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Although tax evasion and auditing are dynamic processes, they have been approached in a dynamic framework only recently. We argue that the decision to avoid taxes is dynamically embedded with consumption decisions, which in turn are driven by consumption habits. The model is cast in a dynamic context with an infinite horizon. Our paper makes several contributions to the existing literature on tax evasion: 1) habit formation has a dampening effect on tax evasion; 2) as the representative consumer grows older, the gap between habit and consumption decreases and his tax evasion decreases; 3) the effect of an increase in tax evasion depends on the ration of habit to capital, i.e. the presence of the Yitzhaki (1974) paradox depends on such a ratio; 4) we show that in the long run the ratio increases while the relationship between evasion and the tax rate changes from being positive to being negative; 5) the model has policy implications: other things being equal, it is better to induce people to reduce their level of tax evasion with controls rather than fines.

Keywords: dynamic tax evasion, habit

JEL Codes: H26, H30

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1 Introduction

Tax evasion behaviour is a severe plague in many countries. Some recent estimates (Peige and Cebula, 2011) show that intentional under-reporting of income is about 18-19% of the total reported income in the US, leading to a tax gap of about 500 billion Dollars, while in Europe the level of tax evasion is about 20% of GDP, accounting for a potential loss of about 1 trillion Euros each year (Buehn and Schneider, 2012; Murphy, 2014).

Although tax evasion and auditing are dynamic processes (Engel and Hines, 1999), they have been approached in a dynamic framework only recently (Wen-Zhong and Yang 2001; Nieper 2005; Dehumashev and Gahramanov 2011; Levaggi and Menoncin 2012, 2013; Bernasconi et al. 2014). In this paper, we study optimal consumption and evasion in a dynamic framework, where consumer decisions depend on habit formation. Accordingly, the usual trade-off between present and future consumption is made more strict because, in any period, the past stream of consumption builds a new habit. We model the optimal choice of a representative consumer who aims at maximising the expected present value of his intertemporal utility of consumption in a dynamic framework with an infinite horizon, given a dynamic capital accumulation. The consumer gets utility (in the form of a Hyperbolic Absolute Risk Aversion – HARA) from consumption that exceeds a time dependent threshold given by his habit. Such habit is computed as an exponentially weighted sum of his own past (optimal) consumption.\footnote{The model of intrinsic linear habit formation with HARA preferences is sometimes called "additive" habit formation (Chapman, 1998).} Capital is accumulated through savings, i.e. yield which is not consumed, and the evaded taxes on such yield. The consumer optimally chooses how much to evade by taking into account that if evasion is detected a fee must be paid (we modeled this fee as an affine transformation of the evaded yield). We are able to obtain a closed-form solution to this optimal dynamic problem and we perform comparative statics with respect to the fiscal parameters. Our paper makes several contributions to the existing literature on tax evasion:

- we show that in a dynamic setting with habit the optimal evasion strategy does not simply depend on future expected taxpayer's incomes and on the instantaneous risk of being audited (as postulated by the traditional literature), but it also depends on the time at which the risk is evaluated. This is because the risk for the taxpayer depends on his level of habit;

- we show that the attitude towards risk changes over time according to the speed of consumer's habit formation. If the process is "sufficiently" rapid, the consumer becomes less and less willing to take the risk associated with tax evasion. This leads to a reduction in tax evasion for elders;

- we show that the reaction to a change in tax rate depends on the ratio of habit to income; this finding provides us with a possible explanation of the so called Yitzhaki (1974)’s paradox.
The prediction that, for a sufficiently strong habit effect, young taxpayers complain less than older ones is a distinctive result of the model. Quite interestingly, the prediction is consistent with most of the empirical literature which studies the demographic factors affecting tax evasion (Jackson and Milliron, 1986; Andreoni et al., 1998; Richardson and Sawyer, 2001) without being able to explain the reasons for such a behaviour.

The comparative static analysis of optimal tax evasion with respect to changes in fiscal parameters sheds some light on the Yitzhaki (1974) paradox. In particular, the presence of habit in consumption allows our model to be consistent with both signs in the derivative of optimal evasion with respect to the tax rate. Thus, we are able to reconcile theory with the empirical evidence. Finally, we study the differential effect of audits and fees. A change in the level of fees has a larger proportional impact on evasion than a change in the number of audits; some policy implications of this result are discussed.

Understanding behavioural aspects of tax compliance decision is important (Hashimzade et al., 2013) and in our model we propose to use an habit formation approach.

Habit formation has been introduced in the economic literature since the early 1970’s to analyse several issues in economics and finance, including in the study of consumption behaviour (Dybvig, 1995; Chapman, 1998; Carroll et al., 2011), financial decisions (Constantinides, 1990; Campbell and Cochrane, 1999; Chen and Ludvigson, 2009), economic growth (Carroll et al., 2000; Boldrin et al., 2001), monetary policies (Fuhrer, 2000), and optimal taxation (Kochh and Kühn, 2015).

Habit formation implies a form of time-nonseparability of preferences which is very intuitive: mainly, consumer’s today utility depends positively on his current consumption and negatively on a time-varying subsistence level which in models of intrinsic habit corresponds to the consumer’s past consumption (as in Constantinides, 1990; Chapman, 1998; Rozem, 2010).2

Preferences with habit formation are also known to have a psychological underpinning in theory of reference dependent preferences as developed from Kahneman and Tversky (1979) and Tversky and Kahneman (1991). In particular, models with intrinsic habit formation can be viewed as specifications of reference-dependent preferences where the reference point corresponds to the habit. An important difference, however, is that the reference dependent literature emphasises the attitude of individuals to avoid losses, defined as outcomes below the reference point. In this event, the individual experiences a negative utility. In our approach of habit formation utility cannot be negative, which implies that the optimal consumption cannot fall below the level of habit. Here, habit formation is described by the same differential equation used in Con-

---

2Alternatively, the level of habit may depend on the aggregate past consumption, as in Abel (1990)’s model of “catching up with the Joneses”, also known as models with external habit. Models with extrinsic habit are analytically simpler than those with internal habit since the former do not include terms in marginal utility by which an increase in current consumption raises tomorrow habit. For this reason they are more typically used in equilibrium analysis, rather than in studying one agent’s decision problem.
stantinides (1990) which allows, through an appropriate choice of parameters, to model several degrees of persistence in habit formation.

Reference dependent preferences have been previously applied to the study of tax evasion in a static context (Yaniv, 1999; Bernasconi and Zanardi, 2004; Dhami and Al-Nowaihi, 2007). However, this approach has been recently questioned (e.g., Pirolato and Rablen, 2014). We argue that the role of reference dependent preferences in studying tax evasion behaviour is more naturally cast in a dynamic framework where the reference point is replaced by habit.

The paper is organised as follows. Section 2 presents the model of tax evasion, capital accumulation, and preferences with habit formation. Section 3 derives optimal consumption and evasion both in closed form. Section 4 presents simulations and Section 5 concludes. Some technical passages are gathered in the appendices.

2 The model

Over an infinite horizon, we model the behaviour of a representative consumer whose income is produced by capital through a production function. In each period the consumer decides the percentages of income to consume and to evade in order to maximise his expected inter-temporal utility which contains habit.

2.1 Accumulation of capital

In an economy, over the period $[t_0, \infty]$, total income $y_t$ is produced through a deterministic, linear production function of accumulated capital $k_t$:

$$y_t = A k_t,$$

where $A$ measures total factor productivity.

Without taxation, the capital accumulation is described by the following differential equation:

$$dk_t = (y_t - c_t) \, dt,$$

where the expected increment in capital is given by saving, i.e. income $y_t$ net of consumption $c_t$.

The government levies a proportional tax $0 \leq \tau \leq 1$ on income. Without evasion, the net change in capital becomes

$$dk_t = ((1 - \tau) \, y_t - c_t) \, dt.$$

As in Dzhumashev and Gahramanov (2011) and Wen-Zhung and Yang (2001), capital accumulation is deterministic, but it becomes stochastic if the agent decides to engage in tax evasion activities.

We assume that the agent may hide a proportion $e_t \in [0, 1]$ of his income $y_t$. If evasion is detected, a fine $\eta(\tau)$ must be paid on the evaded income (where $\eta(\tau)$ is a non-decreasing function of $\tau$). Thus, the total fine is

$$\eta(\tau)e_t y_t.$$
The specification of $\eta(\tau)$ allows us to consider several fine regimes:

- $\eta(\tau) = \eta_0$: the fine is computed on evaded income as in Allingham and Sandmo (1972);
- $\eta(\tau) = \eta_1 \tau$: the fine is computed on evaded tax as in Yitzhaki (1974);
- $\eta(\tau) = \eta_0 + \eta_1 \tau$: the fine is a combination of the two previous cases.

Evasion introduces risk in Equation (3) because the fine must be paid if evasion is audited. If a proportion $e_t$ of income is evaded, the amount $\tau e_t y_t$ remains unpaid and can be added to the capital accumulation.

As in Levaggi and Menoncin (2012, 2013), and Bernasconi et al. (2015), we model auditing as a Poisson jump process $d\Pi_t$ with expected value and variance given by$^3$

$$E_t[d\Pi_t] = \lambda_t dt,$$
$$V_t[d\Pi_t] = \lambda_t dt,$$  
(5)  
(6)

where $E_t[\cdot]$ and $V_t[\cdot]$ are the expected value and the variance operators, conditional on the information set at time $t$, and $\lambda_t \in [0, \infty]$ is the “intensity” of the process and determines the frequency of audits within a time interval. When $\lambda_t = 0$, the probability of being caught is zero, while when $\lambda_t$ tends towards infinity, the probability of being caught tends towards 1.

Finally, the stochastic process of capital accumulation can be written as

$$dk_t = ((1 - \tau + \tau e_t) y_t - c_t) dt - \eta(\tau) e_t y_t d\Pi_t,$$  
(7)

from which,

$$E_t[dk_t] = (((1 - \tau + \tau \eta(\tau) \lambda_t) e_t) y_t - c_t) dt,$$
$$V_t[dk_t] = \eta(\tau)^2 e_t^2 y_t^2 \lambda_t dt.$$  
(8)  
(9)

2.2 Consumer’s preferences

Consumer’s preferences are represented by a Hyperbolic Absolute Risk Aversion (HARA) utility function with a minimum level of consumption ($h_0$) given by a weighted mean of the past consumptions (i.e. an endogenous habit). Formally, the instantaneous utility function is:

$$U(c_t) = \frac{(c_t - h_t)^{1-\delta}}{1-\delta},$$  
(10)

where $h_t$ is the solution to the following differential equation:

$$dh_t = (\alpha c_t - \beta h_t) dt,$$  
(11)

$^3$This process can be thought of as the limit of a binomial model whose value is 1 with probability $\lambda dt$ and $0$ otherwise.
with an exogenous initial value of $h_t$ given by $h_0$, sometimes called subsistence consumption level, which is assumed to be lower than the initial yield: $h_0 < y_0$. Given $h_0$, the (unique) solution to (11) is:

$$h_t = h_0e^{-\beta t} + \alpha \int_0^t c_se^{-\beta(t-s)}ds.$$

The parameter $\delta$ measures the consumer's risk aversion and we assume it is higher than 1. The model in equations (10)-(11) is sometimes called "addictive" habit formation (see e.g. Chapman 1998, p. 1224). The utility function (10) exists only when $c_t \geq h_t$. Thus, this function works like a reference dependent utility where an infinite penalty is associated to a downward deviation from the habit. This property is equivalent to assume that the consumer is infinitely loss averse. The latter seems a natural assumption to make in the present context, in which the initial subsistence level $h_0$ is lower than the initial wealth $y_0$ and the only source of uncertainty which could bring consumption below the reference point derives from the fines due to tax evasion, which, however, is a choice variable of the agent. Thus, optimal consumption and tax evasion will be sufficiently low to avoid a loss which would bring capital below the level assuring at least the consumption of the habit.

The parameter $\beta$ measures persistence: the higher $\beta$ the lower the weight of the past consumption on the habit. In particular, for $\beta = 0$ past consumptions levels are all taken into account with the same weight, independently of the time they were available. On the other hand, if $\beta \to \infty$ there is no habit formation and the utility function is traced back to the Constant Relative Risk Aversion case (with a subsistence consumption level equal to zero).

The parameter $\alpha$ measures the percentage of the weighted sum of past consumptions which contributes to create the minimum consumption level $h_t$ and it could be interpreted as the intensity at which past consumption becomes a habit. The habit $h_t$ is a positive function of $\alpha$ and a negative function of $\beta$, but the two parameters should be interpreted together to understand their joint role. In general, we should expect $\beta \geq \alpha \geq 0$ (e.g. Constantinides, 1990), nevertheless, some particular cases are worth highlighting:

- $\alpha = 0$: in this case, $h_t$ coincides with $h_0$ discounted at the rate $\beta$. For $\beta$ sufficiently high, this case approximates the classical model of tax evasion without habit or commitments in consumption;

- $\beta = \alpha = 0$: $h_0$ can be interpreted as the classical subsistence consumption level of the linear expenditure models (Varian, 1992; Levaggi and Menoncin, 2013);

- $\alpha = 0$ and $\beta < 0$: in this case there is no habit formation ($\alpha = 0$), but there exists a subsistence consumption level which is increasing over time. If, for instance, we assume that $\beta$ is a (constant) interest or inflation rate, the consumer wants to increase his minimum consumption over time to take into account a kind of opportunity cost of consumption.
Finally, it should be noted that the HARA specification implies decreasing absolute risk aversion in the difference between instantaneous consumption and habit: in particular, with \( z_t \equiv c_t - h_t \) the absolute risk aversion is \(-\frac{\partial^2 \psi}{\partial z^2} \), which is decreasing with respect to \( z_t \). This, of course, implies that the closer the consumption to the habit, the higher the consumer’s risk aversion.

3 Results

The consumer optimises his inter-temporal utility by solving the following problem:

\[
\max_{c_t, h_t} E_0 \left[ \int_0^\infty \frac{(c_t - h_t)^{1-\delta}}{1-\delta} e^{-\rho s} ds \right], \tag{12}
\]

where \( \rho > 0 \) is a subjective discount rate and the state variables \( k_t \) and \( h_t \) solve (7) and (11), respectively.

The solution to Problem (12) is shown in the following proposition.

**Proposition 1.** The optimal consumption and evasion which solve Problem (12), given the capital accumulation (7) and the habit formation (11), are

\[
c_t^* = \frac{1}{\eta A} \left( 1 - \frac{h_t}{k_t} \right) \left( 1 - \frac{1}{(1 - \tau) A + \beta - \alpha} \right) \left( 1 - \frac{\lambda A}{\tau} \right)^{\frac{1}{2}}, \tag{13}
\]

\[
h_t^* = h_t + \frac{k_t ((1 - \tau) A + \beta - \alpha) - h_t}{(1 - \tau) A + \beta} \left( \frac{\rho + \lambda}{\delta} + \frac{\delta - 1}{\delta} \left( \frac{\tau}{\eta} + (1 - \tau) A \right) - \frac{\tau}{\eta} \left( \frac{\lambda A}{\tau} \right)^{\frac{1}{2}} \right). \tag{14}
\]

**Proof.** See Appendix A.

Equation (13) measures the fraction of income that is concealed from the tax authority. Tax evasion negatively depends on \( \frac{h_t}{k_t} \): other things being equal, the higher the ratio, i.e. the closer the level of habit to income, the lower tax evasion. This result follows directly from the hypothesis of addictive habit or infinite loss aversion. In fact, given that the utility function cannot be negative, the consumer is allowed to take the risk associated to tax evasion only if his income is sufficiently higher than his habit, so that even when the taxpayer is discovered evading and convicted to pay a penalty, he/she can nevertheless remain with non-negative utility. On the other hand, when income and habit become closer, the consumer must reduce evasion. In other words, habit in consumption decisions dampen tax evasion. To understand this relationship, it is interesting to compare tax evasion in (13) with the results derived using a HARA (without habit) and a CRRA utility function (Table 1).

With CRRA preferences, evasion is a constant fraction of income. Habit affects the decision to evade because it creates a genuinely dynamic pattern. The
Table 1: Optimal tax evasion under different assumptions on preferences and habit formation

decreasing profile of tax evasion is a specific effect arising when habit accumulates sufficiently rapidly over time. Indeed, when there process of endogenous habit formation is substituted by a fixed subsistence level \(\alpha = \beta = 0\) so that \(h_t = h_0\), equation (13) becomes

\[
e^*_t = \frac{1}{\eta \lambda} \left( 1 - \frac{h_0}{k_t} \left( 1 - \frac{1}{\lambda \eta (1 - \tau)} \right) \right) \left( 1 - \left( \frac{\lambda \eta (1 - \tau)}{\tau} \right)^\frac{1}{\delta} \right),
\]

(15)

which coincides with the result presented in Levaggi and Menoncin (2013).

The evolution of tax evasion over lifetime depends on the ratio \(k_t/h_t\). To study its behaviour over time we start by computing the dynamics of \(h_t^{-1}\):

\[
d\left( \frac{1}{h_t} \right) = -\frac{1}{h_t} \left( \alpha \frac{c_t}{h_t} - \beta \right) dt.
\]

Then we substitute (13) and (14) into (3) and compute the differential of the product \(k_t h_t\). On average, the optimal ratio \(k_t/h_t\) moves over time as follows:

\[
\frac{1}{dt} E_t \left[ d \left( \frac{k_t}{h_t} \right) \right] = -\frac{\xi}{\tau} \left( 1 - \frac{\lambda \eta (1 - \tau)}{\tau} \right) \left( 1 - \left( \frac{\lambda \eta (1 - \tau)}{\tau} \right)^\frac{1}{\delta} \right) - \frac{\eta \lambda}{\delta} + \frac{1}{\delta} \left( \frac{\tau}{\eta} + \left( 1 - \frac{\lambda \eta (1 - \tau)}{\tau} \right)^\frac{1}{\delta} \right) k_t \frac{k_t}{h_t}
\]

\[
+ \left( \frac{2 \alpha - (1 - \tau) A + \beta}{(1 - \tau) A + \beta} \left( \frac{\beta + \lambda}{\delta} + \delta - \frac{1}{\delta} \left( \frac{\tau}{\eta} + (1 - \tau) A - \frac{\tau}{\eta} \left( \frac{\lambda \eta (1 - \tau)}{\tau} \right)^\frac{1}{\delta} \right) \right) k_t \frac{k_t}{h_t} \right)
\]

\[
- \frac{\alpha - (1 - \tau) A + \beta - \alpha}{(1 - \tau) A + \beta} \left( \frac{\beta + \lambda}{\delta} + \left( \frac{\tau}{\eta} + (1 - \tau) A - \frac{\tau}{\eta} \left( \frac{\lambda \eta (1 - \tau)}{\tau} \right)^\frac{1}{\delta} \right) \right) \left( \frac{k_t}{h_t} \right)^2
\]

(16)

This is a quadratic function of the ratio \(k_t/h_t\). It is not possible to sign unambiguously the solution of this quadratic form which may be increasing or
decreasing over time according to the relative strength of the habit parameters. For some special cases, it is however possible to determine its evolution.

In particular, equation (16) shows that α plays a crucial role to influence the dynamics of the ratio since it multiplies the quadratic term \((k_t/h_t)^2\). In fact, for \(α = 0\) (with no true habit, but just a subsistence level whose value is changing over time according to \(β\)) the expected differential of \(k_t/h_t\) has the following simplified form:

\[
\frac{1}{dt} E_t \left[ \alpha \left( \frac{h_t}{k_t} \right) \right] = \left( \frac{\beta + \lambda}{\delta} + \frac{1}{\delta} \frac{1}{\tau} + \frac{1}{\delta} (1 - \tau) A + \beta \left( 1 - \left( \frac{\lambda \eta}{\tau} \right) \right) \right) \left( \frac{h_t}{k_t} - \frac{1}{(1 - \tau) A + \beta} \right),
\]

which allows to distinguish two cases:

1. \(κ > 0\) (this is the most likely case and the one we obtain with the values of the parameters chosen for the simulations): if the initial ratio \(k_0/h_0\) is higher (lower) than \((1 - \tau) A + \beta\)^{-1}, then the ratio \(k_t/h_t\) is increasing (decreasing) over time;

2. \(κ < 0\): the process \(k_t/h_t\) exhibits a mean reverting behaviour and, in the long run, it will tend to reach the equilibrium value \((1 - \tau) A + \beta\)^{-1}, thus the process is on average increasing (decreasing) if it starts below (above) this equilibrium value.

In all the other cases (with \(α \neq 0\)), the path cannot be determined from the algebra, and for this reason in Section 4 we present a simulation exercise. We will show that a decreasing evasion over time is optimal for a sufficiently strong habit formation (i.e. a sufficiently high value of \(α\)).

### 3.1 Comparative statics

Because of the presence of habit, preferences are non time separable, i.e. decisions are taken not only on the basis of present values of the state variables, but also on their whole past history. This implies that the optimal behaviour is no longer a simple feedback strategy, and the optimal solution at time \(t\) is a function of the whole optimal path starting from \(t_0\). In spite of this general difficulty, a closed form solution for the relationship between tax rate and evasion can be computed.

**Proposition 2.** The elasticity of optimal tax evasion to a change in the tax rate may be either positive or negative according to the sign of the following
inequality:

\[
\frac{\partial c^*_t}{\partial \tau} c^*_t \underset{\text{WV}}{\Leftrightarrow} 0 \quad \text{if} \quad \frac{\delta \lambda \kappa}{\kappa} \frac{1}{1 - \frac{\delta \lambda}{\kappa} (1 - \tau) \frac{A \beta - \alpha}{A \tau}} \frac{\left( 1 - \frac{\partial \eta(\tau)}{\partial \tau} \right) \tau}{\eta(\tau)} \left( 1 + \frac{1}{\delta \left( 1 - \frac{\lambda \eta(\tau)}{\tau} \right)^{\frac{1}{\delta}}} \right).
\]

\[\text{Proof.}\] The elasticity of (13) with respect to \(\tau\) is

\[
\frac{\partial c^*_t}{\partial \tau} c^*_t = \frac{\partial}{\partial \tau} \left( \frac{1}{\kappa} (1 - \tau) A \right) + \frac{\partial}{\partial \tau} \left( \frac{1 - \frac{\delta \lambda}{\kappa} (1 - \tau) A \beta - \alpha}{1 - \frac{\delta \lambda}{\kappa} (1 - \tau) A \beta - \alpha} \right) \frac{\partial \eta(\tau)}{\partial \tau} \tau + \frac{\partial}{\partial \tau} \left( 1 - \frac{\lambda \eta(\tau)}{\tau} \right)^{\frac{1}{\delta}} \tau,
\]

and if the derivatives at the numerators are computed, we have

\[
\frac{\partial c^*_t}{\partial \tau} c^*_t = -\frac{\partial \eta(\tau)}{\partial \tau} \tau + \frac{\delta \lambda}{\kappa} (1 - \tau) A \beta - \alpha \frac{1}{1 - \frac{\delta \lambda}{\kappa} (1 - \tau) A \beta - \alpha} + \frac{1}{\delta \left( 1 - \frac{\lambda \eta(\tau)}{\tau} \right)^{\frac{1}{\delta}}} \left( 1 - \frac{\partial \eta(\tau)}{\partial \tau} \right) \tau,
\]

from which the result of the proposition is obtained. \(\Box\)

While most of the current literature on tax evasion have either \((\eta_0 = 0\text{ and } \eta_1 > 0)\) or \((\eta_0 > 0\text{ and } \eta_1 = 0)\), our model is more general since both \(\eta_0 \text{ and } \eta_1\) may be greater than zero. This allows to better interpret fines in the real world. Our results are summarised in Table 2.

A CRRA model replicates, in a dynamic context, the results of most of the traditional literature on tax evasion and, accordingly, it can be considered as a benchmark (Levaggi and Menoncin, 2013). In particular, with CRRA preferences, the shape of the penalty function determines the sign of the elasticity (column 2, Table 2). If the fine is proportional to evaded income \((\eta_1 = 0)\), the model shows a positive effect, originally found by Allingham and Sandmo (1972); while if the fine is proportional to evaded taxes \((\eta_0 = 0)\), the Yitzhaki (1974)'s paradox of a negative impact is confirmed. If the penalty is an affine transformation of the tax rate (with \(\eta_0 > 0\text{ and } \eta_1 > 1\)), the sign of the elasticity depends on the relative strength of both \(\eta_0\) and \(\eta_1\) in particular, the higher \(\eta_0\) relative to \(\eta_1\), the more likely a positive effect prevails over Yitzhaki's.

The results carry over to the case of HARA preferences without habit (column 3, Table 2), with the additional complexity that even when the fine is proportional to evaded income \((\eta_1 = 0)\), the elasticity can be negative if the ratio of the subsistence consumption level \((\kappa_0)\) to capital \((\kappa_2)\) is greater than a threshold. As explained in Levaggi and Menoncin (2013), the ambiguity is consistent with the classical Allingham and Sandmo (1972) model and arises because HARA
Table 2: Sign of the elasticity of optimal tax evasion with respect to tax under different functional forms of the fee

<table>
<thead>
<tr>
<th>( \eta(\tau) )</th>
<th>CRRA ( (h_0 = \alpha = \beta = 0) )</th>
<th>HARA without habit ( (\alpha = \beta = 0 \Rightarrow h_t = h_0) )</th>
<th>HARA with habit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_0 )</td>
<td>( &gt; 0 )</td>
<td>( &gt; 0 ) if ( \frac{h_0}{h_0} &lt; \frac{1}{1+\tau} \left( \frac{\lambda h_0}{\tau} \right)^{-\frac{1}{2}} - 1 )</td>
<td>( &gt; 0 ) if ( \frac{h_0}{h_0} &lt; \frac{1}{1+\tau} \left( \frac{\lambda h_0}{\tau} \right)^{-\frac{1}{2}} - 1 )</td>
</tr>
<tr>
<td>( \eta_1 \tau )</td>
<td>( = -1 )</td>
<td>( &lt; 0 )</td>
<td>( &lt; 0 ) with ( (1-\tau) A + \beta - \alpha &gt; 0 )</td>
</tr>
<tr>
<td>( \eta_0 + \eta_1 \tau )</td>
<td>( &gt; 0 ) if ( 1 &lt; T_{CRRA} ) ( \frac{h_0}{h_0} \frac{1}{1-\left( \frac{\lambda h_0}{\tau} \right)^{-\frac{1}{2}} + \tau} )</td>
<td>( &gt; 0 ) if ( 1 &lt; T_{HARA} ) ( \frac{h_0}{h_0} \frac{1}{1-\left( \frac{\lambda h_0}{\tau} \right)^{-\frac{1}{2}} + \tau} )</td>
<td>( &gt; 0 ) if ( 1 &lt; T_H ) ( \frac{h_0}{h_0} \frac{1}{1-\left( \frac{\lambda h_0}{\tau} \right)^{-\frac{1}{2}} + \tau} )</td>
</tr>
</tbody>
</table>

\[
T_{CRRA} = \frac{1}{\tau} \left( \frac{\lambda h_0}{\tau} \right)^{-\frac{1}{2}} \frac{h_0}{h_0} 
\]
\[
T_{HARA} = \frac{1-\tau}{\tau} \left( \frac{\lambda h_0}{\tau} \right)^{-\frac{1}{2}} \frac{h_0}{h_0} \frac{1}{\eta_0 + \eta_1 \tau} - \frac{\eta_1 \tau}{\eta_0 + \eta_1 \tau} 
\]
\[
T_H = \frac{(1-\tau) A + \beta - \alpha}{\lambda \tau} \left( \frac{\lambda h_0}{\tau} \right)^{-\frac{1}{2}} \frac{h_0}{h_0} \frac{1}{\eta_0 + \eta_1 \tau} - \frac{\eta_1 \tau}{\eta_0 + \eta_1 \tau} 
\]
preferences with a positive subsistence level $h_0$ imply decreasing relative risk aversion.\footnote{The Arrow-Pratt (AP) measure of relative risk aversion for the utility function in equation (10), is $AP = \frac{\partial^2 u}{\partial x^2} \frac{x}{u_x}$, so that $\frac{\partial AP}{\partial x} = -\frac{\partial h_0}{(x_k - h_0)^2}$, which is negative for any positive $h_t$.}

When habit is taken into account (column 4, Table 2), the intensity and time preference parameters enter the specification of the threshold. However, these parameters do not seem able to change the qualitative sign of the results (as long as, at least, $(1 - \tau) A + \beta - \alpha > 0$). In this case, when the fine is an affine penalty function, for the elasticity to be positive the ratio of habit to capital $(h_t/k_t)$ must be lower than a constant threshold. Both the numerator and the denominator of the ratio move over time. Thus, if we combine this result with Proposition 1, we can conclude that as the ratio $h_t/k_t$ increases tax evasion decreases so that it may fall below the threshold. The consumer might then start reacting to a change in the tax rate with an opposite change in tax evasion.

The latter result is quite interesting and indicates that in a dynamic context with habit the relationships between the tax rate and evasion may become sneaky. In particular, it means that a typical taxpayer will increase evasion as a response to an increase in tax rate when evasion is already high, because $h_t/k_t$ is low; and he will respond reducing tax evasion when $h_t/k_t$ is high so that evasion is already low. This pattern can also contribute to explain why the empirical evidence on the Yitzhaki’s paradox is often not clear cutting and, in particular, why the Yitzhaki’s prediction of a negative relationship between evasion and tax rate may be very difficult to refute even when it could be in fact false.

Another important issue that policy-makers face in designing a fiscal system is the relative effectiveness of the two audit instruments: the level of fines and the number of controls. The effects of a change in the audit parameters can be found by comparing the following elasticities computed with respect to $\lambda$, $\eta_0$ and $\eta_1$:

$$\frac{\partial c_t^*}{\partial \lambda} = \frac{1}{\delta} \left( \frac{1}{1 - \left( \frac{\Delta \tau}{\tau} \right)^\frac{1}{\delta}} \right) < 0, \quad (18)$$

$$\frac{\partial c_t^*}{\partial \eta_0} = - \left( 1 + \frac{1}{\delta} \left( \frac{\Delta \tau}{\tau} \right)^\frac{1}{\delta} \right) \eta < 0, \quad (19)$$

$$\frac{\partial c_t^*}{\partial \eta_1} = - \left( 1 + \frac{1}{\delta} \left( \frac{\Delta \tau}{\tau} \right)^\frac{1}{\delta} \right) \eta \tau < 0. \quad (20)$$

Since all these elasticities are negative, we can compare their absolute values.
on the base of the following results:

\[
\left| \frac{\partial e^*_t \eta_0}{\partial \eta_0 e^*_t} \right| = \left( 1 + \left| \frac{\partial e^*_t \lambda}{\partial \lambda e^*_t} \right| \frac{\eta_0}{\eta} \right) \eta_0 \eta,
\]

\[
\left| \frac{\partial e^*_t \eta_1}{\partial \eta_1 e^*_t} \right| = \left( 1 + \left| \frac{\partial e^*_t \lambda}{\partial \lambda e^*_t} \right| \frac{\eta_1 \tau}{\eta} \right) \eta_1 \tau.
\]

**Proposition 3.** Given the elasticities computed in (18)-(20),

1. the elasticity of optimal evasion w.r.t. \( \eta_0 \) is higher than that w.r.t. to \( \eta_1 \) if and only if \( \eta_0 > \eta_1 \tau \);

2. the elasticity of optimal evasion w.r.t. \( \eta_0 \) is higher than that w.r.t. \( \lambda \) if and only if \( \frac{\eta_0 \lambda}{\eta_0 \tau} > \frac{1}{\delta} \left( \frac{\lambda}{\eta_0 \tau} \right)^{\frac{1}{2}} \left( 1 - \left( \frac{\lambda}{\eta_0 \tau} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \);

3. the elasticity of optimal evasion w.r.t. \( \eta_1 \) is higher than that w.r.t. \( \lambda \) if and only if \( \frac{\eta_1 \lambda}{\eta_0 \tau} > \frac{1}{\delta} \left( \frac{\lambda}{\eta_0 \tau} \right)^{\frac{1}{2}} \left( 1 - \left( \frac{\lambda}{\eta_0 \tau} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \).

These results imply that in the extreme cases, when either \( \eta = \eta_0 \) or \( \eta = \eta_1 \tau \), the elasticity w.r.t. the fee is always greater than the elasticity w.r.t. the frequency of controls. Accordingly, evasion can be fought more effectively by increasing the fee, rather than increasing the number of controls. This result is in line with recent empirical evidence (Feige and Cebula, 2011) which shows that tax evasion is decreasing in the audit rate, but it may also explain why the number of audits are decreasing over time (Slemrod, 2007). If fines are more effective in reducing tax evasion and less costly than controls, it may make sense to reduce the latter. On the other hand, fines should be credible: if they are very high, their social cost may be too high to be enforced.

### 4 Simulation

In this section we propose a simulation exercise that allows to study the optimal dynamic behaviour of consumption, tax evasion, habit and capital accumulation of a representative consumer.

We start by presenting a basic scenario where parameters have been initialised as in Table 3. The results are presented in Figure 1. The upper-left and upper-right diagrams show the optimal path of the control variables, respectively evasion and consumption, both as a percentage of income; the bottom-left and bottom-right diagrams show the path for \( \frac{\lambda}{\eta_0} \) and \( \frac{\lambda}{\eta_1} \), respectively.

Over time, the optimal consumption tends towards a given (constant) percentage of the habit that, in our model, can be interpreted as a steady state. The presence of habit reduces tax evasion over the life span. In fact, without habit, i.e. for \( \alpha = \beta = \lambda_0 = 0 \) (CRRA preferences), evasion would be equal to 83% of total income (the values in Table 3 are substituted in the optimal
Table 3: Values of the parameters in the base scenario

<table>
<thead>
<tr>
<th>A</th>
<th>τ</th>
<th>λ</th>
<th>ρ</th>
<th>α</th>
<th>β</th>
<th>δ</th>
<th>T</th>
<th>N</th>
<th>dt</th>
<th>k₀</th>
<th>h₀</th>
<th>η₀</th>
<th>η₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3</td>
<td>0.3</td>
<td>0.25</td>
<td>0.05</td>
<td>1/3</td>
<td>0.25</td>
<td>2.5</td>
<td>35</td>
<td>100</td>
<td>1/250</td>
<td>100</td>
<td>Δ₁₀₀</td>
<td>0.1</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 1: Base scenario
evasion of Table 1) and it would be, as shown in Proposition 1, constant over time. The ratio $h_t/k_t$ increases over time, hence the fraction $e_t^* \frac{h_t}{k_t}$ of evaded income decreases because the consumer becomes more risk averse (due to the property of the HARA utility function illustrated above), and therefore reduces his tax evasion. A slightly different interpretation, driven by the same economic rationality, is that in the first periods consumer is far away from his long run equilibrium. In order to speed up the process of reaching the steady state he also uses tax evasion. Thus, from the optimal tax evasion in Equation (13), we can conclude that compliance is likely to increase with age and that an older consumer tends to evade less than a younger one. This result is in line with the literature on risk taking (Holt and Laury, 2002). However, the policy implications of our model are different from the standard literature. In the risk taking literature, people become more and more risk averse while they grow older. Instead, in our model their risk aversion parameter $(d)$ does not change over time, and they become more conservative because their consumption is closer to their habit. As already noted in the introduction, many empirical studies have reported evidence consistent with such a behaviour, usually without any reference to a theoretical model able to explain the forces driving this effect.

The empirical literature on habit does not agree on its persistence. Dynan (2000) and Fuhrer (2000) argue that there is a very limited persistence while Kueng and Yakovlev (2014) show that persistence is very high. In our framework it is possible to take into account very short lived habit formation by assuming a high value of $(\beta)$ (we use $\beta = 1$). Accordingly, a higher value for $(\alpha)$ is needed in order to prevent a too low level of habit (we take $\alpha = 0.8$). The results are presented in Figure 2.

When habits are short lived, evasion is higher, as one might expect. As shown in the previous section, habit formation is a barrier to tax evasion: in order to keep up with his life style the consumer has to reduce the risk of falls in income due to auditing. The path over time is also quite interesting: tax evasion decreases, but much less than in the base scenario and the variance is considerably higher. It is interesting to note that in this case, even if in the long run the model converges towards a constant level of tax evasion, the adjustment is not immediate and the value the model converges towards is farther from zero. In fact, it takes more than 10 years to reach a "stable" solution, even though habits are short lived.

Let us now study the effects of a long lived habit. In this case, habit slowly forms over time and agents adjust very slowly to new life styles. In simulations we have assumed $\alpha = 0.05$ and $\beta = 0$. The results are presented in Figure 3.

The path in this case is much smoother. Evasion decreases as in the base

---

Figure 2: Short persistence of habit: $\alpha = 0.8, \beta = 1$

Evasion as % of yield

Consumption as % of yield

Habit as % of yield

Habit as % of consumption
Figure 3: Long living habits: $\alpha = 0.05, \beta = 0$

- Evasion as % of yield
- Consumption as % of yield
- Habit as % of yield
- Habit as % of consumption
scenario, but it reaches about 50% (instead of 20%) in 35 years.

Finally, we present the case for a HARA utility function without habit ($\alpha = \beta = 0 \Rightarrow h_t = h_0$) as in Figure 4.

Evasion is lower than in the traditional models, but it increases over time to reach a level very close to the CRRA case.

In general, given that $h_t/k_t$ decreases over time, the elasticity to a change in the tax rate is more likely to be negative at the end rather than at the beginning of the time span considered.

5 Conclusions

Tax evasions decisions have an important intertemporal dimension that the traditional literature has been ignoring until the recent past. We introduce habit formation in consumption and we are able to compute closed form solutions for the optimal dynamic tax evasion and for consumption. In this way, we embed both the habit formation and the dynamic dimension in studying tax evasion.

Our model shows that risk aversion is not the only determinant of the propensity of consumers to evade. In fact, even for a constant risk aversion, the consumer tends to reduce his level of tax evasion over time because of the habit formation. In fact, in the long run, the consumer wants to keep his stand-
ard of living and he is less willing to bear the risk that the payment of a fine reduces his income preventing him from consuming at least his habit.

The standard comparative statics of the model shows that the response of tax evasion to an increase in tax rate is ambiguous. In particular, the response can be positive when the ratio of habit to capital is high, while it can be negative when the ratio is low. The dynamics of the optimal evasion strategy shows that the former case is more (less) likely when the taxpayers are younger (older). Optimal tax evasion is more responsive to changes in the level of fine rather than in the frequency of audit: other things being equal it is better to induce people to reduce their level of tax evasion with fines rather than controls. The intuitive explanation for this result is that higher fines produce a more pronounced income loss by consumers that are found cheating, and it is more difficult to recover from these losses (i.e. a longer period of evasion is needed).

References


A Proof of Proposition 1

Given Problem (12), the value function $J_t(h_t, k_t)$ which solves it can be defined as

$$J_t(h_t, k_t) e^{-\rho t} = \max_{c_t, e_t} E_t \left[ \int_t^\infty \frac{(c_s - h_s)_{1-\delta}}{1-\delta} e^{-\rho s} ds \right],$$

and $J_t(h_t, k_t)$ must solve the following Hamilton-Jacobi-Bellman equation:

$$0 = \frac{\partial J_t}{\partial t} - \rho J_t + \max_{c_t} \left[ \frac{(c_t - h_t)_{1-\delta}}{1-\delta} + \frac{\partial J_t}{\partial h_t}(\alpha c_t - \beta h_t) - \frac{\partial J_t}{\partial k_t} c_t \right]$$

$$+ \max_{e_t} \left[ (J_t(h_t, k_t - \eta e k_t) - J_t) \lambda + \frac{\partial J_t}{\partial k_t}(1 - \tau + \tau e_t) A k_t \right].$$

22
The first order conditions on $c_t$ and $e_t$ are:

$$c_t^* = h_t + \left( \frac{\partial J_t}{\partial k_t} - \frac{\partial J_t}{\partial h_t} \right)^{-1} - \frac{1}{\delta} \frac{\partial J_t}{\partial e_t} (h_t, k_t - \eta e_t' A_k t) \lambda + \frac{\partial J_t}{\partial h_t} \tau A k_t = 0.$$

In this case the so-called “guess function” is

$$J_t = F_t^t (k_t - h_t h_t)^{1-\delta} \quad 1 - \delta,$$

where the functions $F_t$ and $H_t$ must be found in order to solve the HJB equation.

The two transversality conditions are

$$\lim_{t \to \infty} F_t = 0,$$

$$\lim_{t \to \infty} H_t = 0.$$

Given the guess function, the optimal consumption and evasion are

$$e_t^* = \frac{k_t - h_t h_t}{\eta A k_t} \left( 1 - \left( \frac{\lambda H_t}{\tau} \right)^{\frac{1}{\delta}} \right),$$

$$c_t^* = h_t + \frac{k_t - h_t h_t}{F_t} (1 + \alpha H_t)^{-\frac{1}{\delta}}.$$

After plugging these values into the HJB, it becomes

$$0 = \delta F_t^{1-\delta} (k_t - h_t h_t)^{1-\delta} \frac{\partial F_t}{\partial t} - h_t F_t^d (k_t - h_t h_t)^{-\delta} \frac{\partial H_t}{\partial t} - \rho F_t^d (k_t - h_t h_t)^{1-\delta} \frac{1}{\delta}$$

$$+ \frac{1}{1 - \delta} \left( 1 + H_t \alpha \right)^{-\frac{1}{\delta}} - h_t F_t^d (k_t - h_t h_t)^{-\delta} (\alpha h_t - \beta h_t)$$

$$- \frac{\alpha H_t}{F_t^{1-\delta}} (k_t - h_t h_t)^{1-\delta} (1 + \alpha H_t)^{-\frac{1}{\delta}} F_t^d (k_t - h_t h_t)^{-\delta} h_t$$

$$- \frac{\alpha H_t}{F_t^{1-\delta}} (k_t - h_t h_t)^{1-\delta} (1 + \alpha H_t)^{-\frac{1}{\delta}} F_t^d (k_t - h_t h_t)^{-\delta} (1 - \tau) A k_t$$

$$+ F_t^d (k_t - h_t h_t)^{-\delta} \left( \frac{\lambda H_t}{\tau} \right)^{\frac{1}{\delta}} + F_t^d (k_t - h_t h_t)^{-\delta} \left( 1 - \left( \frac{\lambda H_t}{\tau} \right)^{\frac{1}{\delta}} \right),$$

which can be simplified as two differential equations (one in $F_t$ and one in $H_t$) as follows

$$0 = \frac{\partial F_t}{\partial t} - F_t \left( \frac{\rho + \lambda}{\delta} - \frac{1 - \delta}{\delta} \left( \frac{\tau}{\eta} + \left( 1 - \tau \right) A \right) - \frac{\lambda H_t}{\tau} \left( \frac{\lambda H_t}{\tau} \right)^{\frac{1}{\delta}} \right) + \left( 1 + \alpha H_t \right)^{1-\frac{1}{\delta}},$$

$$0 = \frac{\partial H_t}{\partial t} - H_t \left( \left( 1 - \tau \right) A + \beta - \alpha \right) + 1.$$
Given the transversality conditions, the solutions of these two differential equations are:

\[ H_t = \int_t^\infty e^{-f_s^t ((1-\tau)A + \beta - \alpha)ds} = \frac{1}{(1-\tau)A + \beta - \alpha}, \]

\[ F_t = \int_t^\infty (1+\alpha H_s)^{1-\frac{1}{\beta}} e^{-f_s^t \left( \frac{\alpha x}{s} - \frac{1}{2}\left( \frac{\xi}{s} + (1-\tau)A - \frac{\xi}{s} \left( \frac{\lambda}{\tau} \right)^{\frac{1}{\beta}} \right) \right) ds \]

\[ = \frac{\left( \frac{(1-\tau)A + \beta}{(1-\tau)A + \beta - \alpha} \right)^{1-\frac{1}{\beta}}}{e^{\frac{x-A}{\beta} - \frac{1}{2}\left( \frac{\xi}{\tau} + (1-\tau)A - \frac{\xi}{\tau} \left( \frac{\lambda}{\tau} \right)^{\frac{1}{\beta}} \right)}.} \]

If \( F_t \) and \( H_t \) are substituted into the first order conditions, the result of Proposition is obtained.