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Keywords
Monopolistic competition, variable markups, optimal taxation, business cycles

JEL Codes
E1, E2, E3

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Abstract

The neoclassical macroeconomic framework is extended to general preferences over a variety of goods supplied under monopolistic, Bertrand or Cournot competition to derive implications for business cycle, market inefficiencies and optimal corrective taxation. When markups are endogenously countercyclical the impact of shocks on consumption and labor supply is magnified through new intertemporal substitution mechanisms, and the optimal fiscal policy requires a countercyclical labor income subsidy and a capital income tax that is positive along the growth path and converging to zero in the long run. With an endogenous number of goods and strategic interactions, entry affects markups and the optimal fiscal policy requires also a tax on profits. We characterize the equilibrium dynamics and derive explicit tax rules for a variety of intratemporal preference aggregators including the quadratic, directly additive, indirectly additive and homothetic classes.

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We develop a flexible price macroeconomic model which departs from the ubiquitous assumption of constant elasticity of substitution (CES) preferences over a variety of goods and adopts general intratemporal preferences to examine business cycle implications and inefficiencies emerging under imperfect competition. The model generates markups variable over time with the consumption level (and the number of goods provided in the economy when this is endogenized). This is in sharp contrast with standard flexible price models,\footnote{Few existing real business cycle models with imperfect competition feature endogenously variable markups. These are usually obtained introducing collusion (Rotemberg and Woodford, 1992, 2000) or deep habits (Ravn, Schmidt-Grohe and Uribe, 2006). We obtain variable markups within a standard neoclassical framework just by generalizing the demand side.} and allows us to reproduce countercyclical markups over the business cycle; the classic empirical work on the countercyclicality of markups is Bils (1987), while more recent ones include Rotemberg and Woodford (2000) and Nekarda and Ramey (2013). Markup movements deliver new intertemporal inefficiencies in labor supply and in the savings choice, which affect the process of capital accumulation and (when endogenous) the process of business creation. Optimal taxation in this environment requires a variable labor income subsidy, a capital income tax which vanishes only in the long run, and (when entry is endogenous) a tax on dividends. We characterize equilibrium and optimal taxes for a variety of preferences, some of which are largely unexplored in macroeconomic analysis with imperfect competition: in particular, these include the general classes of directly and indirectly additive aggregators.

Our dynamic stochastic general equilibrium (DSGE) model is standard on the supply side, with a Cobb-Douglas production function for intermediate goods. These are sold under perfect competition to final goods' producers that can be engaged in monopolistic, Bertrand or Cournot competition. The main novelties are on the demand side, where preferences are assumed either directly additive, as in the general version of the Dixit and Stiglitz (1977) model, or of any other (symmetric) type, as in the static microfoundation of imperfect competition with product differentiation introduced in Bertoletti and Etro (2016). This allows our framework to nest a variety of flexible price models, including the standard neoclassical real business cycle (RBC) model with perfect competition in homogenous goods started by Kydland and Prescott (1982), models based on CES preferences and monopolistic competition started by Blanchard and Kiyotaki (1987), models with endogenous entry based on homothetic preferences and monopolistic competition (Bibile, Ghironi and Melitz, 2012) or CES preferences and Bertrand or Cournot competition (Etro and Colciago, 2010), and even a version of the endogenous growth model with monopolistic competition over a number of goods increasing at a constant rate (Romer, 1990). Our microfoundation, however, covers any symmetric preferences over differentiated products and the main forms of imperfect competition.

To exemplify the positive and normative implications of endogenous markups, we start from models with a fixed number of goods, as usual in general equilibrium analysis. In this case, the markups depend on the elasticity of the demand function implicit in the intratemporal preferences, which is determined
by the consumption level. When this elasticity is increasing in consumption, markups are countercyclical and the real wage is procyclical. This implies that the propagation of any temporary shock is magnified through new intertemporal substitution mechanisms: in practice consumers anticipate consumption and work more to gain from temporarily lower prices and temporarily higher wages. The opposite happens with procyclical markups. Instead, constant markups generate the same propagation mechanisms as under perfect competition: this is why traditional flexible price models of monopolistic competition based on CES (or homothetic) preferences over a fixed number of goods behave just as under perfect competition except for a reduced labor supply (easily fixed with a constant labor subsidy).

The variability of markups due to market power is inefficient in an aggregate perspective, therefore labor and capital income taxes variable with consumption over time are needed to restore optimality whenever preferences are not homothetic. For instance, preferences represented by a quadratic direct utility (à la Melitz and Ottaviano, 2008) require variable subsidies on both labor and capital income along the growth path. Directly additive aggregators (i.e. with a separable direct utility à la Dixit-Stiglitz) require a countercyclical labor subsidy and a positive capital income tax if and only if the relative risk aversion is decreasing in consumption. Instead, indirectly additive aggregators (i.e. with a separable indirect utility as in Bertoletti and Etro, 2017) obtain the same results if the demand elasticity is decreasing in the prices. We present a variety of functional forms that are largely unexplored in macroeconomics and that can be useful for quantitative and normative analysis, such as the nested power preferences and others which generate countercyclical markups (see Cavallari and Etro, 2016, for a quantitative assessment in closed and open economies).

When entry is endogenous (as in Ghironi and Melitz, 2005), markups depend on the consumption of each good, namely on the production of each firm, as well as on the number of goods. Investments are split between capital accumulation and business creation to equate their expected returns, and the steady state is typically reached through an increase in the stock of capital and the number of firms, with the production of each good declining over time. This is what maintains the equality of the returns, with the return on capital decreasing because the marginal productivity of capital goes down, and the return on business creation decreasing because the profits on each new variety diminish while the number of varieties expands. Also in this case the markups are typically variable over the business cycle and the entry process, which generates an additional channel of propagation of the shocks, can be characterized by either excessive or insufficient entry. We characterize the social planner solution for this framework and derive the optimal taxation that restores it. This requires labor and capital income taxes as well as a tax on dividends (or profits): all of them depend on both the production of each good and the number of the firms.

3 Under extreme conditions, the model can also deliver a) cycling behavior of the number of firms (with high exit rates or a low discount factor), or b) unbounded growth of the number of firms (when the return on business creation is constant à la Romer, 1986). However, this is not the focus of our analysis.
Besides deriving general tax rules, we characterize them for particular classes of preferences. For instance, under directly (indirectly) additive preferences the optimal profit tax is positive if and only if the elasticity of the subutility is decreasing in consumption (increasing in the price). With homothetic preferences, we know from Bilbiie, Ghironi and Melitz (2016) that the optimal taxes depend only on the number of firms, but in the case of Kimball (1995) aggregators we can derive expressions for the optimal taxes and find conditions for the profit tax to be positive or negative.

We complete the analysis with the cases of Bertrand and Cournot competition, which add procompetitive effects of entry and require amendments to the optimal taxation rules. We can summarize the general principles emerging from the analysis in a simple way. As long as markups are countercyclical due to demand side mechanisms (changes in demand elasticity) or supply side mechanisms (changes in the strength of competition), labor income taxation should be countercyclical, capital income taxation should be positive and decreasing toward zero in the long run, and dividend taxation should be positive if markups are too high.

Our analysis builds on recent advances in microeconomic and macroeconomic theory. On the first front, the wide literature on dynamic consumption theory in partial equilibrium has already analyzed a variety of preference specifications (for an interesting treatment with direct additivity see, for instance, Browning and Crossley, 2000), but has usually neglected implications for pricing under imperfect competition and its feedback on consumption. We mainly build on the industrial organization literature which has recently provided more general microfoundations to the analysis of imperfect competition. In particular, while Dixit and Stiglitz (1977) formalized monopolistic competition when the direct utility is additive, Benassy (1996) has considered homothetic preferences and Bertoletti and Etro (2017) have analyzed the case of an indirect utility which is additive - notice that these three classes of preferences have in common only (and nothing less than) the CES preferences. Bertoletti and Etro (2016) have put together these and more general symmetric preferences in a unique framework studying monopolistic, Bertrand and Cournot competition, and we employ their framework in a dynamic model.

Concerning the literature on macroeconomics with imperfect competition and optimal corrective taxation, most of the dynamic models with monopolistic competition have adopted the CES formulation, which delivers constant markups and implies that optimality can be reached with a constant subsidy to labor income. Notable exceptions include Kimball (1995), who has used a class of implicitly additive homothetic aggregators (originally due to Hanoch, 1974), and Bilbiie, Ghironi and Melitz (2008), who have used translog preferences, but both these works have focused on sticky prices and monetary policy. As far as we know, we provide the first characterization of the basic RBC model with general non-homothetic preferences and monopolistic competition. Moving to models with endogenous entry, the closest work in our spirit is by Bilbiie, Ghironi and Melitz (2012, 2016) who have analyzed a dynamic entry model with flexible prices, monopolistic competition and homothetic aggregators to study business
cycle and optimal taxation. \(^4\) Besides differences in modeling intermediate and final goods and entry costs, our main contribution compared to Bilbiie, Ghironi and Melitz (2012) is to depart from homothetic aggregators and consider general intratemporal preferences. As we have noticed, this has radical implications for the difference between perfect and imperfect competition, for the propagation of the shocks and also for the analysis of the optimal taxation. \(^5\) Few other works on optimal taxation with imperfect competition and endogenous entry are only limited to the case of CES preferences and ignore capital accumulation and, therefore, capital income taxation. For instance, Lewis and Winkler (2015) have analyzed optimal taxation in a related but static environment, Bilbiie, Ghironi and Melitz (2016) have analyzed optimal corrective taxation in a dynamic environment without capital and with homothetic preferences, and Colciago (2016) has analyzed the optimal Ramsey taxation in an economy without capital (in the tradition of Lucas and Stockey, 1983) assuming CES preferences with monopolistic, Cournot and Bertrand competition. We will extend some of their results.

The theoretical analysis is organized through subsequent generalizations. Section 1 presents an introductory example with a fixed number of goods and directly additive preferences. Section 2 extends the example to general symmetric preferences considering in detail some relevant classes. Section 3 discusses the most general environment with endogenous entry and different forms of imperfect competition under general preferences. Section 4 is the conclusion.

### 1 An example

In this section we start by considering an example of our general model based on a consumer with additive preferences à la Dixit and Stiglitz (1977):

\[
\hat{U} = \mathbb{E} \sum_{t=1}^{\infty} \beta^{t-1} \left[ \sum_{j=1}^{n} u(C_{jt}) - \frac{uL_t^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}} \right]
\]

where \(\beta \in (0, 1)\) is the discount factor, \(\mathbb{E}[\cdot]\) is the expectations operator, \(n \in \mathbb{Z}^{++}\) is the number of final goods \(j = 1, 2, \ldots, n\), assumed exogenous in this section, \(u(\cdot)\) is the subutility of consumption \(C_{jt}\) of each good \(j\) at time \(t\) and satisfies \(u'(C) > 0\) and \(u''(C) < 0\), and the disutility from labor \(L_t\) is isoelastic with Frish elasticity \(\varphi \geq 0\) and with \(v \geq 0\). We have assumed symmetry of the intratemporal preferences across goods, but none of the qualitative results of this section depends on that.\(^6\)

\(^4\)In the literature on dynamic entry see also Cavallari (2013), La Croce and Rossi (2014) and Poutineau and Vermandel (2015) for interesting related investigations with imperfections in the goods’ market and in the financial market.

\(^5\)Etro (2016a) has derived the generalized Euler equation for a Ramsey model with monopolistic competition, but without analyzing the business cycle implications in the presence of endogenous labor supply or endogenous entry of firms.

\(^6\)As well known, homothetic preferences that are identical across agents are essential for aggregation of the demand functions into a single demand function (of a so-called “represent-
Capital $K_t$ and labor supply $L_t$ are entirely employed by a perfectly competitive sector producing an intermediate good with a Cobb-Douglas production function:

$$ Y_t = A_t K_t^\alpha L_t^{1-\alpha} $$

(2)

where $A_t$ is total factor productivity and $\alpha \in [0,1)$. The intermediate good is the *numeraire* of the economy and can be used to invest in a standard process of capital accumulation with depreciation rate $\delta \in (0,1)$, or to produce final goods with a linear technology.

Each variety $i$ is sold at price $p_{it}$ generating profits $\pi_{it} = (p_{it} - 1)C_{it}$. The consumer receives all the profits as dividends, $\Pi_t = \sum_{j=1}^{n} \pi_{jt}$, and the remuneration of the inputs. The markets for the factors of production are perfectly competitive. The labor market implies the wage $w_t = (1-\alpha)A_t K_t^\alpha L_t^{-\alpha}$, and the capital market implies the rental rate $r_t = \alpha A_t K_t^{\alpha-1} L_t^{-\alpha}$, always in units of intermediate good. In each period, the consumer chooses spending on each variety $C_{jt}$ for $j = 1, 2, ..., n$, labor supply $L_t$ and the future stock of capital $K_{t+1}$ to maximize utility under the resource constraint:

$$ K_{t+1} = K_t(1-\delta) + w_t L_t + r_t K_t + \Pi_t - \sum_{j=1}^{n} p_{jt} C_{jt} $$

(3)

where total profits $\Pi_t$ and prices of final goods and inputs are taken as given.

The FOCs for $C_{jt}$, $L_t$ and $K_{t+1}$ are:

$$ u'(C_{jt}) = \lambda_t p_{jt} \quad \text{for } j = 1, ..., n, $$

(4)

$$ v L_t^1 = \lambda_t w_t $$

(5)

and:

$$ \lambda_t = \beta \mathbb{E}[R_{t+1} \lambda_{t+1}] $$

(6)

where the Lagrange multiplier $\lambda_t$ corresponds to the marginal utility of income and $R_{t+1} = 1 + r_{t+1} - \delta$ is the gross return. Notice that the marginal utility of income can be computed as $\lambda_t = \sum_{j} C_{jt} u'(C_{jt}) / E_t$, where $E_t = \sum_{j} p_{jt} C_{jt}$ is total expenditure in period $t$.

Perfect competition in the production of each differentiated good would imply $p_{jt} = 1$ for each good in each period, so that consumption would be also symmetric, $C_{jt} = C_t$ for any $j$, and the equilibrium equations would be the same of a standard RBC model extended to multiple goods:

$$ u'(C_t) = \beta \mathbb{E} \left\{ \left[ 1 + \alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} - \delta \right] u'(C_{t+1}) \right\} $$

(7)

$$ L_t = \left[ \frac{(1-\alpha)A_t K_t^\alpha u'(C_t)}{v} \right]^{\frac{1}{\alpha}} $$

(8)

tative agent” with average income). Since heterogeneity in preferences and the computational problem of aggregating demand functions are not our concern here, we assume that there is a single agent in the economy.
for a given initial level of capital. Symmetry implies that total consumption $\bar{C}_t \equiv nC_t$ is equally divided between multiple goods. As well known, the concavity of the $u(C)$ function and the transversality condition are necessary and sufficient to guarantee the existence of a unique deterministic steady state with a saddle-path stable equilibrium. Examples include isoelastic, exponential, quadratic and translated power (Stone-Geary) subutilities (see Section 2.3.2 for details on some examples). The perfectly competitive benchmark is useful because it is represents the efficient allocation of resources.

1.1 Monopolistic competition

Under monopolistic pricing with firms producing a single good we need additional conditions for the existence of a saddle-path stable equilibrium. In particular, let us assume that the function $u'(C)C$ is increasing and concave. This is equivalent to:

$$u'(C) + Cu''(C) > 0 \text{ and } 2u''(C) + u'''(C)C < 0 \quad (10)$$

Under this assumption we can show that the equilibrium satisfies:

$$u'(C_t) [1 - \epsilon(C_t)] = \beta E \left\{ [1 + \alpha A_{t+1} K_t^{\alpha} L_t^{1-\alpha}] [u'(C_{t+1}) [1 - \epsilon(C_{t+1})]] \right\}$$

$$L_t = \left[ (1 - \alpha) A_t K_t^\alpha u'(C_t) [1 - \epsilon(C_t)] \right]^{\frac{1}{1+\alpha}} \quad (11)$$

and (9), where $\epsilon(C) = -u''(C)/u'(C)$ is the index of relative risk aversion. The equilibrium has a unique deterministic steady state which is saddle-path stable. To verify this, notice that each firm producing a variety $i$ has profits:

$$\pi_{it} = \frac{u'(C_{it})}{\lambda_t} - 1$$

where we used $p_{it} = u'(C_{it})/\lambda_t$ from (4) and the marginal utility of income $\lambda_t$ is taken as given by each firm under monopolistic competition à la Dixit and Stiglitz (1977). Each monopolist $i$ selects $C_{it}$ to maximize profits, which provides the FOC $u'(C_{it})/\lambda_t + u''(C_{it}) C_{it}/\lambda_t = 1$. Notice that the SOC for profit maximization requires the marginal revenue $u'(C) + u''(C)C$ to be decreasing, or $u'(C)C$ to be concave, as assumed in (10). Rearranging, we obtain the same equilibrium price for each firm, with:

$$p_t = \frac{1}{1 - \epsilon(C_t)} \quad (13)$$

The relative risk aversion $\epsilon(C)$ is smaller than unity under (10). Replacing (13) in (4) and then in (6) and (5), we can rewrite the modified Euler equation for profit maximization requires the marginal revenue $u'(C) + u''(C)C$ to be decreasing, or $u'(C)C$ to be concave, as assumed in (10). Rearranging, we obtain the same equilibrium price for each firm, with:

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The relative risk aversion $\epsilon(C)$ is smaller than unity under (10). Replacing (13) in (4) and then in (6) and (5), we can rewrite the modified Euler equation
and the labor supply as above after using the equilibrium input prices. Finally, since total profits are \( \Pi_t = n(p_t - 1)C_t \) and total expenditure is \( E_t = np_tC_t \), replacing in the resource constraints and using the equilibrium input prices we obtain (9). The dynamic system under monopolistic competition with utility \( u(C) \) is the same of the dynamic system under perfect competition with utility \( u'(C)C \), which has a unique steady state and is stable if and only if the latter is concave, as it holds if and only if (10) holds.

The main difference due to market power is that the marginal utility of consumption \( u'(C_t) \) is replaced in both the Euler condition and the labor supply condition with \( u'(C_t)[1 - \epsilon(C_t)] = u'(C_t) + C_t u''(C_t) \), which can be interpreted as the marginal revenue of each monopolistic firm (Dixit and Stiglitz, 1977). This is smaller than the marginal utility, but positive under (10), which generates a reduction in labor supply and in the production level due to the intratemporal distortion of the labor choice. Moreover, the marginal revenue is decreasing under (10) and variable over time, which creates an intertemporal distortion of the choice of savings determined by the modified Euler equation, and affects capital accumulation.

Introducing monopolistic competition affects crucially the equilibrium path and the propagation of shocks. If markups are endogenously countercyclical (the empirically relevant case according to Bils, 1987, and part of the subsequent empirical literature), the propagation of an expansionary shock is amplified by both intertemporal substitution of consumption and labor supply because final good prices are temporarily lower and real wages are temporarily higher. This happens, for instance, with nested CES preferences. If markups are procyclical, instead, the propagation is smoothed compared to perfect competition, as it happens with CARA or quadratic subutilities.

The model without capital To relate the model with a well known macro-economic framework, let us consider the simple case without capital. With \( \alpha = 0 \), all output is consumed, and total consumption is \( C_t = Y_t = A_tL_t \), which implies \( C_t = A_t^{1+\varphi}[u'(C_t)][1 - \epsilon(C_t)]/u\varphi/n \). Loglinearizing around the steady state we get the impulse response function:

\[
\hat{Y}_t = \frac{1 + \varphi}{1 + \epsilon(C)\varphi + \kappa(C)\epsilon(C)\varphi} \hat{A}_t \tag{14}
\]

where \( \kappa(C) \equiv \epsilon'(C)C/\epsilon(C) \) is the elasticity of the relative risk aversion. The multiplier of the technology shock is always larger than one under decreasing

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8 The modified Euler equation generates a novel impact of the relative risk aversion on risk premiums for assets with uncertain returns, but we will not investigate this issue here. I am thankful to Fabio Ghironi and Andrea Colciago for noticing the implications for asset pricing.

9 It should be now clear that only with isoelastic subutilities, that is in the traditional case of \( u(C) = C^{1/\theta} - 1/\theta < 1 \), markups are constant and monopolistic competition is irrelevant for the propagation of the business cycle (as noticed by Blanchard and Kiyotaki, 1987).

relative risk aversion. The mechanism of propagation relies on the variability of the prices of the final goods. More formally, if we loglinearize the price equation around the steady state and use obvious notation for expectations, we obtain:

$$\hat{p}_{t+1} - \hat{p}_t = \kappa(C) \epsilon(C) \left( \hat{Y}_{t+1} - \hat{Y}_t \right)$$

which relates changes in output growth to relative price changes. The slope of the relation is the elasticity of prices with respect to consumption. This emphasizes how markup inflation depends on a measure of output-gap growth when preferences are not homothetic (without introducing any form of price rigidity familiar from the New-Keynesian literature). In particular, if technology follows an AR(1) process, the case of countercyclical markups implies that consumer price inflation decreases in response to a technology shock, which is in line with the VAR evidence (see for instance Dedola and Neri, 2007).\(^\text{11}\)

### 1.2 Optimal taxation

Monopolistic competition introduces an intratemporal and an intertemporal distortion compared to the perfectly competitive equilibrium, which corresponds to the efficient allocation of resources. As well known, the intuition for the difference between monopolistic competition equilibrium and optimality goes back to the insights of Lerner (1934). Market power introduces a wedge between intermediate and final goods prices. The size of the wedge determines the extent of the suboptimality of labor supply (since leisure is the only good provided without a monopolistic wedge), but it is the variability over time of the wedge that determines the inefficiency in the savings decision and, consequently, in capital accumulation.

The decentralized equilibrium could be easily augmented with taxation. In case of lump sum transfers/taxes available to balance the budget, we can find the tax rates on capital income and labor income that restore the first best allocation of resources obtained above. Let us introduce a subsidy on labor income \(\tau^L_t\) and a tax rate on gross capital income \(\tau^K_t\) at time \(t\). The equilibrium conditions change as follows:

$$u'(C_t) [1 - \epsilon(C_t)] = \beta \mathbb{E} \left\{ (1 - \tau^K_{t+1}) R_{t+1} u'(C_{t+1}) [1 - \epsilon(C_{t+1})] \right\}$$

$$L_t = \left[ \frac{1 - \alpha}{v} (1 + \tau^L_t) A_t K_t^\alpha u'(C_t) [1 - \epsilon(C_t)] \right]^{\frac{\tau^K_t}{1+\tau^K_t}}$$

where \(R_{t+1} = 1 + \alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} - \delta\). We only need to find the sequence of tax rates \(\tau^L_t\) and \(\tau^K_t\) which equalize these conditions to the efficient conditions.

\(^{11}\)As a “back of the envelope” calculation on the impulse response function (14), assume \(\varphi = 1\) and, to match steady state markups of 20%, \(\epsilon(C) = 1/6\). In case of constant markups, the model predicts \(\hat{Y}_2 \approx 1.74 \hat{A}_2\). However, in case markups decrease by 1% after an increase of consumption of 1%, the model delivers \(\hat{Y}_2 \approx 2.07 \hat{A}_2\), with an amplification of the shock on output of about 20% on impact. Instead, if markups increase by 1% after an increase of consumption of 1%, the model delivers \(\hat{Y}_1 = 1.5 \hat{A}_1\). Of course, the amplification can be larger in the presence of savings and capital accumulation.
Immediate computations deliver the optimal taxation system. The optimal labor income subsidy is:

$$\tau_L^t = \frac{\epsilon(C_t)}{1 + \epsilon(C_t)} > 0$$

and it is decreasing (increasing) with consumption if the relative risk aversion $\epsilon(C)$ is decreasing (increasing). The optimal capital income tax rate is:

$$\tau_K^t = \frac{\epsilon(C_{t-1}) - \epsilon(C_t)}{1 - \epsilon(C_t)} < 1$$

and it is positive (negative) if the relative risk aversion is decreasing (increasing) and consumption is growing, but it converges to zero in steady state.

This confirms that in the traditional case of CES preferences a constant and negative labor income tax in each period is sufficient to establish optimality even in the presence of monopolistic distortions in the goods’ market. In other words, tax smoothing à la Barro (1979) occurs if and only if the relative risk aversion is constant. However, this is not the case in general due to monopolistic distortions, and not due to tax distortions. Countercyclical tax rates on labor income emerge when the markups are countercyclical, because a boom reduces prices increasing the real wages. Instead, a procyclical labor taxation becomes optimal when preferences exhibit a procyclical relative risk aversion.

The fact that the optimal capital income tax rate is zero in the steady state is reminiscent of the famous Chamley (1986) result on zero capital income taxation in the long run, but it emerges here due to monopolistic distortions, and not due to tax distortions. Actually, along the growth process, if markups are decreasing over time, capital income should be taxed to promote consumption and slow down capital accumulation. Instead, a negative capital income tax becomes optimal when preferences exhibit a procyclical relative risk aversion.

1.3 Looking forward

Our example may appear as the natural extension of the RBC framework to multiple goods. However, direct additivity is a strong restriction on preferences, and the fact that the shape of the same subutility function governs both intratemporal and intertemporal substitutability is not without consequences under monopolistic competition. The requirement of a relative risk aversion below unity imposes a substantial restriction on intertemporal substitutability in this example. The next objective of this work will be to move toward general intratemporal preferences that can depend in any way on the consumption vector, as analyzed for the first time in Bertoletti and Etro (2016) within a

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12 The same outcome could be reached with commodity taxation (in case of asymmetric preferences such a commodity taxation would need to be differentiated across goods). Notice that perfect competition in the credit market insures that Ricardian equivalence holds; introducing imperfect competition in the credit market would break up such equivalence (see Etro, 2016b).
static context. This will generate prices \( p_t = p(C_t, n) \) depending on both the consumption level and the number of goods, because the latter can affect substitutability between goods through a pure demand side effect. This will be the topic of Section 2.

Another limit of our example is its monopolistic competition assumption, which is unrealistic for markets characterized by a small number of firms producing differentiated goods. Our second objective will be to extend the analysis to other forms of imperfect competition. This will add a supply side dimension in which the number of firms/goods affects markups. Within our example, Cournot prices \( p_t = \frac{n}{(n-1)[1-\epsilon(C_t)]} \) increase compared to monopolistic competition in the same proportion for any consumption level, therefore there are no substantial consequences on the equilibrium dynamics.\(^{13}\) Instead, under Bertrand competition, prices \( p_t = \frac{\epsilon(C_t) + n-1}{(n-1)[1-\epsilon(C_t)]} \) are distorted upward in a way that depends on the consumption level.\(^{14}\) Both the optimal taxes tend to be higher compared to monopolistic competition, because strategic interactions tend to increase both the intratemporal and intertemporal distortions (but the optimality of zero capital income taxation is confirmed).

The final element that our example has neglected is the endogeneity of the number of firms and goods. When we will take this in consideration we will verify that business creation exerts an additional impact on prices \( p_t = p(C_t, n_t) \) through the changes in the number of firms. As we have seen, this impact can work through the demand side (substitution effect) or the supply side (competition effect). The dynamic endogenous market structure will be characterized by a decreasing production of each good while entry takes place and capital accumulates, and the optimal taxation will require an additional tool to optimize entry: a variable tax on profits. The general case will be the topic of Section 3.

\section{A DSGE model with general preferences}

After clarifying the role of market power in the RBC model with additive preferences, we can move to the case of general intratemporal preferences. The supply side is the same as in the baseline model. On the demand side, let us consider

\(^{13}\)The optimal taxes under Cournot competition can be amended as follows:

\[
\tau^{LC}_t = \frac{1}{n-1} + \frac{\epsilon(C_t)}{1-\epsilon(C_t)} \quad \text{and} \quad \tau^{KC}_t = \frac{\epsilon(C_{t-1}) - \epsilon(C_t)}{1-\epsilon(C_t)}
\]

\(^{14}\)Under Bertrand competition the optimal taxes are:

\[
\tau^{LB}_t = \frac{\epsilon(C_t)}{(1-\frac{1}{n})[1-\epsilon(C_t)]} \quad \text{and} \quad \tau^{KB}_t = \frac{\epsilon(C_{t-1}) - \epsilon(C_t)}{[1-\epsilon(C_t)] \left(1-\frac{1-\epsilon(C_{t-1})}{n}\right)}
\]
the following generalized direct utility:

\[ \tilde{U} = E \sum_{t=1}^{\infty} \beta^{t-1} \left[ U(C_t, n) - \frac{\nu L_t^{1+\frac{1}{\phi}}}{1 + \frac{1}{\phi}} \right] \]  

(18)

where the period utility from consumption \( U(C_t, n) \) can now be any symmetric function of the \( n \)-dimensional consumption vector \( C_t \equiv [C_{1t}, C_{2t}, \ldots, C_{nt}] \), assumed increasing and quasi-concave but not necessarily additive. This allows us to separate intratemporal and intertemporal substitutability, which is important to investigate imperfect competition without imposing undue restrictions on the consumption dynamics. Nevertheless, we retain intertemporal additivity, which is crucial for two-stage budgeting and time consistency of the consumption decisions.

For a given expenditure \( E_t = \sum_{j=1}^{n} p_{jt} C_{jt} \) at time \( t \), the first order conditions for the consumption of each good can be written in terms of the Hotelling-Wold identity:

\[ p_{it} = \frac{U_i(C_t, n) E_t}{\sum_{j=1}^{n} C_{jt} U_j(C_t, n)} \quad i = 1, \ldots, n \]

where \( U_i(C_t, n) = \partial U(C_t, n) / \partial C_{it} \) is the marginal utility of consumption of good \( i \) in period \( t \). The above system for inverse demand functions can be inverted to obtain the system of Marshallian demand functions as a function of the price vector \( p_t \equiv [p_{1t}, p_{2t}, \ldots, p_{nt}] \) and of the expenditure \( E_t \). The corresponding vector \( C_t(p_t/E_t) \) allows us to define the intratemporal preferences also in terms of the indirect utility function:

\[ V \left( \frac{p_t}{E_t}, n \right) \equiv U \left( C_t \left( \frac{p_t}{E_t} \right), n \right) \]

where expenditure is the fruit of intertemporal allocation across periods. The demand for each good can be derived through the Roy’s identity:

\[ C_{it} = \frac{V_i \left( \frac{p_t}{E_t}, n \right) E_t}{\sum_{j=1}^{n} \frac{p_{jt}}{E_t} V_j \left( \frac{p_t}{E_t}, n \right)} \quad i = 1, \ldots, n \]

where \( V_i \left( \frac{p_t}{E_t}, n \right) \equiv \partial V \left( \frac{p_t}{E_t}, n \right) / \partial p_{it} \). In this perspective, the preferences in (18) can be expressed equivalently with the mixed utility:

\[ \tilde{U} = E \sum_{t=1}^{\infty} \beta^{t-1} \left[ V \left( \frac{p_t}{E_t}, n \right) - \frac{\nu L_t^{1+\frac{1}{\phi}}}{1 + \frac{1}{\phi}} \right] \]  

(19)

in which case total expenditure and labor supply become the choice variables for the consumer in each period, with consumption allocated to goods according to the Roy’s identity.
2.1 Generalized monopolistic competition

Under monopolistic competition, each firm producing a variety $i$ maximizes its profits $\pi_{it} = (p_{it} - 1)C_{it}$ either with respect to the consumption level $C_{it}$ or the price $p_{it}$ considering only its direct effects on the demand. As shown by Bertoletti and Etro (2016, Prop. 1), the symmetric equilibrium price can be derived as:

$$p_t = \frac{1}{1 - \epsilon(C_t, n)}$$

where the relevant elasticity is the Morishima elasticity of complementarity between goods (see Blackorby and Russell, 1981):

$$\epsilon(C, n) = -\frac{\partial(p_i/p_j)}{\partial C_i} \frac{C_i}{(p_i/p_j)}$$

with $p_i = p_j$. Direct computation from $p_i/p_j = U_i(C_t, n)/U_j(C_t, n)$ allows one to obtain an expression for this elasticity, which is assumed less than unitary to guarantee a well defined price:

$$\epsilon(C, n) = \frac{CU_{ij}(C, n)}{U_j(C, n)} - \frac{CU_{ii}(C, n)}{U_i(C, n)} < 1$$

where, given the symmetric utility $U(C_t, n) \equiv U(C_t u, n)$, we defined the marginal utility of consumption of one good as $U_i(C_t, n) \equiv \frac{\partial U(C_t u, n)}{\partial C_{it}} > 0$, and the second derivative $U_{ij}(C_t, n) \equiv \frac{\partial^2 U(C_t u, n)}{\partial C_{it} \partial C_{jt}}$, with $u$ as the $n$-dimensional unit vector.$^{15}$

If preferences are expressed in terms of the indirect intratemporal utility, Bertoletti and Etro (2016, Prop. 1) show that the relevant elasticity can be also expressed as $\epsilon(C, n) = \frac{1}{\theta(s, n)}$ where $s \equiv p/E$ is the price-income ratio and is the Morishima elasticity of substitution between goods:

$$\theta(s, n) = -\frac{\partial(C_i/C_j)}{\partial s_i} \frac{s_i}{(C_i/C_j)}$$

with $C_i = C_j$. Direct computation from $C_i/C_j = V_i(s, n)/V_j(s, n)$ shows that this corresponds to:

$$\theta(s, n) \equiv \frac{sV_{ji}(s, n)}{V_j(s, n)} - \frac{sV_{ii}(s, n)}{V_i(s, n)} > 1$$

under symmetry, where we defined $V(s_t, n) \equiv V(s_t u, n)$ and its derivatives $V_i(s_t, n) \equiv \frac{\partial V(s_t u, n)}{\partial s_{ti}} > 0$ and $V_{ij}(s_t, n) \equiv \frac{\partial^2 V(s_t u, n)}{\partial s_{ti} \partial s_{jt}}$. In this perspective the equilibrium price solves:

$$p_t = \frac{\theta(s_t, n)}{\theta(s_t, n) - 1}$$

$^{15}$It is natural to notice that with directly additive preferences the cross effect $U_{ij}(C, n)$ is null and the Morishima elasticity reduces to the relative risk aversion.
where the symmetric budget constraint implies \( s_t = p_t / E_t = 1 / nC_t \).

Notice that, as long as the number of goods is fixed, prices change only with aggregate consumption. Nevertheless, the more general nature of preferences does affect the equilibrium dynamics as summarized in what follows:

**Proposition 1.** In a dynamic model with general intratemporal preferences and monopolistic competition with an exogenous number of goods the equilibrium satisfies:

\[
U_i(C_t, n) [1 - \epsilon(C_t, n)] = \beta \mathbb{E} \{ R_{t+1} U_i(C_{t+1}, n) [1 - \epsilon(C_{t+1}, n)] \} 
\]

\[
L_t = \left[ \frac{1 - \alpha}{\nu} A_t K_t^\alpha U_i(C_t, n) [1 - \epsilon(C_t, n)] \right]^{\frac{\phi}{\phi - \alpha}} \tag{23}
\]

\[
K_{t+1} = K_t (1 - \delta) + A_t K_t^\alpha L_t^{1-\alpha} - nC_t \tag{24}
\]

where \( \epsilon(C, n) \) is the Morishima elasticity of complementarity.

The condition that \( U_i(C, n) [1 - \epsilon(C, n)] \) is decreasing in consumption will be verified in the examples below. Essentially, this requires that the function \( U(C, n) \) is concave enough in the symmetric consumption level. The business cycle properties of the model are qualitatively similar to the baseline case, but the general formulation is much more flexible in modeling intertemporal substitution, which governs savings, separately from intratemporal substitution, which governs markups. This allows us to improve the quantitative performance of the macroeconomic model in matching realistic impulse response functions compared to the case of perfect competition (see Cavallari and Etro, 2016).

### 2.2 Social planner problem and optimal taxation

To evaluate the inefficiencies of the general model, let us consider the relevant social planner problem:

\[
\max_{K_{t+1}, L_t} \mathbb{E} \sum_{t=1}^{\infty} \beta^{t-1} \left[ U \left( \frac{A_t K_t^\alpha L_t^{1-\alpha} - K_{t+1} + K_t (1 - \delta)}{n}, n \right) - \frac{v L_t^{1+\gamma}}{1 + \frac{\phi}{\gamma}} \right] \tag{26}
\]

where we have solved the symmetric resource constraint \( K_{t+1} = K_t (1 - \delta) + A_t K_t^\alpha L_t^{1-\alpha} - nC_t \) for consumption of each good and replaced the latter in the symmetric utility \( U(C_t, n) \). The FOCs are:

\[
\frac{\partial U(C_t^*, n)}{\partial C_t^*} = \frac{n}{A_t K_t^\alpha L_t^{1-\alpha}} \left[ 1 + \alpha A_{t+1} K_{t+1}^{\alpha} L_{t+1}^{1-\alpha} - \delta \right] \frac{\partial U(C_t^*, n)}{\partial C_t^*} \tag{27}
\]

and:

\[
L_t^* = \left[ \frac{1 - \alpha}{\nu} A_t K_t^\alpha \frac{\partial U(C_t^*, n)}{\partial C_t^*} \right]^{\frac{\phi}{\phi - \alpha}} \tag{28}
\]
An intuitive property of symmetric functions is that \( \frac{\partial U(C, n)}{\partial C} = \frac{\partial U(C, n)}{\partial C_j} \)
which implies \( nU_i(C, n) = \frac{\partial U(C, n)}{\partial C} \)
and allows us to rewrite the optimality conditions as:

\[
U_i(C^*_t, n) = \beta E \left\{ 1 + \alpha A_t K^*_t L^*_t - \delta \right\} U_i(C^*_{t+1}, n) \quad (27)
\]
and:

\[
L_t^* = \left[ 1 - \frac{\alpha}{\upsilon} A_t K^*_t U_i(C^*_t, n) \right] \frac{1}{\pi W_0} \quad (28)
\]

These correspond, as expected, to the equilibrium conditions under perfect competition. Accordingly, the optimal taxation can be easily generalized as follows:

**Proposition 2.** In a dynamic model with general intratemporal preferences and monopolistic competition with an exogenous number of goods the optimal labor and capital income taxes are:

\[
\tau^L_t = \frac{\epsilon(C_t, n)}{1 - \epsilon(C_t, n)} \quad \text{and} \quad \tau^K_t = \frac{\epsilon(C_{t-1}, n) - \epsilon(C_t, n)}{1 - \epsilon(C_t, n)}
\]

where \( \epsilon(C, n) \) is the Morishima elasticity of complementarity.

The optimal tax rates can be directly computed as functions of the number of goods available and the production level of each firm once the Morishima elasticity if computed. Some examples will clarify how one can characterize and analyze equilibria in this generalized environment under monopolistic competition.

### 2.3 Functional forms for macroeconomic analysis

Our model nests the traditional preferences used in most macroeconomic applications. These involve what we can define as log-CES intratemporal preferences:

\[
U(C, n) = \log \left[ \sum_{j=1}^{n} \frac{C_j^{\theta - 1}}{\theta - 1} \right] \quad (29)
\]

with \( \theta > 1 \). Introducing this specification in (18) allows one to distinguish the intertemporal elasticity of substitution, which here is unitary due to the logarithmic transformation, from the intratemporal elasticity of substitution, which is \( \theta \). As well known, this specification leads to constant markups, therefore market power induces only a suboptimal labor supply without adding any intertemporal distortion. The optimal taxation is simply given by a constant labor income subsidy.

In what follows we consider a variety of examples that are largely new in macroeconomic applications. First, we analyze a quadratic utility function which delivers markups depending on both consumption and the number of firms. Then, we analyze three general classes of preference aggregators with
polar properties: markups will depend only on consumption under direct additivity, only on the number of firms under homotheticity or only on their product under indirect additivity. Remarkably CES aggregators belong to each one of these three classes of preferences but are the only ones to do that, therefore these three classes generalize the traditional CES case along different dimensions. We will also present specific examples for each class of preferences.

2.3.1 Quadratic direct utility

Let us start with an example of non-homothetic and non-separable preferences. The most familiar one is probably the case of a quadratic direct utility function:

\[ U(C, n) = \alpha \sum_{j=1}^{n} C_j - \gamma \sum_{j=1}^{n} \frac{C_j^2}{2} - \eta \left( \sum_{j=1}^{n} C_j \right)^2 \]  

(30)

where \( \eta \geq 0 \) parametrizes the cross-substitutability; notice that only for \( \eta = 0 \) we obtain an additive quadratic direct utility which fits in the example of Section 1. Similar preferences are often employed in trade models with heterogeneous firms (see Melitz and Ottaviano, 2008, for a quasilinear version) and have been already used by Ottaviano (2012) in a dynamic model, but without capital accumulation or endogenous labor supply.\(^{16}\)

In this case we have \( U_i(C, n) = \alpha - (\gamma + \eta n) C \) and the Morishima elasticity is \( \epsilon(C, n) = \frac{\gamma C}{\alpha - (\gamma + \eta n) C} \), which is increasing in individual consumption as well as in the number of goods, due to non-additivity. This delivers the monopolistic price:

\[ p_t = \frac{\alpha - (\gamma + \eta n) C_t}{\alpha - (2\gamma + \eta n) C_t} \]

which requires \( C_t < \alpha/(2\gamma + \eta n) \) for any \( t \), and is increasing in consumption and in the number of goods.\(^{17}\) Under monopolistic competition we have the modified Euler equation:

\[ \alpha - (2\gamma + \eta n) C_t = \beta \mathbb{E} \left[ R_{t+1} [\alpha - (2\gamma + \eta n) C_{t+1}] \right] \]  

(31)

It is useful to notice that, as in the celebrated analysis of Hall (1978) with perfect competition in a single good, these quadratic preferences can generate the result for which consumption is a martingale also with monopolistic competition.\(^{18}\)

\(^{16}\)Bertoletti and Etro (2016) have proposed more general versions of preferences with quadratic direct and indirect utility functions, and these could be easily employed here.

\(^{17}\)This shows that an increase in the number of goods does not induce a markup reduction under quadratic utility unless it also forces lower consumption of each good, a point already noticed by Bertoletti and Epifani (2014).

\(^{18}\)In case of a constant interest rate, we have:

\[ C_{t+1} = \beta_0 + \beta_1 C_t + \varepsilon_t \]

with \( \beta_0 = \frac{\alpha}{\alpha - (2\gamma + \eta n)} \left( 1 - \frac{1}{\beta R} \right), \beta_1 = \frac{1}{\beta R} \) and \( \varepsilon_t \) white noise. The only difference compared to perfect competition is in the constant term.
In this case, the optimal taxation can be easily derived as:

\[ \tau_L = \frac{\gamma C_t}{\alpha - (2\gamma + n\eta)C_t} \quad \text{and} \quad \tau^K = \frac{\alpha\gamma(C_{t-1} - C_t)}{[\alpha - (2\gamma + n\eta)C_t][\alpha - (\gamma + n\eta)C_{t-1}]} \]

which provides a procyclical labor subsidy and a negative capital income taxation on the growth path.

### 2.3.2 Directly additive aggregators

Let us consider the case where the intratemporal utility is a monotonic transformation \( U(\cdot) \) of a directly additive aggregator:

\[ U(C, n) = U\left( \sum_{j=1}^{n} u(C_j) \right) \]  

(33)

Of course, the baseline model of Section 1 corresponds to this when \( U \) is a linear function and we have the traditional case (29) when \( U \) is a logarithmic function and \( u \) is a power function. Introducing a non-linear transformation, the intertemporal utility (18) is not anymore additive in the consumption of each good, while the intratemporal elasticity of substitution is the same as in the baseline model. This implies that in each period monopolistic pricing remains:

\[ p_t = \frac{1}{1 - \epsilon(C_t)} \]

with Morishima elasticity \( \epsilon(C) = -u''(C)C/u'(C) \). The modified Euler condition can be expressed as:

\[ U'(nu(C_t))u'(C_t)[1 - \epsilon(C_t)] = \beta E\{R_{t+1}U'(nu(C_{t+1}))u'(C_{t+1})[1 - \epsilon(C_{t+1})]\} \]

(34)

which differs from its perfectly competitive version for the Morishima elasticity present on each side.\(^\text{19}\) Accordingly, the implications for optimal taxation are exactly the same as in the baseline model, with:

\[ \tau_L = \frac{\epsilon(C_t)}{1 - \epsilon(C_t)} \quad \text{and} \quad \tau^K = \frac{\epsilon(C_{t-1}) - \epsilon(C_t)}{1 - \epsilon(C_t)} \]

(35)

which is independent from the transformation function \( U \). However, the mechanism of intertemporal substitution is now governed by the shape of the transformation function and not just by the subutility. This allows us to separate the roles of intertemporal substitutability and market power in determining the reaction of aggregate variables to shocks. Let us consider some simple specifications for the subutility function (see Cavallari and Etro, 2016, for the additional case of Stone-Geary preferences, which delivers countercyclical markups).

\(^{19}\)Since directly additive aggregators have not been used in macroeconomic applications (besides the log-CES specification), it is useful to express also the labor supply equation:

\[ L_t = \left\{ \frac{1 - \alpha}{\alpha} A_t K_t^{\alpha} U'(nu(C_t))u'(C_t)[1 - \epsilon(C_t)] \right\}^{\frac{\alpha}{1 - \alpha}} \]
**CARA subutility**  With the subutility:

\[ u(C) = 1 - e^{-\chi C} \]  

where \( \chi > 0 \), we have \( \epsilon(C) = \chi C \) (see Bertoletti, 2006). Assuming \( C_t < 1/\chi \) for any \( t \), the monopolistic price is:

\[ p_t = \frac{1}{1 - \chi C_t} \]

and is procyclical. Therefore, the propagation of shocks is dampened by market power relative to the case of perfect competition. Consequently, the optimal labor subsidy is procyclical and the optimal capital income tax is negative on the growth path.\(^{20}\)

**Nested power subutility**  Let us consider a linear combination of power functions (Bertoletti, Fumagalli and Poletti, 2008). The simplest example is the following:

\[ u(C) = \alpha C + \frac{\theta}{\theta - 1} C^{\frac{\theta - 1}{\theta}} \]

for \( \alpha \geq 0 \). This specification nests the CES case for \( \alpha = 0 \). Otherwise, it provides a countercyclical elasticity \( \epsilon(C) = \frac{1}{\theta(1 + \alpha C^1/\theta)} \). Monopolistic prices are:

\[ p_t = \frac{\theta(1 + \alpha C_{t+1}^{1/\theta})}{\theta(1 + \alpha C_{t}^{1/\theta}) - 1} \]

and decrease in consumption. This implies that the propagation of any expansionary shock is magnified by monopolistic competition compared to perfect competition whenever \( \alpha > 0 \). Moreover, optimal taxation requires a countercyclical labor subsidy and a positive capital income tax on the growth path.\(^{21}\)

### 2.3.3 Indirectly additive aggregators

A general class of preferences recently introduced in a static analysis of monopolistic competition is characterized by an indirect utility that is additive

\(^{20}\)The optimal taxes with CARA subutility are:

\[ \tau_L^t = \frac{\chi C_t}{1 - \chi C_t} \quad \text{and} \quad \tau_K^t = \frac{-\chi(C_t - C_{t-1})}{1 - \chi C_t} \]

\(^{21}\)The optimal taxes with translated power subutility are:

\[ \tau_L^t = \frac{1}{\theta - 1 + \alpha C_{t+1}^{1/\theta}} \quad \text{and} \quad \tau_K^t = \frac{\alpha (C_{t+1}^{1/\theta} - C_{t-1}^{1/\theta})}{\theta \left( 1 + \alpha C_{t-1}^{1/\theta} \right) \left( \theta - 1 + \alpha C_{t}^{1/\theta} \right)} \]
(Bertoletti and Etro, 2017). In particular, let us assume that the intratemporal indirect utility in (19) is additively separable as in:

$$V \left( \frac{P}{E} \right) = U \left( \sum_{j=1}^{n} v \left( \frac{p_j}{E} \right) \right) \quad (40)$$

where \(v(s)\) is decreasing and convex in the price-expenditure ratio \(s = p/E\) and \(U(\cdot)\) is always the monotonic transformation that insures concavity in income.

In this case, we can reformulate easily the decentralized equilibrium. The demand for each good \(i\) in period \(t\) derives from the Roy’s identity, and the corresponding profits are:

$$\pi_{it} = \frac{(p_{it} - 1) v' \left( \frac{p_{it}}{E_t} \right) E_t}{\sum_{j=1}^{n} v' \left( \frac{p_{jt}}{E_t} \right) \left( \frac{p_{jt}}{E_t} \right)}$$

whose denominator is directly related to the marginal utility of income and is taken as given under monopolistic competition. The monopolistic price satisfies:

$$p_t = \frac{\theta(p_t/E_t)}{\theta(p_t/E_t) - 1} \quad (41)$$

where \(\theta(s) \equiv -v''(s)s/v'(s)\) is the demand elasticity in function of the price-expenditure ratio, and corresponds to what we defined above as the Morishima elasticity of substitution.\(^{23}\)

However, symmetry implies \(E_t = np_tC_t\), therefore the price can be seen as a function of the product of consumption and number of firms through \(\theta(1/nC)\), and it is increasing in the consumption level if and only if \(\theta' > 0\). The case of CES preferences emerges again if \(\theta' = 0\), namely with \(v(s) = s^{1-\theta}\) for \(\theta > 1\).

In each period, since the direct demand of each good is already implicit in the specification of preferences, the consumer chooses only total spending \(E_t\) and labor supply \(L_t\) to maximize intertemporal utility (19) under the resource contraint. The problem:

$$\max_{E_{t+1}, L_t} \tilde{U} = E \sum_{t=1}^{\infty} \beta^{t-1} \left[ U \left( \sum_{j=1}^{n} v \left( \frac{p_{jt}}{E_t} \right) - \frac{vL_{t+1}^{1+\frac{1}{\gamma}}}{1 + \frac{1}{\gamma}} \right) \right] \quad (42)$$

$$K_{t+1} = K_t(1 - \delta) + w_tL_t + r_tK_t + \Pi_t - E_t \quad (43)$$

leads to the Euler condition:\(^{24}\)

$$\frac{U'[nv \left( \frac{p_t}{E_t} \right) v' \left( \frac{p_t}{E_t} \right) p_t}{E_t^2} = \beta E \left\{ R_{t+1}U'[nv \left( \frac{p_{t+1}}{E_{t+1}} \right) v' \left( \frac{p_{t+1}}{E_{t+1}} \right) p_{t+1}} \right\} \quad E_{t+1}^2$$

\(^{22}\)More precisely, \(\lambda_tE_t = -\sum_{j=1}^{n} v' \left( \frac{p_{jt}}{E_t} \right) \left( \frac{p_{jt}}{E_t} \right) \).

\(^{23}\)It is natural to notice that with indirectly additive preferences the cross effect \(V_{ij}(s, n)\) is null.

\(^{24}\)A similar result is obtained in Etro (2016,b) to analyze a Ramsey model of consumption growth and used by Boucekkine et al. (2016) for an interesting analysis of endogenous growth.
This is a particular case of our general framework once we recognize that in a symmetric equilibrium \( p_t/E_t = 1/nC_t \).

Since a well behaved indirect utility function is increasing in income but not necessarily concave, the transformation \( U \) must insure enough concavity for the equilibrium to be well defined. For instance, if \( U \) is linear, one can verify that this is the case if and only if \( \theta(s) \in (1,2) \), which is quite restrictive for our analysis of monopolistic competition.\(^{25}\) Therefore, in what follows we focus on the case of a logarithmic transformation, with \( U(\cdot) = \log(\cdot) \), whose concavity insures saddle-path stability under more general conditions.

Defining \( \eta(s) = -v'(s)s/v(s) \) as the elasticity of the subutility, we have \( \eta(p/E) = \eta(1/nC) \) and \( \epsilon(C,n) = 1/\theta(1/nC) \). This allows us to rewrite the modified Euler condition as a particular case of the general model:

\[
\eta \left( \frac{1}{nC} \right) C \left[ 1 - \theta \left( \frac{1}{nC} \right)^{-1} \right] = \beta E \left\{ R_t \eta \left( \frac{1}{nC_{t+1}} \right) C_{t+1} \left[ 1 - \theta \left( \frac{1}{nC_{t+1}} \right)^{-1} \right] \right\}
\]

With CES preferences both elasticities are constant and we are back to the same results as with perfect competition, as under the log-CES specification (29). When the demand elasticity is variable, instead, market power has bite and affects the business cycle properties of the model.\(^{26}\)

The optimal taxation can be derived from the general principles stated above as follows:

\[
\tau^L_t = \frac{1}{\theta(s_t) - 1} \quad \text{and} \quad \tau^K_t = \frac{\theta(s_t - \theta'(s_{t-1}))}{\theta(s_t) - 1}
\]

where \( s_t = p_t/E_t = 1/nC_t \). The labor subsidy is countercyclical and the capital income taxation is positive if and only if \( \theta' < 0 \).

This class of preferences can be easily analyzed in macroeconomic models. To show this, we will now present some examples deriving the relevant Euler equations (see Etro, 2016a, for the additional case of translated power preferences, which delivers countercyclical markups).

**Linear demand preferences** As a first example, let us consider the preferences introduced by Bertoletti and Etro (2017) in a static context with subutility:

\[
v(s) = \frac{(a - s)^{1+\gamma}}{1+\gamma}
\]

\(^{25}\)I am thankful to Paolo Bertoletti for insightful discussions on this condition and on the concept of intertemporal elasticity of substitution. On the latter, a useful reading is Browning and Crossley (2000).

\(^{26}\)Since indirectly additive utility functions have never been used in macroeconomics, it is useful to express also the general labor supply equation:

\[
L_t = \left\{ \frac{1-\alpha}{vC_t} A_t K_t^\alpha \left[ 1 - \theta \left( \frac{1}{nC_t} \right)^{-1} \right] \right\} \frac{\omega}{\gamma + \omega}
\]
where \( a > 0 \) represents the maximum willingness to pay (demand is zero for normalized prices above this level) and \( \gamma > 0 \) parametrizes the demand elasticity. This parameter is estimated as unitary by Bertoletti et al. (2016) in a multicountry context, which supports a linear demand function. In general, the demand is a power function of a linear demand, with elasticities 
\[
\theta(s) = \frac{\gamma s}{a-s} \quad \text{and} \quad \eta(s) = \frac{(1+\gamma)s/(a-s)}{a-n},
\]
which are increasing in \( s \). Therefore the elasticity \( \epsilon(C) = \frac{\gamma}{a-s} \) is linearly increasing in consumption under the regularity condition \( 1 < anC_t < 1 + \gamma \). The Euler equation can be derived as:
\[
\frac{1}{(anC_t - 1) p_t C_t} = \beta \mathbb{E} \left\{ \frac{R_{t+1}}{(anC_{t+1} - 1) p_{t+1} C_{t+1}} \right\}
\]
where under perfect competition we should set \( p_t = 1 \) and under monopolistic competition we should set:
\[ p_t = \frac{\gamma}{1 + \gamma - anC_t} \tag{47} \]
Notice that the markup is procyclical, therefore market power tends to enhance consumption smoothing compared to perfect competition. The optimal taxation requires a procyclical labor income subsidy and a negative tax on capital income.\(^{27}\)

**Log-linear demand preferences** Let us consider the exponential indirect subutility:
\[
v(s) = e^{-bs} \tag{48}
\]
with \( b > 0 \) parametrizing the semi-elasticity of demand, which is loglinear in this case. Notice that \( \theta(s) = \eta(s) = bs \) which is increasing in \( s \), therefore \( \epsilon(C) = \frac{nC}{b} \) is increasing in individual consumption. Under monopolistic competition, the equilibrium price is:
\[
p_t = \frac{b}{b - nC_t} \tag{49}
\]
which is procyclical under the regularity condition \( C_t < b/n \). The Euler condition becomes:
\[
\frac{1}{C_t^2 p_t} = \beta \mathbb{E} \left\{ \frac{R_{t+1}}{C_{t+1}^2 p_{t+1}} \right\}
\]
Under perfect competition we would have a very simple variation of the traditional Euler condition, isomorphic to the case of utility isoelastic in a CES aggregator (with intertemporal elasticity of substitution equal to 2). However, the modified Euler equation under monopolistic competition depends on demand elasticity through the parameter \( b \), as well as on the number of goods. Also in

\(^{27}\)The optimal taxes are:
\[
\tau^L_t = \frac{anC_t - 1}{\gamma + 1 - anC_t} \quad \text{and} \quad \tau^K_t = \frac{an(C_t - 1) - C_t}{\gamma + 1 - anC_t}
\]
this case market power dampens the propagation of shocks compared to perfect competition, and the first best is restored subsidizing both labor and capital income.\textsuperscript{28}

### 2.3.4 Homothetic aggregators

Finally, let us assume that intratemporal preferences for consumption in (18) can be expressed as:

\[ U(C, n) = U[H(C, n)] \]  

(50)

where, without loss of generality, \( H(C, n) \) is a consumption index that is homogenous of degree one, and \( U(\cdot) \) is an appropriate concave monotonic transformation. Again, we have as a particular case the specification (29) when \( U \) is a logarithmic function and \( H(C, n) \) is a CES aggregator.

Beyond the CES case, the class of preferences with homothetic aggregators includes other examples often used in macroeconomics, such as implicitly additive functions (Kimball, 1995)\textsuperscript{29} or translog preferences (Feenstra, 2003, Bilbiie, Ghironi and Melitz, 2012). It is well known (see for instance Benassy, 1996) that the symmetric price of monopolistic competition for these preferences is a function of the number of goods \( n \) only. Since here the number of goods is exogenous, the price of the final goods:

\[ p_t = \frac{1}{1 - \epsilon(n)} \]

is indeed a constant.\textsuperscript{30} Therefore the Euler equation remains identical as under perfect competition. Using homogeneity, we have \( H(C_t u, n) = C_t H(u, n) \) under symmetry, therefore the Euler equation can be written as:

\[ U'(C_t H(u, n)) = \beta \mathbb{E} \{ R_{t+1} U'(C_{t+1} H(u, n)) \} \]

(51)

while the labor supply is distorted downword by a constant in every period. This has an important implication: under homothetic preferences, it is always optimal to adopt tax smoothing on labor income and zero taxation on capital income:

\[ \tau^L = \frac{\epsilon(n)}{1 - \epsilon(n)} \quad \text{and} \quad \tau^K = 0 \]  

(52)

\textsuperscript{28}The optimal taxes are:

\[ \tau^L_t = \frac{nC_t}{b - nC_t} \quad \text{and} \quad \tau^K_t = \frac{n(C_{t-1} - C_t)}{b - nC_t} \]

\textsuperscript{29}Kimball (1995) considered implicitly additive production functions satisfying constant returns to scale, but the reinterpretation in terms of homothetic preferences is common in the literature.

\textsuperscript{30}We should remark that markups are variable in Kimball (1994) because prices are not flexible and consumption varies across goods, and in Bilbiie, Ghironi and Melitz (2012) because the number of firms is variable.
Translog preferences  Feenstra (2003) has analyzed preferences based on a translog expenditure function. This implies the price \( p_t = 1 + \frac{1}{\sigma n} \), where \( \sigma > 0 \) is a parameter related to substitutability between goods. In this case \( \epsilon(n) = \frac{1}{1+\sigma n} \) and the optimal labor income subsidy can be easily derived as \( \tau_L = \frac{1}{\sigma n} \).

Hanoch-Kimball preferences  The implicitly additive aggregators of Kimball (1995) belong to a more general class introduced by Hanoch (1974). Let us consider a direct utility represented by a homogenous aggregator \( H(C_t, n) \) which is implicitly defined by:

\[
\sum_{j=1}^{n} u \left( \frac{C_j}{H(C, n)} \right) = 1
\]

where the function \( u(x) \) as the same properties as a direct subutility. Using symmetry and homogeneity, we must have \( u(C/H(C, u, n)) = u(x(n)) = 1/n \) where \( x(n) = H(u, n)^{-1} \) is a decreasing function of the number of goods. For instance, in the CES case we have \( u(x) = x^{1-\theta}/\theta \) and \( x(n) = 1/n \), and additivity becomes explicit. Under flexible prices we can easily characterize the equilibrium markups.

By duality principles, the intratemporal problem can be seen as minimizing spending \( E = \sum_{j=1}^{n} p_j C_j \) with respect to each consumption level under the utility constraint (53). This provides the inverse demand:

\[
p_j = u' \left( \frac{C_j}{H(C, n)} \right) \frac{\mu}{H(C, n)}
\]

where \( \mu \) is the Lagrange multiplier. Applying (21), we can compute the symmetric elasticity \( \epsilon(n) = -\frac{\theta u'(x(n))z(n)}{\theta u'(z(n))} \). Notice that this is can be either increasing or decreasing in the number of goods. However, here the number of goods is exogenous, therefore markups and optimal taxes are constant. The optimal labor income subsidy can be easily derived as \( \tau_L = \frac{u'(x(n))}{u'(x(n))z(n) + u''(x(n))z(n)} \).

3 Dynamic endogenous market structures

Our last step is to endogenize the number of firms producing each differentiated variety and engaged in imperfect competition when there is a fixed cost of creating new firms à la Romer (1990). The dynamic endogenous market structures that emerge are not efficient in general and will be compared to the optimal allocation of resources to characterize the optimal taxation.

\[\text{Hanoch (1974) has also analyzed the case of an indirect utility } \tilde{H}(p/E, n) \text{ that is homogenous and implicitly additive, as with } \sum_{j=1}^{n} v \left( \frac{p_j}{\tilde{H}(p/E, n)} \right) = 1, \text{ where the function } v(s) \text{ as the same properties as an indirect subutility. Of course, in the CES case we have } v(s) = s^{1-\theta} \text{ and additivity is explicit. As above, one can compute the constant symmetric elasticity } \theta(n) = -\frac{\theta v'(x(n))z(n)}{v'(z(n))} \text{ where } z(n) = \tilde{H}(u, n)^{-1}, \text{ and derive the constant optimal labor subsidy.}\]
The initial stock of capital and number of firms \((K_t, n_t)\) are given. The Cobb-Douglas production function (2) is the same as before. Following Ghironi and Melitz (2005), let us assume that the number of firms/goods follows the law of motion:

\[
n_{t+1} = (1 - \delta_n) (n_t + n_t^\pi)
\]

where \(\delta_n \in (0, 1)\) is an exogenous exit probability and \(n_t^\pi \geq 0\) is the endogenous number of entrants in period \(t\).

The consumer chooses how much to spend in final goods, how much to invest in stocks of existing and new firms, and how much to invest in physical capital, as already examined in entry models by Ghironi and Melitz (2005), Etro and Colclia (2010), Bilbiie, Ghironi and Melitz (2012) and others. All this must match the sum of labor and capital income, the profits of the existing firms, and their current value. Expressing the budget constraint of the agent in terms of the intermediate good, the consumer problem becomes:

\[
\max_{c_t, k_{t+1}, l_t, x_{t+1}} \quad \bar{U} = \mathbb{E} \sum_{t=1}^{\infty} \beta^{t-1} \left[ U(C_t, n_t) - \frac{w_t L_t^{1 + \frac{\nu}{\gamma}}}{1 + \frac{\nu}{\gamma}} \right] \\
\text{s.t.:} \\
K_{t+1} + x_{t+1} \sum_{j=1}^{n_{t+1}} V_{jt} = K_t(1 - \delta) + w_t L_t + r_t K_t + \sum_{j=1}^{n_t} \left[ x_t (\pi_{jt} + V_{jt}) - p_{jt} C_{jt} \right]
\]

where \(x_t\) is the share of (mutual fund investing in) stocks of the firms, \(\pi_{jt}\) are the profits/dividends and \(V_{jt}\) the value of firm \(j\) at time \(t\) and the other variables are the same as before.

### 3.1 General equilibrium dynamics

The utility maximization problem leads to the same FOCs as before for \(C_{jt}\). Under monopolistic, Bertrand and Cournot competition we obtain the respective prices (see Bertoletti and Etro, 2016, Prop. 1, 2, 3):

\[
p_t = \frac{1}{1 - \epsilon (C_t, n_t)}, \quad p_t^B = \frac{\epsilon (C_t, n_t) + n_t - 1}{(n_t - 1)[1 - \epsilon (C_t, n_t)]}, \quad p_t^C = \frac{n_t}{(n_t - 1)[1 - \epsilon (C_t, n_t)]}
\]

with profits:

\[
\pi_t = \frac{\epsilon (C_t, n_t) C_t}{1 - \epsilon (C_t, n_t)}, \quad n_t^B = \frac{n_t \epsilon (C_t, n_t) C_t}{(n_t - 1)[1 - \epsilon (C_t, n_t)]}, \quad n_t^C = \frac{[1 + (n_t - 1) \epsilon (C_t, n_t)] C_t}{(n_t - 1)[1 - \epsilon (C_t, n_t)]}
\]

which we assume increasing in individual production/consumption \(C_t\).\(^{32}\)

Denoting the generic price as \(p(C_t, n_t)\), the FOCs for \(K_{t+1}\) and \(L_t\) deliver the modified Euler equation and the labor supply equation as natural extensions of our earlier results:

\[
\frac{U_t(C_t, n_t)}{p(C_t, n_t)} = \beta \mathbb{E} \left\{ R_{t+1} \frac{U_t(C_{t+1}, n_{t+1})}{p(C_{t+1}, n_{t+1})} \right\}
\]

\(^{32}\)This is always the case with CES preferences, with any directly additive aggregator under (10) or with a Morishima elasticity \(\epsilon(C_t, n_t)\) increasing in \(C_t\).
Let us assume that entry requires a sunk cost $F_t$ in units of intermediate good at time $t$.\footnote{The simplest case is the deterministic case where the entry cost is a constant $F$ (see the Appendix). Another realistic case is the one in which an increase in total factor productivity makes the business creation sector more productive, as with $F_t = f/A_t$ for a constant parameter $f > 0$, so that a productivity shock affects the economy through a double channel, leading to higher productivity for the production of both intermediate goods and firms.} We will leave unspecified the dynamics of the fixed cost $F_t$, assuming only that this is low enough to allow entry around the steady state. The reason is that the exogenous process determining the fixed costs will not affect the optimal tax rules around the steady state (even if the business cycle properties of the model do, of course, depend on the nature of the entry costs). Free entry requires that in every moment the number of entrants is such that the value of firms equates the fixed entry cost or zero if the fixed costs are higher than the value of new firms.

To investigate how many firms enter in the market we first need to derive the value of the firms, which is the present discounted value of their expected profits. The new FOC of the consumer is the one for $x_{t+1}$:

$$
\lambda_t V_t (n_t + n_{t+1}) = \beta \mathbb{E} \{ \lambda_{t+1} (\pi_{t+1} + V_{t+1}) n_{t+1} \}
$$

Using the equation of motion for the number of firms and the modified Euler equation, this provides a standard recursive asset pricing formula for $V_t$:

$$
V_t = \beta (1 - \delta_n) \mathbb{E} \left\{ \frac{U_i(C_{t+1}, n_{t+1})}{U_i(C_t, n_t)} \cdot \frac{p(C_t, n_t)}{p(C_{t+1}, n_{t+1})} \cdot (\pi_{t+1} + V_{t+1}) \right\}
$$

If the initial conditions are such that investing in capital accumulation provides a higher return than creating new firms (namely if initially the stock of capital is low, the number of goods relatively high and the entry cost high enough), all investment is initially in capital and the number of goods decreases according to $n_{t+1} = (1 - \delta_n) n_t$, while output and consumption in each good must increase gradually. If, instead, investing in capital accumulation provides initially a lower return (namely if the stock of capital is high and the number of goods relatively low), all investment is initially in business creation and the stock of capital decreases according to $K_{t+1} = (1 - \delta) K_t$. The equilibrium interest rate decreases in both situations but none of them is compatible with a steady state equilibrium with positive consumption. Indeed, both these phases must end when the returns of both forms of investment are equalized, and savings start being allocated between them along a growth path for both the stock of capital and the number of firms.

In what follows, we will focus on the neighborhood of a steady state where both entry and capital investment occur in each period. Accordingly, the free entry condition $V_t = F_t$ in each period and the general expression for profits
\[ \pi_t = [p(C_t, n_t) - 1] C_t \] allow us to rewrite the asset pricing equation as follows:

\[ \frac{U_t(C_t, n_t)}{p(C_t, n_t)} = \beta \left( 1 - \frac{\delta_n}{F_t} \right) E \left\{ \frac{U_t(C_{t+1}, n_{t+1})}{p(C_{t+1}, n_{t+1})} \right\} \left[ C_{t+1} [p(C_{t+1}, n_{t+1}) - 1] + F_{t+1} \right] \]

In equilibrium with \( x_t = 1, V_t = F_t, w_t = (1 - \alpha) A_t K_t^\alpha L_t^{-\alpha} \) and \( r_t = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} \), the resource constraint becomes:

\[ K_{t+1} - K_t (1 - \delta) + n_t^* F_t = A_t K_t^\alpha L_t^{1-\alpha} - n_t C_t \]

which can be solved for the number of new firms \( n_t^* \) and replaced in the equation of motion for the total number of firms \( (54) \). This allows us to summarize the equilibrium around a steady state as follows:

**Proposition 3.** In a dynamic model with general intratemporal preferences and imperfect competition between an endogenous number of firms the equilibrium satisfies:

\[ \frac{U_t(C_t, n_t)}{p(C_t, n_t)} = \beta E \left\{ \frac{U_t(C_{t+1}, n_{t+1})}{p(C_{t+1}, n_{t+1})} \right\} \left[ 1 + \alpha A_{t+1} K_t^{\alpha-1} L_t^{1-\alpha} - \delta \right] \]

\[ L_t = \left[ \frac{(1 - \alpha) A_t K_t^\alpha U_t(C_t, n_t)}{v p(C_t, n_t)} \right]^{\frac{\delta_n}{1-\alpha}} \]

\[ n_{t+1} = (1 - \delta_n) \left[ n_t + \frac{A_t K_t^\alpha L_t^{1-\alpha} - n_t C_t - K_t + K_t (1 - \delta)}{F_t} \right] \]

around the steady state, where the price function \( p(C, n) \) is given by \( (55) \).

This dynamic system governs the evolution of the stock of capital \( K_t \), labor supply \( L_t \), consumption/production of each good \( C_t \) and the number of firms \( n_t \). It is immediate to reduce the system to a three-dimensional system for \( (C_t, K_t, n_t) \) with labor supply determined residually by \( (57) \). Moreover, equating the right hand sides of \( (56) \) and \( (59) \), which represent the expected return rate respectively on investment in capital and firms, one can express consumption in each period \( t \) as a function of the stock of capital and the number of firms in the same period, \( C_t = C(K_t, n_t) \). This allows us to reduce the equilibrium system to a bi-dimensional system for \( (K_t, n_t) \), or equivalently for \( (C_t, n_t) \). Moreover, since the profit function is increasing in consumption, we obtain that the production/consumption of each good declines over time while capital accumulates (and the equilibrium return rate declines). At this level of generality we cannot draw insightful conditions on the uniqueness of the steady state and on saddle-path stability, but they hold under standard functional forms and calibrations (see the Appendix for examples in the deterministic case).\(^{34}\) Therefore we now

\(^{34}\)However, under special assumptions, the model can also deliver a) cycling behavior of the number of firms (under high exit rates or a low discount factor), or b) unbounded growth of the number of firms (when the return on business creation is constant à la Romer, 1986). See the Appendix for examples.
move to the welfare analysis and derive the optimal tax rules, which hold around any well defined steady state.

3.2 Optimal dynamic market structures

For any decentralized equilibrium path characterized in the earlier section, we can make a comparison with the social planner solution and determine the tax system that restores the latter. Replacing the equation of motion for the number of firms (54) in the resource constraint (60) and solving for the symmetric consumption of each good \( C_{t} \), we can express the symmetric intratemporal utility \( U(C_{t}, n_{t}) \) and state the social planner problem as follows:

\[
\max_{K_{t+1}, L_{t}, n_{t+1}} \mathbb{E} \sum_{t=1}^{\infty} \beta^{t-1} \left[ U \left( \frac{A_{t}K_{t}^{\alpha}L_{t}^{1-\alpha} - K_{t+1} + K_{t}(1-\delta) - \frac{n_{t+1}F_{t}}{1-\delta n}}{n_{t}} + F_{t}, n_{t} \right) - \frac{nL_{t}^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \right]
\]

Using the properties of the symmetric functions, the FOCs for \( K_{t+1}, L_{t} \) and \( n_{t+1} \) can be rearranged as:

\[
U_{t}(C_{t}^{*}, n_{t}^{*}) = \beta \mathbb{E} \left\{ [1 + \alpha A_{t+1}K_{t+1}^{\alpha-1}L_{t+1}^{1-\alpha} - \delta] U_{t}(C_{t+1}^{*}, n_{t+1}^{*}) \right\}\]

\[
L_{t}^{*} = \left[ 1 - \frac{\alpha}{\psi} A_{t}K_{t}^{\alpha}U_{t}(C_{t}^{*}, n_{t}^{*}) \right]^{\frac{\psi}{\psi - \alpha}}
\]

which extend to a dynamic context the analysis of optimal market structures first presented in Bertoletti and Etro (2016) for a static context. As there, the optimal number of firms derives from the trade-off between the costs of producing new varieties and the benefits of enjoying them (in the future) net of the reduction of consumption (needed to invest in replacing the lost varieties). In both contexts, this trade-off depends on the elasticities of the welfare function \( U(C, n) \) with respect to consumption and the number of goods:

\[
\psi^{C}(C, n) \equiv \frac{U_{t}(C, n)C}{U(C, n)} \quad \text{and} \quad \psi^{n}(C, n) \equiv \frac{U_{t}(C, n)n}{U(C, n)}
\]

Indeed, the last FOC can be rearranged as follows:

\[
U_{t}(C_{t}^{*}, n_{t}^{*}) = \beta \left( 1 - \frac{\delta n}{F_{t}} \right) \mathbb{E} \left\{ U_{t}(C_{t+1}^{*}, n_{t+1}^{*}) \left[ C_{t+1}^{*} \left[ \psi^{C}(C_{t+1}^{*}, n_{t+1}^{*}) + 1 - F_{t+1} \right] \right] \right\}
\]

whose right hand side emphasizes the social return from creating new varieties, which is general different from the private return.
3.3 Optimal taxes with monopolistic competition

The comparison between the optimality conditions and the decentralized free entry conditions augmented with taxes leads to the optimal corrective taxation. This requires three time-changing taxes: in each period $t$, the labor subsidy turns the effective wage into $(1 + \tau_L^t)w_t$, the capital income tax turns the effective gross return rate on capital investment into $(1 - \tau_K^t)R_t$, and we need to introduce a tax on profits/dividends $\tau_D^t$ which turns the net profits/dividends into $(1 - \tau_D^t)\pi_t$. Amending the equilibrium equations under monopolistic competition and equating them with the social optimum equations we obtain:

**Proposition 4.** In a dynamic model with general intratemporal preferences and monopolistic competition between an endogenous number of firms the optimal taxes around the steady state are:

$$
\tau_L^t = \frac{\epsilon(C_t, n_t)}{1 - \epsilon(C_t, n_t)}
$$

$$
\tau_K^t = \frac{\epsilon(C_{t-1}, n_{t-1}) - \epsilon(C_t, n_t)}{1 - \epsilon(C_t, n_t)}
$$

$$
\tau_D^t = \frac{1 - [1 - \epsilon(C_t, n_t)]\psi^n(C_t, n_t)}{\epsilon(C_t, n_t)}
$$

where $\psi^n(C, n)$ and $\psi^n(C, n)$ are the elasticities of utility with respect to number of goods and aggregate consumption.

The simplicity of the optimal tax system relies on the fact that the tax rates can be directly computed as (non-linear) functions of the number of firms active in each period and the production level of each firm. Some remarks are in order. First, the traditional case of a constant labor subsidy and a zero capital income tax emerges only under preferences that deliver a symmetric Morishima elasticity independent from consumption and from the number of goods. A well known case is the one of CES aggregators, but Bertoletti and Etro (2016) have shown that there are other possible cases.35

Second, except for these special cases, general preferences generate variable optimal tax rates over the business cycle (for the case of homothetic preferences see Bilbiie, Ghironi and Melitz, 2016). The general principle is that countercyclical markups require a countercyclical labor subsidy and a positive capital income tax along the growth path. In the long run, however, the optimal capital income tax is zero for any preferences.

Third, the optimal tax on profits is aimed at restoring the efficient entry process. The traditional case of a CES aggregator with intratemporal preferences (29) implies $\epsilon(C, n) = 1/\theta$ and $\psi^n(C, n)/\psi^n(C, n) = \theta/(\theta - 1)$, therefore

35One of them is based on the generalized linear direct utility introduced by Diewert (1971): in special cases of this class of homothetic preferences the equilibrium markup of monopolistic competition is constant and therefore capital income taxation is not needed to reach optimality and the optimal labor subsidy is constant. For instance, this requires intratemporal utility $U(C, n) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sqrt{C_i C_j}$. See Bertoletti and Etro (2016) for details.
the optimal profit tax is zero in each period: both entry and capital accumulation are efficient, therefore only labor should be subsidized to restore optimality in the CES case. More in general, however, dividend taxation is needed to reach the efficient number of entrants in each period. The underlying principle is that dividend taxation should be positive if markups are too high. To see what determines whether markups are too high or too low and characterize the optimal dividend taxation we need to re-examine the main classes of preferences.

3.3.1 Quadratic direct utility

We can directly compute the optimal tax system for our example of quadratic preferences (30). The welfare function \( U(C, n) = \alpha n C - \frac{\eta n^2 C^2}{2} - \frac{\gamma^2 n^2 C^2}{2} - \frac{\eta n C^2}{2} \) allows one to compute \( \psi^C(C, n) \) and \( \psi^n(C, n) \) and derive the optimal tax rates:

\[
\tau^L_t = \frac{\gamma C_t}{\alpha - (2\gamma + \eta n_t)C_t}, \quad \tau^K_t = \frac{\alpha \gamma (C_{t-1} - C_t) - \gamma C_t C_{t-1} (n_t - n_{t-1})}{[\alpha - (2\gamma + \eta n_t)C_t] [\alpha - (\gamma + \eta n_{t-1})C_{t-1}]} \\
\text{and} \quad \tau^D_t = \frac{\alpha \gamma - \alpha \eta n_t + \gamma n_t C_t + \eta^2 n^2_t C_t}{2 \gamma [\alpha - (\gamma + \eta n_t)C_t]} \tag{63}
\]

These tax rates depend on both the production/consumption of each good and the number of firms producing these goods, which complicates things compared to the earlier case with a given number of firms. An increase in both the number of goods and the individual production/consumption tend to increase the labor income subsidy and to determine a negative income tax. However, this specification is consistent with both positive or negative values for the optimal capital income and dividend taxes.\(^{36}\)

3.3.2 Directly additive aggregators

With directly additive aggregators (33), the welfare function \( U(nu(C)) \) implies that \( \psi^n(C, n)/\psi^C(C, n) = 1/\psi(C) \), where \( \psi(C) \equiv u'(C) C/u(C) > 0 \) is the elasticity of the sub-utility function \( u(C) \). Computing the derivative \( \psi'(C) = \psi(C)^{1 - \epsilon(C) - \psi(C)} \), we can rearrange the optimal tax system as follows:

\[
\tau^L_t = \frac{\epsilon(C_t)}{1 - \epsilon(C_t)}, \quad \tau^K_t = \frac{\epsilon(C_{t-1}) - \epsilon(C_t)}{1 - \epsilon(C_t)} \quad \text{and} \quad \tau^D_t = \frac{-C_t \psi'(C_t)}{\epsilon(C_t) \psi(C_t)^2} \tag{64}
\]

Notice that the optimal taxation is independent from the form of the monotonic transformation \( U(\cdot) \). The tax rates on the inputs are the same as in the baseline model, while the profit tax is positive if and only if the elasticity of the subutility function is decreasing in consumption. This is a dynamic generalization of the static principle found in Dixit and Stiglitz (1977) for which excess entry occurs in this case. Examples with CARA subutilities require a positive profit taxation because their elasticities are decreasing in consumption and the decentralized equilibrium features too many firms, while the nested

\(^{36}\)For instance, for \( \eta \) low enough we obtain a positive dividend taxation.
power subutilities require a negative profit taxation because there are too few firms in equilibrium.

3.3.3 Indirectly additive aggregators

With aggregators represented by an indirectly additive utility (40), the welfare function $U(\theta(1/nC))$ implies $\psi^n(C, n)/\psi^C(C, n) = 1 + 1/\eta(1/nC)$, where $\eta(s)$ is always the elasticity of the sub-utility function. Computing the derivative $\eta'(s) = \eta(s)\frac{1-\theta(s)+n(s)}{s}$, we can rearrange the optimal tax system as:

$$
\tau_t^L = \frac{1}{\theta(s_t)} - 1, \quad \tau_t^K = \frac{\theta(s_t) - \theta(s_{t-1})}{\theta(s_t) - 1 - \theta(s_{t-1})} \quad \text{and} \quad \tau_t^D = \frac{s_t\eta'(s_t)}{\eta(s_t)^2} \quad (65)
$$

where $s_t = p_t/E_t = 1/n_tC_t$. This optimal scheme is related to static results in Bertoletti and Etro (2017), where excess entry occurs if and only if the elasticity of the subutility is increasing in the price. This requires a positive profit taxation in each period to restore the optimal entry process, as in our examples with linear demand or exponential subutility, whose price elasticities are increasing and excess entry occurs in the decentralized equilibrium.

3.3.4 Homothetic aggregators

Finally, consider intratemporal preferences that are homothetic with a welfare function $U(CH(x, n))$. It is standard to verify that $\psi^n(n)/\psi^C(n)$ depends only on the number of goods. Therefore, the optimal taxes change over time in function of the endogenous number of firms only.

The case of translog preferences of Feenstra (2003) generates countercyclical prices $\rho_t = 1 + \frac{1}{\sigma n_t}$. It is easy to derive also $\psi^n(C, n)/\psi^C(C, n) = 1 + 1/2\sigma n$. This allows one to compute the optimal taxes as:

$$
\tau_t^L = \frac{1}{\sigma n_t}, \quad \tau_t^K = \frac{n_t - n_{t-1}}{1 + \sigma n_{t-1}} \quad \text{and} \quad \tau_t^D = \frac{1}{2} \quad (66)
$$

which deliver, during the entry process, a decreasing labor subsidy, a positive and decreasing capital income tax and a constant tax on dividends. These results are consistent with those of Bibliole, Ghironi and Melitz (2016).

We can also derive explicit expressions for the optimal tax system within the general class of homothetic preferences of Kimball (1995). Consider the implicitly additive aggregator (53). In equilibrium the price $p_t = \frac{1}{1-\epsilon(n_t)}$ is countercyclical if and only if the elasticity $\epsilon(n) = \frac{-u''(x(n))x(n)}{u'(x(n))}$ is decreasing in the number of firms. Under symmetry the homogenous aggregator $H$ is implicitly defined by $nu(C/H) = 1$, that allows to derive $\psi^n(C, n)/\psi^C(C, n) = 1/\psi(x(n))$. Direct computation gives $\psi'(x) = \psi(x)\frac{1-\epsilon(x)/\psi(x)}{x}$ and allows us to rearrange the optimal tax system as:

$$
\tau_t^L = \frac{\epsilon(n_t)}{1-\epsilon(n_t)}, \quad \tau_t^K = \frac{\epsilon(n_{t-1}) - \epsilon(n_t)}{1-\epsilon(n_t)} \quad \text{and} \quad \tau_t^D = \frac{-x(n_t)\psi'(x(n_t))}{\epsilon(n_t)\psi(x(n_t))^2} \quad (67)
$$
The optimal labor subsidy is countercyclical and the optimal capital tax is positive if and only if the markups are decreasing in the number of firms. Instead, the dividend tax is positive if and only if the elasticity of the \( u(\cdot) \) function is decreasing, and in general it depends on the number of firms.\(^{37}\)

### 3.4 Optimal taxes with Cournot and Bertrand competition

Dynamic endogenous market structures in the presence of strategic interactions can be analyzed in a similar way. The equilibrium in Proposition 3 must incorporate the prices under either Bertrand or Cournot competition from (55). The case of log-CES preferences is similar to the model in Etro and Colciago (2010), but we can now explore any other microfoundation of the demand side. The dynamic properties of the equilibrium are similar to the case with monopolistic competition except for the pro-competitive effects due to the strategic interactions.

We can characterize the optimal taxation for this environment matching equilibrium and efficiency conditions. We report the optimal taxation results starting from the case of competition in quantities:

**Proposition 5.** In a dynamic model with general intratemporal preferences and Cournot competition between an endogenous number of firms the optimal taxes around the steady state are:

\[
\begin{align*}
\tau_{t}^{LC} &= \frac{1}{n_t-1} + \epsilon(C_t, n_t) \\
\tau_{t}^{KC} &= \frac{\epsilon(C_{t-1}, n_{t-1}) - \epsilon(C_t, n_t)}{1 - \epsilon(C_t, n_t)} \\
\tau_{t}^{DC} &= n_t - (n_t - 1) \frac{1 - \epsilon(C_t, n_t)^{\theta}}{[1 + (n_t - 1)\epsilon(C_t, n_t)]^{\theta}}
\end{align*}
\]

In the case of CES aggregators we obtain a very simple optimal tax system, with a zero capital income tax, a countercyclical labor subsidy \( \tau_{t}^{LC} = \frac{\theta + n_t - 1}{(\theta - 1)(n_t - 1)} \) and a countercyclical profit tax \( \tau_{t}^{DC} = \frac{\theta}{\theta + n_t - 1} \). The latter is consistent with a long run optimal tax obtained by Colciago (2016) under an equivalent assumption on the entry costs.\(^{38}\) Notice that with homogeneous goods (\( \theta \to \infty \)), the case analyzed in Colciago and Etro (2010), we have \( \tau_{t}^{LC} \to \frac{1}{n_t - 1} \) and \( \tau_{t}^{DC} \to 1 \) because it is convenient to incentivize individual production (as long as strategic interactions between a small number of firms restrict production) and disincentivize the creation of new firms (there are no gains from variety). The optimal profit tax is positive because imperfect competition attracts more entry than monopolistic competition (which generates the efficient number of firms under

\(^{37}\)We leave to the reader the analysis of the specular case of an implicitly additive indirect utility.

\(^{38}\)In reality, Colciago (2016) assumes a fixed cost in units of labor, and has no capital. This is equivalent to assume a fixed cost in units of an intermediate good produced with labor only.
In case of competition in prices, we have:

**Proposition 6.** In a dynamic model with general intratemporal preferences and Bertrand competition between an endogenous number of firms the optimal taxes around the steady state are:

\[
\tau_{LB}^t = \frac{\epsilon(C_t, n_t)}{\left(1 - \frac{1}{n_t}\right) [1 - \epsilon(C_t, n_t)]}
\]

\[
\tau_{KB}^t = \frac{\epsilon(C_{t-1}, n_{t-1}) - \epsilon(C_t, n_t)}{[1 - \epsilon(C_t, n_t)] [1 - \frac{1}{n_t}]} \left[1 - \epsilon(C_{t-1}, n_{t-1})\right]
\]

\[
\tau_{DB}^t = \frac{n_t + \epsilon(C_t, n_t) - 1 - (n_t - 1) [1 - \epsilon(C_t, n_t)] \frac{\psi^s(C_t, n_t)}{\psi^{s'}(C_t, n_t)}}{n_t \epsilon(C_t, n_t)}
\]

With CES preferences we obtain again a very simple optimal tax system, with a zero capital income tax, a labor subsidy \(\tau_{LB}^t\) decreasing with the number of firms and the substitutability between goods, and a profit tax which is independent from substitutability and inversely proportional to the number of firms, according to the simple rule \(\tau_{DB}^t = \frac{1}{n_t}\), which is again consistent with Colciago (2016).

In general, strategic interactions increase the markups relative to monopolistic competition, therefore the optimal fiscal wedges are, ceteris paribus, higher.

### 4 Conclusions

Departing from perfect competition and CES preferences, flexible price DSGE models deliver new channels of propagation of the shocks and new inefficiencies that operate through changes in the endogenous markups. Remarkably, these changes depend crucially on the properties of preferences, restoring a novel role for the demand side in determining the propagation of shocks and corrective fiscal policy. We have analyzed these dynamic models and evaluated the optimal taxation that restores the first best allocation of resources through taxes on labor income, capital income and dividends.

Further work should evaluate the empirical properties of particular models after analyzing their impulse response functions and moments under standard calibrations (see Cavallari and Etro, 2016). The framework could be applied to examine Ramsey policies of optimal distortionary taxation (in the absence of lump sum taxes), as done by Colciago (2016) only for the case of CES preferences: optimal taxation should then account for the standard principle of lower taxation on more elastic demand functions and markup synchronization. Finally one could introduce price frictions (in the intermediate good sector) for the analysis of monetary shocks in a New-Keynesian style.

Moreover, one could study shocks and policies in an open economy framework (see Ghironi and Melitz, 2005, and Cavallari, 2013, for open economy entry...
models with CES preferences). A flourishing literature in international trade has been recently departing from CES preferences to investigate the main classes of preferences emphasized here with heterogeneous firms. In particular this is the case of quadratic preferences (Melitz and Ottaviano, 2008), homothetic preferences (Feenstra, 2014), directly additive preferences (Arkolakis et al., 2015) and indirectly additive preferences (Bertoletti et al., 2016). A contamination between these literatures could be fruitful. An interesting tension emerges in an open economy framework: while we have seen that lower markups in booms can contribute to better explain the business cycle, the cross-country evidence suggests that markups are higher in richer countries. Apparently, one may think that matching both features is difficult within a general framework. Instead, countercyclical markups due to entry effects on the supply side can be perfectly consistent with higher markups in richer countries due to lower demand elasticities for high income countries (as, for instance, with addilog preferences and strategic interactions). This is also consistent with the evidence of different reactivity of markups in front of supply and demand shocks (Nekarda and Ramey, 2013).

Our ultimate objective would be to provide a general microfoundation of DSGE models to expand their ability to replicate empirical findings on international business cycles, incorporate realistic imperfections in the labor and credit markets and be used for the analysis of fiscal and monetary policy. Many of the limits of the standard macroeconomic framework are deeply linked with the ubiquity of CES preferences.

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Appendix: Deterministic Equilibria with Monopolistic Competition and Dynamic Entry

To make progress in the analytical characterization of the equilibria with endogenous entry and their inefficiencies under different conditions, let us focus on monopolistic competition in a deterministic environment, with constant productivity $A$ and constant fixed cost $F$, as well as a rigid labor supply ($\varphi = 0$). Let us also assume for simplicity that $\delta_n = \delta$. It will be useful to distinguish three cases depending on the technological conditions: $\alpha = 0$ that corresponds to the limit case of an economy without capital, $\alpha \in (0, 1)$ which is the standard case with decreasing marginal productivity of each factor, and $\alpha = 1$ which can be regarded as the limit case of an AK technology.

Case 1. The model without capital

Consider the technology $Y_t = AL_t$, so that the output is used for consumption and to create new firms. Let us start from the baseline case of directly additive preferences used in Section 1. The dynamics of consumption is summarized by the equation:

$$u'(C_t) [1 - \epsilon(C_t)] F = \beta(1 - \delta) \left\{ u'(C_{t+1}) [1 - \epsilon(C_{t+1})] \left[ \frac{\epsilon(C_{t+1})C_{t+1}}{1 - \epsilon(C_{t+1})} + F \right] \right\}$$

whose peculiarity is the independence from the number of firms. This simplifies drastically the evolution of the economy. It is easy to verify that the difference equation for consumption, which is the only jump variable, is unstable around the steady state, namely $\frac{dC_{t+1}}{dC_{t}} \mid_{C_t = \tilde{C}} > 1$ under the assumption (10). Therefore the equilibrium must imply that the consumption of each good, which here is the production of each firm, is constant at the steady state level $\tilde{C}$ that solves:

$$\frac{\tilde{C}\epsilon(\tilde{C})}{1 - \epsilon(\tilde{C})} = \frac{[1 - \beta(1 - \delta)]F}{\beta(1 - \delta)}$$

Since the left hand side is the per period profit and is monotonic increasing in consumption, there is always a unique steady state for any fixed cost low enough. The remaining equation of motion for the number of firms is unidimensional and linear:

$$n_{t+1} = (1 - \delta) \left( n_t + \frac{A - n_t\tilde{C}}{F} \right)$$

whose intercept is positive and whose slope around the steady state is:

$$\frac{dn_{t+1}}{dn_t} = (1 - \delta) \left( \frac{1 - \tilde{C}}{\tilde{C}} \right) = \frac{\beta(1 - \delta) + \epsilon(\tilde{C}) - 1}{\beta\epsilon(\tilde{C})} < 1$$

The left hand side is the steady state expression for profit, which is monotonic increasing in steady state consumption under (10). This implies that, for any fixed cost low enough, there exist always a unique steady state.
The last inequality is equivalent to $\beta(1 - \delta - \epsilon(\tilde{C})) < 1 - \epsilon(\tilde{C})$, which holds for any $\beta$, $\delta$, $\epsilon(\tilde{C}) < 1$. As long as $\epsilon(\tilde{C}) > 1 - \beta(1 - \delta)$, that is if the markup is high enough and the exit rate low enough, the slope is positive and the number of firms converges monotonically to the steady state. For low markups, a high exit rate and a low discount factor, however, equilibrium convergence takes place through cycles in the number of goods.

In case of CES preferences $u(C) = C^{\frac{\theta - 1}{\theta}}$ we can solve explicitly for the steady state endogenous market structure with:

$$\hat{n} = \frac{\beta(1 - \delta)A/F}{\beta(1 - \delta) + (\theta - 1)[1 - \beta(1 - \delta)\tilde{C}]}$$

and

$$\hat{C} = \frac{[1 - \beta(1 - \delta)](\theta - 1)F}{\beta(1 - \delta)}$$

This case is similar to the one analyzed by Bilbiie, Ghironi and Melitz (2012), with the difference that in their model labor is used to produce either final goods or firms, while in our model labor is used to produce intermediate goods, which are in turn used to produce final goods and firms. Notice that along the growth path the number of firms increases and the production/consumption of each variety remains constant, but aggregate consumption $\tilde{C}_t = n_t \hat{C}$ (or the standard consumption index $n_t^{\frac{\theta}{\theta - 1}}\hat{C}$) is increasing toward its steady state value.

This example is peculiar in generating constant consumption of each good and therefore constant markups, but this happens only under the baseline assumption of a linear transformation of a directly additive aggregator. Just by adding a non-linear monotonic transformation or moving to any other general symmetric preferences $U(C_t, n_t)$, the equilibrium dynamics becomes bidimensional in $(C_t, n_t)$, with variable markups along the equilibrium paths. This can be easily verified in the traditional case of the log-CES aggregator (29), whose free entry condition is:

$$\frac{F}{C_t n_t} = \beta(1 - \delta) \left[ \frac{1}{C_{t+1} n_{t+1}} \left( \frac{C_{t+1}}{\theta - 1} + F \right) \right]$$

Accordingly, the equilibrium bi-dimensional system becomes:

$$C_{t+1} = \frac{\beta F C_t}{\frac{A}{n_t} + F - (1 + \frac{\delta}{\theta - 1})C_t}$$

$$n_{t+1} = (1 - \delta) \left[ n_t + \frac{A - n_tC_t}{F} \right]$$

The steady state is the same as above. Convergence holds under standard calibration.

**Case 2. The model with both capital and labor** Consider the neoclassical technology $Y_t = AK_t^\alpha L_t^{1-\alpha}$ with $\alpha \in (0, 1)$ under the log-CES aggregator (29). The equilibrium system reduces to:

$$\frac{C_{t+1} n_{t+1}}{C_t n_t} = \beta \left( 1 + \alpha AK_t^{\alpha - 1} - \delta \right)$$

$$= \beta(1 - \delta) \left( 1 + \frac{C_{t+1}}{(\theta - 1)F} \right)$$

$$K_{t+1} + \frac{F}{1 - \delta} n_{t+1} = AK_t^\alpha + (1 - \delta) \left[ K_t + \frac{F}{1 - \delta} n_t \right] - n_tC_t$$

37
where the last equation is just rewritten emphasizing the dynamics of total investment in capital and firms.

The steady state endogenous market structure \((\tilde{C}, \tilde{K}, \tilde{n})\) can be easily characterized. In particular, long run production/consumption of each good is always given by:

\[
\tilde{C} = \frac{[1 - \beta(1 - \delta)](\theta - 1)F}{\beta(1 - \delta)}
\]

which is always independent from technological parameters \((\alpha \text{ and } A)\) because it does not depend on the number of firms (this holds for all directly additive aggregators).

The stock of capital is:

\[
\tilde{K} = \left(\frac{\alpha A}{\frac{1}{\beta} - 1 + \delta}\right)^{1-\alpha}
\]

which is independent from the intratemporal preference parameters (here \(\theta\)). The number of firms is:

\[
\tilde{n} = \frac{(1 - \delta)[1 - \beta + \delta(1 - \alpha)]}{\alpha[\beta \delta + [1 - \beta(1 - \delta)](\theta - 1)]F}\left(\frac{\alpha A}{\frac{1}{\beta} - 1 + \delta}\right)^{1-\alpha}
\]

which depends on both supply and demand parameters.

To analyze the dynamics of the equilibrium notice that the Euler condition and the free entry condition provide a unique relation between capital and consumption of each good:

\[
C(K_t) = \frac{\alpha(\theta - 1)AF}{(1 - \delta)K_t^{1-\alpha}}
\]

While stock of capital and number of firms increase toward the steady state, the consumption/production of each single variety is reduced. This is what maintains the equality of the returns: the return on capital decreases because of the decreasing marginal productivity of capital, and the return on business creation decreases because the profits on each new variety diminish while the number of consumed varieties expands.

Replacing \(C(K_t)\) in the equilibrium system above, we obtain the bidimensional system:

\[
K_{t+1} = K_t \left[\frac{n_{t+1}}{\beta n_t (1 + \alpha AK_t^{\alpha-1} - \delta)}\right]^{1-\alpha}
\]

\[
n_{t+1} = (1 - \delta) \left[n_t \left(1 - \frac{\alpha(\theta - 1)AK_t^{\alpha-1}}{1 - \delta}\right) + \frac{AK_t^{\alpha} - K_{t+1} + K_t(1 - \delta)}{F}\right]
\]

and this is saddle-path stable under standard calibrations. As we argued earlier, this implies that for any initial conditions, the economy reaches in a finite time the equilibrium path for \((K_t, n_t)\), which in turn converges to the steady state.

What happens beyond the CES case? Under directly additive aggregators, the analysis is qualitatively the same as above because markups and profits depend on
consumption only. Under more general preferences \( U(C_t, n_t) \), markups and profits depend also on the number of firms, with profits decreasing in the latter. But the equality of returns around the steady state establishes always a relation between consumption and the predetermined variables (capital and number of firms) in each period, leading to a bidimensional system as above. In general, markups are variable over time.

**Case 3. The AK model with endogenous growth**  Finally, consider the technology \( Y_t = AK_t \), as the limit for \( \alpha \to 1 \). Let us focus again on the log-CES aggregator (29) as a benchmark. Since the marginal productivity of capital is constant, the return of the investments in capital and new firms can be the same only if profits are constant, which requires a constant consumption level for each good. This level can be computed as:

\[
\tilde{C} = \frac{(\theta - 1)AF}{1 - \delta}
\]

which is now increasing in productivity. The resource constraint requires output and the stock of capital to grow at the same rate as the number of firms. This pins down the constant ratio between the number of firms and the stock of capital in each period:

\[
n_t \frac{K_t}{n_t} = \frac{(1 - \beta)(1 + A - \delta)(1 - \delta)}{[1 - \beta(1 + A - \delta) + A\theta]F}
\]

Accordingly, the economy generates endogenous growth of output and of the number of goods à la Romer (1986, 1990), with:

\[
\frac{n_{t+1}}{n_t} = \beta (1 + A - \delta) \text{ for any } t
\]

assuming that this is above unity. What drives unbounded growth in the number of goods is the fact that the return on business creation remains constant due to the AK technology and the constant fixed cost of firms’ creation.\(^{40}\)

What happens beyond the CES case? Under directly additive aggregators, the growth rate of the number of goods is the same, and the consumption level and the ratio between number of firms and stock of capital change only qualitatively because markups depend uniquely on consumption. Under more general symmetric preferences \( U(C_t, n_t) \), the profits depend on the number of firms through the variable markups. However, as long as the elasticity \( \epsilon(C, n) \) is bounded below (namely \( \lim_{n \to \infty} \epsilon(C, n) > 0 \)) perpetual growth can be reached asymptotically in a similar way. Instead, if the elasticity drops to zero with an indefinite increase in the number of firms (for instance this is the case with translog preferences) unbounded growth in the number of goods is impossible and a finite steady state number of firms must emerge.

\(^{40}\)With the Romer (1986) microfoundation, this would be due to a productivity increasing with the aggregate stock of capital. Unbounded growth in the number of firms could also emerge in case of productivity increasing with the number of consumed varieties.