1/2)} \left\{ |t-s|^{H(t)-\frac{1}{2}} \mathbb{1}_{(-\infty,t]}(s) - |s|^{H(t)-\frac{1}{2}} \mathbb{1}_{(-\infty,0]}(s) \right\}$ where $V_{H(t)}$is a normalizing function, $K(t)$ is a scaling function and $B(\cdot)$ is the Brownian motion (see [5] for details). Notice that when $H(t) = H$ the fBm is recovered as special case of the mBm.

Basically, two main approaches have been developed to estimate $H(t)$: 1) the generalized quadratic variations ([26]); 2) the absolute moments of a Gaussian random variable ([39] and [5]). For our purpose, we will adopt a well-known estimator introduced in [6] (see [7] for details), that measures the Hölder pointwise regularity at point t along the trajectory of process $X_t = \log(P_t)$, where P_t is the stock price. In particular, sampling in discrete time t, $X_n (n = 1,...,N)$, and denoting by K the unit time variance, the estimator (of order k and lag q) is defined in terms of moving average of size δ as

$$\hat{H}^k_{\delta,q,N,K}(t) = \frac{\log\left(\sqrt{\pi} S_{\delta,q,N}(t) / \left(2^{k/2}\Gamma\left(\frac{k+1}{2}\right)K^k\right)\right)}{k \log\left(\frac{q}{N-1}\right)} \tag{2}$$

where $S_{\delta,q,N}(t) = \frac{\sum_{j=t-\delta}^{t-q} |X(j+q) - X(j)|^k}{\delta - q + 1}$ and $t = \delta + 1, \cdots, N + 1 - q$.

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[1] The stochastic process $\{X(t), t \in T\}$ is said *self-similar* with parameter H if for any $a > 0$ $\{X(at) \overset{d}{=} a^H X(t)\}$, where the equality holds for the finite-dimensional distributions of the process (see, e.g. [16]).

[2] Let $\{Y(t), d_Y\}$ and $\{Z(t), d_Z\}$ be two metric spaces, the function $f : Y \to Z$ is called Hölder function with exponent $\gamma > 0$ if, for each $x, y \in Y$ such that $d_Y(x,y) < 1$ there exists a constant k such that $d_Z(f(x), f(y)) \leq k d_Y(x,y)^\gamma$.

In this regard, some issues deserve a concise discussion in light of empirical applications. It has been shown that $k = 2$ and $q = 1$ are the optimal values which minimize the variance of estimator (2) (see e.g. [6]) while the choice of the size of δ is critical. From a financial point of view, δ represents the window in which information is assumed not to change significantly and this means assuming the normal distribution for data. Since the variance of the estimator (2) decreases with δ, the larger δ the smoother the estimates but – at the same time – the larger the pointwise information lost. Empirical evidence for financial time series (see, e.g. [7] and [19], among others) along with the good rate of convergence ($O(\delta^{-1/2} log^{-1} N)$) of estimated $H(t)$ suggest a range of variation for δ of 20-30 datapoints. In this study, we set $\delta = 21$, which represents about a trading month.

As $H(t)$ is the punctual Hölder exponent of mBm at point t, the process is locally asymptotically self-similar with index $H(t)$ (see e.g. [3]) in the sense that

$$\lim_{h\to\ 0^+} \frac{M_{H(t+hu),K(t+hu)}(t+hu) - M_{H(t),K(t)}(t)}{h^{H(t)}} \stackrel{d}{=} B_{H(t),K(t)}(u)$$

with $u \in \Re$. The previous distributional equality means that at any point t there exists a fBm with parameter $H(t)$ tangent to mBm. This indicates that mBm locally behaves like a fBm of a given Hölder exponent. For this reason, in light of the meaning of the Hurst parameter of a fBm previously discussed, it is natural to interpret $H(t)$ as a measure of volatility. In order to reinforce our consideration, fig. 1 displays the exponential interpolation of the standard deviation of log-variations against the corresponding estimates of $H(t)$ calculated for the Nikkei 225 index that will be used in the following applications. Table 1 reports the results of the goodness of fit. When the market volatility increases (decreases) the corresponding pointwise exponent decreases (increases) following an exponential trend. The advantage of using the pointwise Hölder exponent instead of the simple standard deviation as a measure of volatility relies in the fact that the estimation of $H(t)$ is linked to the concept of market efficiency in the sense of Fama [18] (see [7] for an exhaustive discussion) and this allows to measure the efficiency of the analyzed stock or index for all time t belonging to the time domain.

Table 1. Exponential regression: $std_t = a \cdot exp^{b \cdot H_t}$

a	b	sse	$rmse$	R^2
0.703 (0.694, 0.712)	−8.259 (−8.277, −8.231)	0.0032	0.0009	0.982

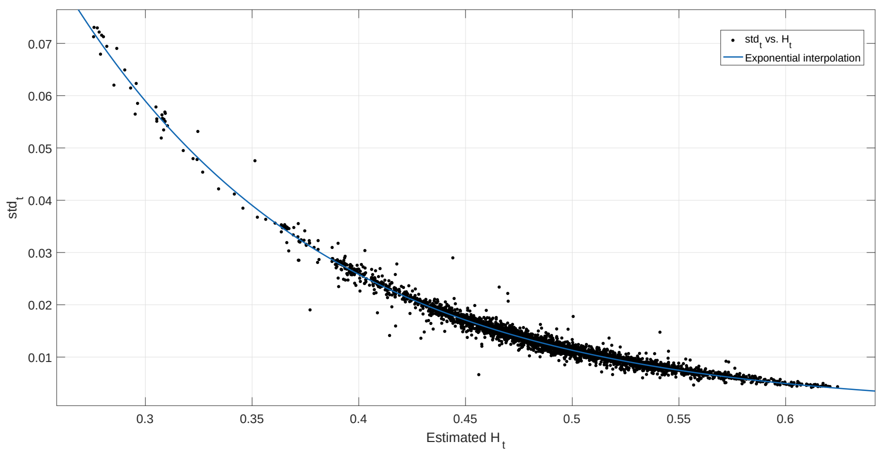

Fig. 1. Exponential interpolation: standard deviation versus the estimated pointwise Hölder exponent.

3 Empirical applications

3.1 Data and preliminary analysis.

The analysis concerns the daily closing prices and trading volumes of Nikkei 225 index from July 8th, 2002 to November 2nd, 2017. Let $\Delta P_t = \ln \frac{p_t}{p_{t-1}}$ denote the log-variations, where p_t represents the daily closing stock index. In line with previous literature, in order to stabilize the variability of the trading volume series and to reduce the non-normality of distributions, the raw trading volumes are firstly transformed by natural logarithm (see e.g. [10]) and then filter out[3] by the their trend to avoid spurious regressions, according to a consolidated procedure (see e.g. [9], [38] and [21] among others). Furthermore, for each trading month we calculate the corresponding mean of detrended volume (v_t). As to the estimates of the pointwise Hölder exponent[4], these are obtained applying equation (2) with $\delta = 21$, $k = 2$ and $q = 1$, as discussed in the previous paragraph. The Augmented Dickey-Fuller (ADF) test for the stationarity of series as well as descriptive statistics of variables are reported in Table 2 [5].

It can be seen that log-variations behave similarly to what we usually observe in the literature: they exhibit an excess of kurtosis and are left-skewed. In

[3] In fact, as pointed out by Tauchen et al. [42]: *"any variance-volume study should include preliminary tests for trend in the volume of trading"*.

[4] We adopt an in-house software running in MatLab R2015a and developed by the QuantLab (Laboratory of Applied Mathematics) at the UCLAM.

[5] For the estimated H_t, the Augmented Dickey-Fuller test with drift variant has been developed. Critical values for ADF test are: -3.44 (1%), -2.87 (5%) and -2.57 (10%).

Table 2. Descriptive Statistics

	Log-variations	Estimated H_t	Log-volumes (detr.)
mean	1.7705^{*}	0.4904	-3.4824^{**}
max	0.1323	0.6245	1.4479
min	-0.1211	0.2741	-1.6291
std	0.0151	0.0505	0.3428
skew	-0.4822	-0.4630	0.0324
kurt	10.2939	4.2641	3.4805
ADF	-63.5825	-4.3062	-18.4714

$*\times 10^{-4}$, $**\times 10^{-15}$.

addition, the mean value of the estimated pointwise Hölder exponent is approximately $\frac{1}{2}$; this is strongly consistent with the literature claiming that markets are efficient over long time span. Clearly, the hypothesis of a unit root process is rejected for each series, allowing for a vector-autoregressive analysis. Fig. 2 shows the log-variations (top panel) and the corresponding estimates of H_t (bottom panel).

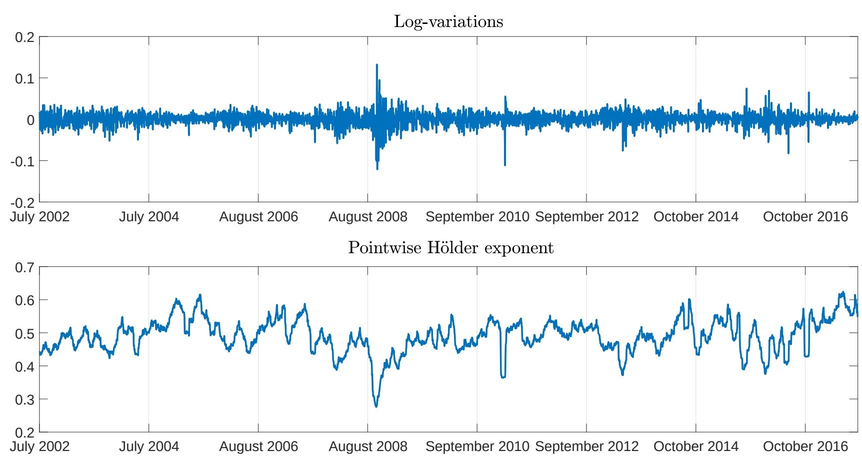

Fig. 2. Log-variations (top panel). Pointwise Hölder exponent (bottom panel) for Nikkei 225 index.

The mean-reverting property displayed by the pointwise Hölder exponent – which swings around 0.5 (market efficiency) – reveals its capability to capture the stylized fact of *volatility clustering*. Indeed, the estimates of $H(t)$ range from significant antipersistence (the minimum value is about 0.27) to significant persistence (the maximum value is about 0.62). An evident feature of this pattern is that quite flat and relatively long periods alternate to sudden and large downward movements which are followed in their turn by a gradual upward movements. This mechanism admits the following explanation: when uncertainty dominates the market, the confidence of investors reduces and the pointwise Hölder exponent declines itself toward 0.5 or toward lower values. This effect is particularly pronounced in reference to the period of September-October 2008 (Lehman Brother collapse). The great uncertainty of financial market generated high volatility punctually captured by the estimator with low peaks of $H(t)$ around the level 0.30.

3.2 Analysis of the volume-volatility relation

We begin our analysis by investigating the simultaneous relation between volatility and volumes. Fig. 3 presents a scatterplot of (detrended) volumes against the log-variations of index. The scatterplot shows that, for the most part, large log-variations are associated with generally high volume. This behavior is consistent with existing findings on the contemporaneous positive correlation between the magnitude of price movements and volume. Particularly, the V-shape relation strongly corroborates the theoretical (see, e.g. [30]) as well as the empirical (see, e.g. [20]) results achieved in literature.

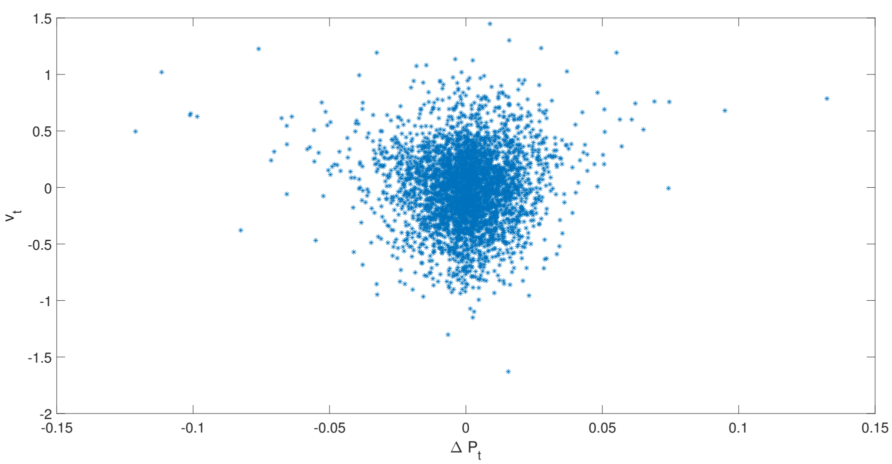

Fig. 3. Scatterplot of volumes against log-variations.

To formalize the previous intuition, we examine the simultaneous volume-volatility relation by running the following simple regression:

$$v_t = \alpha_0 + \alpha_1 H_t + \varepsilon_t \tag{3}$$

The results are reported in table 3 while fig. 4 presents a scatterplot of stock index volatility versus volume. The pattern in the figure supports the contemporaneous positive correlation between the stock index volatility and volume, as required by the Mixture of Distributions Hypothesis model. In fact, the slope parameter α_1 is negative and significant at 1% level. This means that when volatility increases ($H(t)$ decreases) then the volume increases itself, according to MDH. It is interesting to note in fig. 4 that when $H(t)$ is close to 0.5 (market efficiency) the relation seems to be less pronounced. This may deserve future investigations.

Table 3. Linear regression: $v_t = \alpha_0 + \alpha_1 H_t + \varepsilon_t$

α_0	α_1	sse	$rmse$	R^2
1.115	-2.262	11.950	0.261	0.163
(0.737, 1.492)	(-3.026, -1.497)			

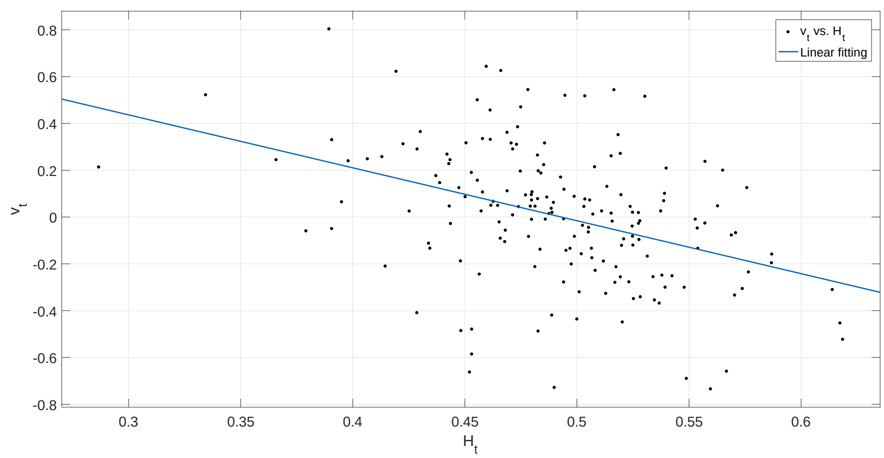

Fig. 4. Dataplot of contemporaneous volume-volatility relation. Blue line represents the linear interpolation through OLS.

In order to analyze the causal relation between volume and volatility, a vector autoregressive analysis is performed. In general, denoted by r_t and v_t the volatility (or return) and volume at time t, the following bivariate VAR model is used for financial applications (see, e.g. [10] and [11]):

$$v_t = a + \sum_i b_i v_{t-i} + \sum_i c_i r_{t-i} + e_t \tag{4}$$

$$r_t = d + \sum_i g_i r_{t-i} + \sum_i h_i v_{t-i} + u_t \tag{5}$$

The lag length in equations (4) and (5) is usually chosen according to a particular information criterion[6]. For our purpose, using monthly measures, we set $i = 1$ and run the following regressions:

$$v_t = a + b_1 v_{t-1} + c_1 H_{t-1} + e_t \tag{6}$$

$$H_t = d + g_1 H_{t-1} + h_1 v_{t-1} + u_t \tag{7}$$

The corresponding outputs are shown in table 4.

Table 4. VAR estimation results

Dep. variable	H_t	v_t
Constant	0.236*	0.335*
	(7.24)	(2.93)
H_{t-1}	0.520*	−0.723*
	(7.86)	(17.95)
v_{t-1}	−0.027*	0.793*
	(−2.31)	(−2.94)
rmse	0.041	0.153
Adj. R^2	0.346	0.711
F stat.	47.60	218.00

Notes: 1) t-statistics are reported in parentheses.
2) * indicates significance at the 1% level.

The results indicate that past trading volume and volatility significantly affect the current volatility and current trading volume (bidirectional effect) but to different extent. In fact, despite the concordance of sign displayed by coefficients of lagged variables, the capability of past volatility to lead current volume is greater than viceversa as one can note by looking at the size of corresponding coefficients. These results corroborate the findings of [25], [32], [33] and [10], among others.

4 Conclusion

The relation between stock price volatility and trading volume has gained a remarkable attention over the past three decades in the field of finance. Using

[6] The most used are the Akaike information criterion (AIC) and/or Bayesian information criterion (BIC).

data for the Nikkei 225 index from July 2002 to November 2017, we study both the contemporaneous and causal relation by measuring the market volatility in a different way with respect to traditional approaches. In fact, assuming the stock index to follow a multifractional Brownian motion, we estimate the market volatility by means of the pointwise Hölder regularity of the price process.

Our findings reveal a contemporaneous positive correlation between volatility and volume, according to the Mixture of Distributions Hypothesis theory. In this regard, further investigations may involve the concept of informational efficiency at the aim to understand the economic mechanisms that governs the link between trades and price adjustments to the news.

As to the causal relation, a vector autoregressive analysis has been developed. Our results show a general bidirectional causal volume-volatility relationship but to different extent. In particular, the past volatility seems to be more appropriate in predicting current volume than viceversa.

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