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## Working Paper

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#### Abstract

The term structure of equity and its cyclicality are key to understand the risks driving equilibrium asset prices. We propose a general equilibrium model that jointly explains four important features of the term structure of equity: (i) a negative unconditional term premium, (ii) countercyclical term premia, (iii) procyclical equity yields, and (iv) premia to value and growth claims respectively increasing and decreasing with the horizon. The economic mechanism hinges on the interaction between heteroskedastic long-run growth - which helps price long-term cash flows and leads to countercyclical risk premia - and homoskedastic shortterm shocks in the presence of limited market participation - which produce sizeable risk premia to short-term cash flows. The slope dynamics hold irrespective of the sign of its unconditional average. We provide empirical support to our model assumptions and predictions.


## Keywords

Term Structure of Equity, Dynamics, General Equilibrium, Expected Growth Volatility

## JEL Codes

D51, D53, E30, G10, G12
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# Dynamic Equity Slope* 

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#### Abstract

The term structure of equity and its cyclicality are key to understand the risks driving equilibrium asset prices. We propose a general equilibrium model that jointly explains four important features of the term structure of equity: (i) a negative unconditional term premium, (ii) countercyclical term premia, (iii) procyclical equity yields, and (iv) premia to value and growth claims respectively increasing and decreasing with the horizon. The economic mechanism hinges on the interaction between heteroskedastic long-run growth - which helps price long-term cash flows and leads to countercyclical risk premia - and homoskedastic short-term shocks in the presence of limited market participation-which produce sizeable risk premia to short-term cash flows. The slope dynamics hold irrespective of the sign of its unconditional average. We provide empirical support to our model assumptions and predictions.


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[^1]Over the last decade, there has been a significant interest in the term structure of equity, which has proved to be a powerful tool to understand equity markets and their connection with economic fundamentals. van Binsbergen, Brandt, and Koijen (2012) have pioneered a recent stream of literature that documents the empirical properties of the term structure of equity and tries to explain it in light of asset pricing theory. While leading asset pricing models have proved to be unable to explain sizeable compensations to short-term claims of equity payouts, a few recent models point to a number of potential explanations for a negative equity term premium (see van Binsbergen and Koijen, 2017). However, van Binsbergen, Hueskes, Koijen, and Vrugt (2013) and Gormsen (2020) document rich conditional dynamics of the term structure of equity, which are still unexplained. These dynamics and their link with economic fundamentals are important because they help understand which risks drive asset price fluctuations. Moreover, the conditional term structure of equity is informative about the economic outlook and discount rates and, thus, has implications for real decisions. ${ }^{1}$

This paper proposes a general equilibrium model that explains the most important properties of the term structure of equity, including (i) a negative unconditional equity term premium, (ii) procyclical equity yields, (iii) countercyclical equity term premium, (iv) premia to claims of value (respectively, growth) payouts that are increasing (decreasing) with the horizon, and (v) a countercyclical value premium. Our model links the dynamics of the term structure of equity to the timing of risk of economic fundamentals and, specifically, sheds light on the pivotal effect played by the volatility of expected growth. We provide supportive empirical evidence of the model assumptions and predictions.

The model mechanism hinges on the interaction of two risk factors steering economic fundamentals. First, a permanent component is driven by time-varying expected growth, gives rise to a stochastic trend, and induces upward-sloping risk with the horizon. Second, a transitory component produces stationary (short-term) fluctuations and leads to downward-

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Short-Term Asset Long-Term Asset

Figure 1: Equity Slope Dynamics. This figure displays the model-implied dynamics of the forward equity yields and the dividend strip risk premium. The short-term and the long-term assets represent the average value of the above quantities respectively over the first five years and the residual infinite-horizon. The state refers to the level of expected growth volatility and, endogenously to the price level.
sloping risk. The upward-sloping risk component is heteroskedastic, whereas the downwardsloping one is homoskedastic. In equilibrium, the relative weight of these two risks determines the slope of equity compensations as a function of the horizon. To generate a counter-cyclical equity term premium, the weight of the two risk factors needs to be time-varying, with most of the variation driven by the upward-sloping risk factor. Thus, when expected growth volatility rises, prices decline and become more volatile, short-term equity yields rise relative to long-term ones, and long-term equity risk premia rise relative to short-term ones. The model jointly explains the cyclical patterns of the term structure of equity documented by van Binsbergen et al. (2013) and Gormsen (2020) and finds support in our empirical analysis. Figure 1 summarizes the model predictions about these dynamics.

We also investigate the cross-sectional predictions of the model. Heterogeneous loadings on expected growth volatility lead to a cross section of equities, whose valuation ratios and risk premia can either decrease or increase with expected growth volatility. Thus, the model generates a positive and countercyclical value premium (Petkova and Zhang, 2005). This result arises from the higher payout cyclicality of value firms relative to growth firms (Koijen, Lustig, and Van Nieuwerburgh, 2017) and from risk premia to claims of value-payouts being
steeper with the horizon than those of growth-payouts (Giglio, Kelly, and Kozak, 2020).
Notably, the equilibrium dynamics of our model are robust to the unconditional properties of the term structure of equity. Namely, two features of our model are noteworthy. First, the model mechanism driving the cyclicality of equity yields and term premia holds irrespective of the sign of their unconditional slope. Second, our model reconciles standard asset pricing moments with sizeable risk premia to short-term assets independently of the sign of the unconditional equity term premium - the main challenge posed by van Binsbergen et al. (2012) to leading models. Thus, our economic mechanism is not affected by the concern posed by Bansal, Miller, Song, and Yaron (2020) about empirical studies focusing on short samples - that is, a short sample may not properly capture the alternation of good and bad economic conditions and lead to biased estimates of the unconditional slope, because the equity term premium switches sign over time. ${ }^{2}$

Our empirical analysis provides tight support to the model mechanism and its predictions. We estimate a simple measure of expected growth volatility (EGV) from survey forecasts of economic growth, which is an observable and genuine measure of investors' expectations. We document that EGV rises during economic downturns, as in Bansal, Kiku, Shaliastovich, and Yaron (2014) - a stylized pattern that we feed into our model.

Then, we document four stylized facts that arise as endogenous outcomes in our equilibrium model, then supporting its economic mechanism. ${ }^{3}$ First, as predicted by the model, we provide evidence that the market price-dividend ratio decreases with EGV, whereas its volatility increases with EGV. This result allows us to link macroeconomic fundamentals with the cyclicality of the equity term premium, estimated through the price-dividend ratio as Gormsen (2020). Second, the slope of the equity yields is strongly negatively correlated with EGV, consistently with the model and with the procyclical dynamics of the equity

[^3]yields slope documented by van Binsbergen et al. (2013). Third, EGV predicts the realized slope of equity returns with positive coefficient. Thus, EGV is a good candidate to drive the countercyclical dynamics of the equity term premium, in accord with the model and with the empirical findings of Gormsen (2020). Fourth, EGV predicts the value firms return and the value-minus-growth return with positive coefficient, whereas it does not predict the growth firms return. Thus, EGV credibly drives the countercyclical dynamics of the value premium, consistent with our model. Moreover, EGV strongly predicts the value firms return at long horizons, in accord with the model mechanism that produces upward-sloping compensations to the claims of value-payouts - as documented by Giglio et al. (2020). To the best of our knowledge, this is the first work that theoretically and empirically connects the dynamics of both equity yields and equity term premia to macroeconomic fundamentals.

Given our focus on the term structure of equity, our assumptions aim at correctly describing the timing of fundamentals' risk and how it transmits to the equilibrium state-price density. Our economy assumes consumption and payouts cointegration and limited market participation (Greenwald, Lettau, and Ludvigson, 2014; Marfè, 2017). These assumptions are supported by the empirical evidence and play a relevant role in shaping fundamentals' risk. Cointegration implies that payout risk is downward-sloping with the horizon (Belo, Collin-Dufresne, and Goldstein, 2015; Marfè, 2016), whereas limited market participation implies that market participants' consumption is much more correlated with payouts than aggregate consumption (Berk and Walden, 2013). ${ }^{4}$ Thus, by affecting the timing of fundamentals' risk, these assumptions can help understand the properties of the term structure of equity in equilibrium. Indeed, we verify that our model calibration matches both the downward-slope of payout risk as well as the predictability of payout growth by the payout-to-consumption ratio across the horizons. Instead, many models disregard such empirical

[^4]patterns and strongly overestimate payout risk at long horizons-further amplified under preferences for the early resolution of uncertainty. Moreover, we should avoid the model explains sizeable risk premia of short-term assets because of a mis-specified state-price density that weighs too much on short-term risk. Therefore, we exploit the state-price density decomposition of Alvarez and Jermann (2005) and Hansen and Scheinkman (2009): We verify that under our calibration, the fraction of state-price density volatility due to its permanent component does not violate the high lower bound estimated by Alvarez and Jermann (2005). This supports the way the timing of fundamentals' risk gets priced in equilibrium.

Several works study the term structure of equity in light of macroeconomic risk. Economic channels that have been investigated are beliefs formation (Croce, Lettau, and Ludvigson, 2015), financial leverage (Belo et al., 2015), disaster recovery (Hasler and Marfè, 2016), labor costs rigidity (Marfè, 2017), investment vintage (Ai, Croce, Diercks, and Li, 2018), alternative preferences (Andries, Eisenbach, and Schmalz, 2019), and reinvestment risk (Gonçalves, 2020). We complement this literature by providing a parsimonious equilibrium framework that explains the rich conditional dynamics of equity slope, which are consistent with recent empirical findings (van Binsbergen et al., 2013; Giglio et al., 2020; Gormsen, 2020).

Our paper is also related to works about the cross-section of equity returns and the value premium. Among others, Berk, Green, and Naik (1999), Gomes, Yaron, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), and Zhang (2005) propose equilibrium models with heterogeneity either in growth options risk or in adjustment costs. We complement this literature by explaining the pricing of value and growth payouts across the horizon (Giglio et al., 2020)—further corroborating the model mechanism about the equity slope dynamics. ${ }^{5}$

The paper is organized as follows. The empirical analysis of Section I supports the main model assumption and predictions. Section II and III describe the theoretical model and investigate its predictions. Section IV concludes. Model derivation and robustness results are in Appendix A and B respectively.

[^5]
## I Stylized Evidence

In this section, we first provide empirical support for the main model assumption about expected growth volatility (EGV). Then, we generate a set of key stylized facts about the dynamics of the term structure of equity, which can be jointly rationalized within our model.

To build our baseline EGV measure, we obtain US mean growth forecasts from the Survey of Professional Forecasters (SPF) maintained by the Federal Reserve Bank of Philadelphia about real gross domestic product (GDP). Survey data allows to more genuinely capture market participants' actual expectations about the economy's fundamentals as opposed to inferring them from realized variables, and can provide important insights into asset price dynamics (e.g., Barberis, Greenwood, Jin, and Shleifer, 2015; Greenwood and Shleifer, 2014). We lever on survey expectations to investigate the drivers of the dynamics of the equity term structure over the business cycle. Building on SPF data, we also construct alternative EGV measures from industrial production (IP) and personal consumption expenditures (PCE) growth forecast, as well as a measure exploiting cross-sectional dispersion of GDP growth forecasts. The time series of SPF forecasts are the longest available, covering the period 1968-2019 at quarterly frequency, with the exception of PCE whose forecasts start in 1981.

Information on actual macroeconomic conditions (e.g., inflation, Treasury rates, recessions) is from Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis. We obtain information on aggregate stock market and equity term structure from a variety of sources. As stock market index, we either rely on the value-weighted index from Center for Research in Security Prices (CRSP) or on the S\&P 500 index as reported on Robert Shiller's webpage. Monthly data on both indices are available throughout the period 1968-2019. Equity yields over different investment horizons are computed from van Binsbergen et al. (2012) and Giglio et al. (2020), who provide information at monthly frequency for the period 1996-2009 and 1975-2016, respectively. Monthly data on value and growth portfolio value-weighted returns for the period 1968-2019 are from Kenneth French's website. All returns and monetary variables are expressed in real terms. Monthly observations are
converted to quarterly frequency by summing them (for returns, which are logarithmic) or by taking the average (for other variables) over the quarter. Detailed information on data sources and variables definitions is in Appendix Table B1.

EGV and Expected Growth. The main assumption of our model is that the expected economic growth is heteroskedastic, so that its conditional volatility (EGV) is time-varying and, in particular, higher when the economic outlook deteriorates. To substantiate this assumption, we construct our baseline EGV measure by first filtering the conditional mean out of GDP growth forecasts with an $\mathrm{AR}(1)$ model and, then, by computing a moving average of the absolute residuals so obtained. In particular, we denote survey-based expected GDP growth as of time $t$ for the period $(t, t+1)$ as $\widehat{g}(t, t+1)$ and estimate an $\operatorname{AR}(1)$ specification:

$$
\begin{equation*}
\widehat{g}(t, t+1)=\theta_{0}+\theta_{1} \widehat{g}(t-1, t)+\epsilon(t) . \tag{1}
\end{equation*}
$$

Residuals from this model are not serially correlated, but their absolute values are. Thus, we build a simple measure of EGV (or conditional volatility):

$$
\begin{equation*}
\sigma(t, t+1)=\frac{1}{n} \sum_{i=0}^{n-1}|\epsilon(t-i)| . \tag{2}
\end{equation*}
$$

A moving average of four lags produces a good fit of residuals. The inferred time series of conditional volatility $\sigma(t, t+1)$ is our main EGV measure and explanatory variable. ${ }^{6}$

Importantly, EGV is negatively and significantly correlated with the expected growth, at $-18 \%$ ( $p$-value of 0.012 ). When investors have low expectations about future growth, forecasts are more volatile. This evidence supports the main assumption of our general equilibrium model: Expected growth is decreasing with its conditional volatility. Another approach to detect the negative relation between expected growth and EGV is to exploit the persistence of

[^6]

Figure 2: Expected Growth Forecastability by EGV. The left panel of the figure shows the slope point estimates as well as $90 \%$ confidence intervals (based on Newey-West standard errors with four lags) from predictive regressions of the SPF growth forecasts of GDP on its EGV over the horizons from one quarter to ten years for the period 1968-2019. The right panel of the figure shows the corresponding $R^{2}$.

EGV and verify whether it positively predicts cumulative expected growth. Figure 2 reports the predictive slope and the Newey-West confidence intervals for any horizon between one quarter and ten years. The limited EGV persistence does not lead to very precise coefficients but a clear positive relation can be observed for horizons beyond one year.

We next document four empirical facts concerning the relation between EGV and (i) macroeconomic conditions, (ii) the equity yield slope, (iii) the equity term premium, and (iv) the returns of value and growth firms.

The Cyclicality of EGV. The negative correlation between EGV and expected growth suggests that EGV is a countercyclical measure of the state of the economy. We confirm this intuition in Table 1 through contemporaneous regressions of EGV on macroeconomic and financial measures. EGV exhibits a negative and significant relation with the Chicago Fed National Activity Index (CFNAI) and the logarithm of the price-dividend ratio of the CRSP value-weighted index. It also exhibits a positive and significant relation with an indicator for National Bureau of Economic Research (NBER) recessions and the default spread.

Based on this evidence, we examine the relation between EGV and the logarithm of the

Table 1: The Cyclicality of EGV

|  | EGV |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) |
| Constant | $\begin{gathered} \hline 0.243^{* * *} \\ (10.32) \end{gathered}$ | $\begin{gathered} 0.219^{* * *} \\ (9.13) \end{gathered}$ | $\begin{aligned} & -0.000 \\ & (-0.01) \end{aligned}$ | $\begin{gathered} 1.153^{* * *} \\ (6.43) \end{gathered}$ |
| CFNAI | $\begin{gathered} -0.049^{* *} \\ (-2.35) \end{gathered}$ |  |  |  |
| NBER recession |  | $\begin{gathered} 0.162^{* * *} \\ (4.53) \end{gathered}$ |  |  |
| Default spread |  |  | $\begin{gathered} 0.226^{* * *} \\ (5.59) \end{gathered}$ |  |
| $\ln (\mathrm{P} / \mathrm{D})$ |  |  |  | $\begin{gathered} -0.249^{* * *} \\ (-5.22) \\ \hline \end{gathered}$ |
| Observations | 205 | 205 | 205 | 205 |
| $R^{2}$ | 0.06 | 0.11 | 0.30 | 0.30 |

Note. This table reports estimates from contemporaneous regressions at quarterly frequency of EGV on selected measures of macroeconomic (the three-month moving average of the CFNAI and an indicator for NBER recessions) and financial conditions (the default spread and the logarithm of the price-dividend ratio of the CRSP value-weighted index) over the period 1968-2019. Coefficient estimates are multiplied by 100 to favor readability. The $t$-statistics are reported in parentheses and are based on Newey-West standard errors with four lags. Significance at the $10 \%, 5 \%$, and $1 \%$ levels is indicated by ${ }^{*},{ }^{* *},{ }^{* * *}$, respectively. Detailed variable definitions are provided in Appendix Table B1.
price-dividend ratio in more depth. Theoretical models predict that the price-dividend ratio is driven by the latent factors that affect the distribution of aggregate cash flows. In the left panel of Figure 3, we consider the following regression:

$$
p d(t)=\alpha+\beta \sigma(t, t+1)+\varepsilon(t)
$$

We observe that EGV decreases with the logarithm of the price-dividend ratio (correlation of $-55 \%$, statistically significant at the $1 \%$ level). This negative correlation can be generated in a model where investors feature an elasticity of intertemporal substitution above one, that is when the substitution effect dominates the wealth effect (e.g., Bansal and Yaron, 2004). We then study the correlation between EGV and the conditional volatility of the logarithm of the price-dividend ratio, which we obtain following the same approach as in equation (2). In the right panel of Figure 3, we estimate this regression:

$$
\sigma_{p d}(t, t+1)=\alpha+\beta \sigma(t, t+1)+\varepsilon(t)
$$



Figure 3: EGV and the Price-Dividend Ratio. This figure shows the scatter plots of either the log price-dividend ratio (left panel) or log price-dividend ratio volatility (right) of the CRSP value-weighted stock market index against the EGV estimated from GDP growth forecasts for the period 1968-2019.

We observe a positive correlation of $57 \%$ between EGV and the conditional volatility of the price-dividend ratio, significant at the $1 \%$ level. These results conform with the literature on macroeconomic volatility and uncertainty (Bansal et al., 2014; Boguth and Kuehn, 2013).

EGV and the Equity Yield Slope. The second empirical pattern regards the relation between EGV and the slope of equity yields, which is defined as the difference between the long- and the short-maturity equity yield at each point in time. van Binsbergen et al. (2013) illustrate that such a slope is procyclical: short-maturity equity yields are lower than long-maturity ones during economic expansions, whereas they exceed long-maturity ones during recessions. Bansal et al. (2020), Gormsen (2020) and Giglio et al. (2020) document similar patterns. We verify if the procyclical nature of the equity yield slope is-at least partially - channeled through EGV, which we have shown to be countercyclical.

Because of the hard-to-observe nature of the equity yield slope, we measure it in a variety of ways. First, we use data by van Binsbergen et al. (2012), who extract information on shortmaturity equity yields from option prices on the S\&P 500 index. We then proxy for the slope by taking the difference between the S\&P 500 dividend yield (ey $(t$, long $)$ ) and short-maturity equity yields $(e y(t$, short $))$, whose maturity ranges between 0.5 and two years. Second, we

Table 2: Equity Yield Slope and EGV

| Panel A |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Equity Yield Slope (BBK12) |  |  |  |
|  | (1) <br> MKT-0.5Y | (2) <br> MKT-1Y | (3) <br> MKT-1.5Y | (4) <br> MKT-2Y |
| Constant EGV | $\begin{gathered} 0.141^{* *} \\ (2.08) \\ -47.448^{* * *} \\ (-3.31) \end{gathered}$ | $\begin{gathered} 0.071^{* *} \\ (2.01) \\ -32.920^{* * *} \\ (-4.70) \end{gathered}$ | $\begin{gathered} 0.045^{*} \\ (1.80) \\ -24.242^{* * *} \\ (-4.69) \end{gathered}$ | $\begin{gathered} \hline 0.036^{*} \\ (1.82) \\ -19.323^{* * *} \\ (-4.71) \end{gathered}$ |
| Observations $R^{2}$ | $\begin{gathered} 55 \\ 0.08 \end{gathered}$ | $\begin{gathered} 55 \\ 0.12 \end{gathered}$ | $\begin{gathered} 55 \\ 0.14 \end{gathered}$ | $\begin{gathered} 55 \\ 0.15 \end{gathered}$ |
| Panel B |  |  |  |  |
|  | Equity Yield Slope (GKK20) |  |  |  |
|  | $\begin{gathered} (1) \\ 10 \mathrm{Y}-2 \mathrm{Y} \end{gathered}$ | $\begin{gathered} (2) \\ 25 \mathrm{Y}-2 \mathrm{Y} \end{gathered}$ | $\begin{gathered} (3) \\ 100 \mathrm{Y}-2 \mathrm{Y} \end{gathered}$ | (4) <br> MKT-2Y |
| Constant EGV | $\begin{gathered} \hline 0.012^{* * *} \\ (6.16) \\ -3.269^{* * *} \\ (-4.07) \end{gathered}$ | $\begin{gathered} \hline 0.019^{* * *} \\ (7.41) \\ -4.571^{* * *} \\ (-4.94) \end{gathered}$ | $\begin{gathered} \hline 0.025^{* * *} \\ (8.80) \\ -5.779^{* * *} \\ (-5.69) \end{gathered}$ | $\begin{gathered} \hline 0.022^{* * *} \\ (8.12) \\ -5.276^{* * *} \\ (-5.40) \end{gathered}$ |
| Observations $R^{2}$ | $\begin{aligned} & 165 \\ & 0.22 \end{aligned}$ | $\begin{gathered} 165 \\ 0.26 \end{gathered}$ | $\begin{aligned} & 165 \\ & 0.30 \end{aligned}$ | $\begin{aligned} & 165 \\ & 0.28 \end{aligned}$ |

Note. This table reports estimates from contemporaneous regressions at quarterly frequency of the equity yield slope on EGV. Panel A uses measures of the equity yield slope based on data by van Binsbergen et al. (2012, BBK12) for the period 1996-2009. Panel B uses measures of the equity yield slope based on data by Giglio et al. (2020, GKK20) for the period 1975-2016. The maturities of the long and short legs considered to compute the equity yield slope are indicated at the top of each column. The $t$-statistics are reported in parentheses and are based on Newey-West standard errors with four lags. Significance at the $10 \%, 5 \%$, and $1 \%$ levels is indicated by ${ }^{*},{ }^{* *},{ }^{* * *}$, respectively. Detailed variable definitions are provided in Appendix Table B1.
use the model-implied equity yields made available by Giglio et al. (2020), which allow us to compute the equity slope at various maturities up to 100 years (ey(t,long)) relative to the two-year yield $(e y(t$, short $))$. For consistency with the first set of proxies based on van Binsbergen et al. (2012), we also compute it using the dividend yield of the CRSP valueweighted index as $e y(t$, long $) .{ }^{7}$ By spanning the period 1975-2016, the equity yields by Giglio et al. (2020) are informative about the slope dynamics across different economic conditions.

Table 2 reports the estimates from the regressions of these measures of the equity yield

[^7]

Figure 4: EGV and the Equity Yield Slope. The left of the figure shows the standardized time-series of the equity yield slope based (solid line) and the EGV (dashed line). The right panel shows the scatter plot of the standardized equity yield slope against the EGV. The equity yield slope is computed as the difference between the dividend yield of the CRSP value-weighted index and the model-implied 2-year equity yield of Giglio et al. (2020) for the period 1975-2016.
slope - based on data by van Binsbergen et al. (2012) in Panel A, and by Giglio et al. (2020) in Panel B- on EGV:

$$
e y(t, \text { long })-e y(t, \text { short })=\alpha+\beta \sigma(t, t+1)+\varepsilon(t)
$$

We observe a negative correlation between EGV and the slope of equity yields. The slope coefficients are negative and significant at the $1 \%$ confidence level across all the horizons. The $R^{2}$ from the regressions lies in the $8-30 \%$ range, pointing to a substantial explanatory ability of EGV as to the business cycle dynamics of the equity yield slope. ${ }^{8}$

Consistently, the left panel of Figure 4, which focuses on a slope measure based on Giglio et al. (2020), documents a strong negative relation between EGV and the slope of equity

[^8]yields. We observe a sharp mirror effect: the slope strongly decreases when EGV increases. The right panel of Figure 4 shows the corresponding scatter plot and linear fit. These results confirm that EGV is a major driver of the procyclical dynamics of the equity yield slope.

EGV and the Equity Term Premium. Third, we study the relation between EGV and the equity term premium - that is, the compensation of long-term equity claims over the compensation of short-term equity claims. Gormsen (2020) finds that the equity term premium is time-varying and countercyclical. Namely, long-term equity premia are more sensitive to price levels than short-term equity premia. This implies that the equity term premium increases in bad times and decreases in good times. We test if EGV helps explain time-variation of the equity term premium in light of macroeconomic risk.

We proxy for the equity term premium by taking the difference between the CRSP valueweighted index return (long-maturity claim) and the return on the two-year dividend strip based on the corresponding model-implied equity yield by Giglio et al. (2020). ${ }^{9}$ Then, we compute the one- to ten-year ahead cumulative equity term premium. Finally, we perform regressions of the future cumulative equity term premium on EGV to verify whether the current conditional volatility of expected growth predicts it. Figure 5 points to a rather strong positive relation between EGV and the equity term premium. The left panel displays that the predictive slope for the current EGV is positive and mostly statistically different from zero for predictive horizons above four years. The right panel shows that $R^{2}$ reaches roughly $8 \%$ for longer horizons. This evidence suggests that EGV is a credible channel through which the term structure of equity incorporates macroeconomic risk. Gormsen (2020) highlights the cyclicality of the equity term premium as measured by its correlation with the price-dividend ratio. We go a step further and show that EGV is a plausible link among the state of the economy, prices, and the term structure of equity. As we show below, this pattern is amenable to be endogenized in a tractable general equilibrium framework.

[^9]

Figure 5: EGV and the Equity Term Premium. The left panel of the figure shows the slope point estimates as well as $90 \%$ confidence intervals (based on Newey-West standard errors with four lags) from predictive regressions of long-minus-short equity returns (equity term premium) on EGV across the horizons from one quarter up to ten years. In our measure of the equity term premium, the long leg is the CRSP value-weighted index return and the short one is the return on the two-year dividend strip obtained from model-implied yields by Giglio et al. (2020) for the period 1975-2016. The right panel of the figure shows the $\mathrm{R}^{2}$ from the predictive regressions for each horizon.

EGV and the Cross-Section of Returns. Fourth, we look at the relation between the EGV and the returns of value firms, growth firms, and the value-minus-growth portfolio (Fama and French, 1992). In Figure 6, we estimate predictive regressions of the cumulative returns from one quarter up to ten years ahead from either value firms, growth firms, or the value-minus-growth portfolio on EGV. We find a strong positive relation between EGV and both the value firms return and the value-minus-growth return. Instead, EGV does not significantly predict the growth firms return over most horizons. The return predictability increases with the horizon for both the value and the value-minus-growth portfolio.

All in all, through EGV, we uncover that the value premium is-at least partially-related to the dynamics of the term structure of equity. Firms more (less) exposed to EGV feature, on the one hand, increasing (decreasing) risk premia and, on the other hand, lower (higher) valuation ratios. Thus, we interpret them as value (growth) firms. This is consistent with the term structure of value and growth portfolios estimated by Giglio et al. (2020). In turn, the value-minus-growth return is positively predicted by EGV and inherits its countercyclical


Figure 6: EGV and the Value Premium. The left panels of the figure show the slope point estimates as well as $90 \%$ confidence intervals (based on Newey-West standard errors with four lags) from the predictive regressions of the value return (upper panels), the growth return (middle panels), and the value-minus-growth return (lower panels) on EGV across the horizons from one quarter up to ten years. The value (growth) returns correspond to the top (bottom) decile of stocks sorted on the book-to-market ratio for the period 1968-2019. The right panels of the figure show the corresponding $R^{2}$.
behavior, in accord with the literature on the value premium (Petkova and Zhang, 2005).

The Cyclicality of the Term Structure of Equity. We now study the cyclical properties of the equity term structure dynamics in light of the recent findings in the literature. To this end, in Appendix Table B3, we estimate univariate specifications of our measures of the equity yield slope and of the equity term premium on business cycle proxies for the period 1975-2016. We are able to confirm the procyclicality of the equity yield slope, both when looking at EGV (column 1) and when looking at macroeconomic measures such as CFNAI and NBER recessions (columns 2 and 3) or at financial variables such as the default spread and the price-dividend ratio (columns 4 and 5).

Moving to the ten-year ahead equity term premium, the picture becomes more nuanced. It appears to correlate negatively with current macroeconomic conditions when these are measured through the EGV (column 6). Yet, the link with CFNAI and NBER recessions is statistically insignificant (columns 7 and 8). Moreover, when capturing the state of the business cycle by means of financial variables like the default spread and the price-dividend ratio, the countercyclicality of the equity term premium stands out again (columns 9 and 10). It is worth drawing a comparison between these findings and those in the literature. On the one hand, the EGV confirms the procyclical behavior of equity yields documented by van Binsbergen et al. (2013) and Bansal et al. (2020), by means of dividend strip prices, which are available only from the early 2000s. On the other hand, the lack of clear evidence with regards to the correlation of the equity term premium with "pure" macroeconomic variables, coupled with its negative correlation with EGV and with the price-dividend ratio, corroborates the analysis of Gormsen (2020) and enhances the role of EGV. Indeed, the long time series of equity yields (1975:2016) from Giglio et al. (2020) points to the countercyclicality of the equity term premium. In addition, our EGV incorporates the component of macroeconomic fundamentals that is relevant for the pricing of equity claims, which is in turn coherent with the negative link between the equity term premium and financial variables.

Alternative Specifications. In Appendix Table B4, we test the robustness of our results to using seven alternative EGV measures. First, we rely on a non-generated measure, namely the cross-sectional dispersion of real GDP growth forecasts. Second, we look at the AR(1)$\operatorname{ARCH}(1)$ specification of EGV of GDP, which, as illustrated in Appendix Table B2, exhibits a good fit of residuals. Third, we create two regression-based measures of EGV of GDP: one capturing only its macroeconomic component, the other capturing both the macroeconomic and the financial markets component. Fourth, we obtain the EGV of GDP growth forecasts computed after filtering out of the latter the short-term business cycle component by means of the (de-trended) labor share of the nonfinancial corporate sector. In addition, we build the EGVs of IP growth and of PCE growth forecasts by applying the baseline approach of equations (1)-(2).

Like the baseline EGV, all these measures are strongly countercyclical as highlighted by the negative and significant relation with the logarithm of the price-dividend ratio and the positive and significant relation with its volatility (columns 1 and 2). Similarly, columns 3 and 4 confirm that even these alternative EGV measures capture the procyclicality of the equity yield slope. At the same time, the positive relation of EGV measures with the equity term premium (columns 5 and 6) remains economically important, especially over long predictive horizons. For the ten-year ahead term premium, only EGV measures based on the AR(1)$\mathrm{ARCH}(1)$ specification and on IP growth forecasts exhibit coefficients that are insignificant at conventional levels ( $p$-value of 0.19 and 0.13 , respectively). We also qualitatively reproduce the result about value, growth, and value-minus-growth portfolios (columns 7-8-9). Although some of the alternative EGV measures have insignificant predictive power with respect to value or value-minus-growth returns, coefficients' sign and magnitude are unchanged.

Summary and Theoretical Underpinnings. Overall, the analysis of this section supports the idea that EGV is a major driver of the term structure of equity. When investors experience a higher conditional volatility of expected growth, (i) economic conditions deteriorate, (ii) prices decline, (iii) the slope of equity yields drops, (iv) the equity term premium
increases, and (v) the slope of value claims becomes steeper relative to the slope of growth claims, leading to an increase in the value premium.

The next section illustrates a parsimonious general equilibrium model that jointly endogenizes all these effects by assuming that expected growth is the negative of a square root process. It is worth discussing the intuition behind the economic mechanism rationalizing the above features (iii) and (iv) -i.e., the two most distinctive facts about the cyclicality of the term structure of equity - as well as (v). With regards to (iii), in our framework, bad economic conditions and, hence, low prices arise when EGV is high and, in turn, short-term expected growth declines. Moreover, in these periods, long-term expected growth is high, and long-term discount rates are low. This implies a negative correlation between EGV and the slope of equity yields. Such a mechanism is further corroborated by the related model prediction that equity yields positively predict future realized growth, in line with the empirical findings of van Binsbergen et al. (2013). With regards to (iv), in our general equilibrium setting, a positive correlation between EGV and the term premium on equity claims can explain the dynamics of the equity slope documented by Gormsen (2020). Indeed, EGV commands an equilibrium compensation that is larger for long-term equity claims than for short-term equity claims. In turn, the former are more sensitive than the latter to movements of EGV. Because EGV moves countercyclically, the premia on long-term equity claims feature more prominent countercyclical behavior than the premia on the short-term equity claims. This leads to a time-varying and countercyclical equity term premium. With regards to (v), a positive relation between EGV and the value premium arises endogenously in our model from the heterogeneous exposure of payouts to heteroskedastic long-run growth across value and growth firms. Thus, we exploit cross-sectional returns to further corroborate the model mechanism about the equity term structure dynamics as well as to explain in general equilibrium the recent findings of Giglio et al. (2020) about the term structures of value and growth returns.

## II The Model

In this section, we describe a parsimonious general equilibrium model that jointly captures the distinctive empirical properties of the term structure of equity, including its unconditional negative slope (van Binsbergen et al., 2012), procyclical yields (van Binsbergen et al., 2013), and countercyclical term premia (Gormsen, 2020). Cross-sectional predictions on value and growth claims (Giglio et al., 2020) provide further support to the model mechanism.

## The Economy

A representative firm produces a cash-flows stream, $C$, which constitutes the revenues from production distributed to workers and shareholders. Workers receive wages, $W$, and shareholders receive payouts, $D$, such that $C=W+D$. In the spirit of Berk and Walden (2013), the model assumes limited market participation. ${ }^{10}$ That is, workers do not access financial markets and consume their wages, whereas shareholders act as a representative agent in the stock market and consume dividends. Shareholders feature recursive preferences in the spirit of Kreps and Porteus (1979), Epstein and Zin (1989), Weil (1989), and Duffie and Epstein (1992). These preferences allow for the separation between the elasticity of intertemporal substitution and relative risk aversion. The utility at each time $t$ is defined as

$$
\begin{equation*}
U_{t} \equiv\left[\left(1-\beta^{d t}\right) \hat{C}_{t}^{\frac{1-\gamma}{\theta}}+\beta^{d t} \mathbb{E}_{t}\left(U_{t+d t}^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}} \tag{3}
\end{equation*}
$$

where $\hat{C}$ is a consumption process, $\beta$ is the time discount factor, $\gamma$ is the coefficient of risk aversion, $\psi$ is the elasticity of intertemporal substitution, and we define $\theta=\frac{1-\gamma}{1-\frac{1}{\psi}}$.

We assume that the dynamics of aggregate consumption depend on two components. The first is a permanent shock, which is driven by time-varying expected growth: It gives rise to a stochastic trend and induces upward-sloping risk with the horizon-i.e., the variance of growth rates increases with the horizon. The second component is a transitory (short-term)

[^10]shock $z_{t}$, that produces stationary fluctuations and induces downward-sloping risk with the horizon. The two shocks jointly allow for flexible term structures of risk. Consistent with the empirical evidence of Section I, we assume that expected growth is heteroskedastic and negatively correlated with its conditional variance. Aggregate consumption dynamics follow:
\[

$$
\begin{equation*}
d \log C_{t}=\left(\mu+\bar{x}-x_{t}\right) d t+d z_{t} \tag{4}
\end{equation*}
$$

\]

where the permanent and the transitory components are governed by:

$$
\begin{align*}
d x_{t} & =\lambda_{x}\left(\bar{x}-x_{t}\right) d t+\sigma_{x} \sqrt{x_{t}} d B_{x, t}  \tag{5}\\
d z_{t} & =-\lambda_{z} z_{t} d t+\sigma_{z} d B_{z, t} . \tag{6}
\end{align*}
$$

The Brownian shocks $B_{x, t}$ and $B_{z, t}$ are assumed to be independent, for the sake of tractability.
The dynamics of aggregate consumption, wages, and payouts are all subject to the permanent shock and, thus, are cointegrated in levels (Lettau and Ludvigson, 2005). Moreover, following previous contributions, we acknowledge that the rigidity of labor costs (Menzio, 2005; Marfè, 2017) with respect to short-term fluctuations lead to a mechanism of income insurance from shareholders to workers, which impose a leverage effect on payouts. ${ }^{11}$ We parsimoniously capture this effect by assuming that wages and payouts respectively satisfy:

$$
W_{t}=\omega\left(z_{t}\right) C_{t} \quad \text { and } \quad D_{t}=\left(1-\omega\left(z_{t}\right)\right) C_{t}
$$

where the share $\omega\left(z_{t}\right)$ is a function of the transitory (short-term) shock $z_{t}::^{12}$

$$
\omega\left(z_{t}\right)=1-\delta e^{(\phi-1) z_{t}} .
$$

The parameter $\phi \geq 1$ denotes the leverage effect on payouts. That is, payouts (respectively, wages) are more (less) exposed to short-term shocks than the firm's total cash flow stream. Consistently, payouts evolve as follows:

$$
\begin{equation*}
d \log D_{t}=\left(\mu+\bar{x}-x_{t}\right) d t+\phi d z_{t} . \tag{7}
\end{equation*}
$$

[^11]Although parsimonious, these dynamics account for many empirical stylized facts, such as: (i) cointegration among consumption, wages, and payouts (Lettau and Ludvigson, 2005),
(ii) excess volatility of payouts over consumption at short horizons (Belo et al., 2015), (iii) variance ratios of payout and wage growth rates, which respectively lie below and above those of consumption (Marfè, 2017), and (iv) countercyclical wage share (Ríos-Rull and Santaeulàlia-Llopis, 2010).

The shock $x_{t}$-the key variable of the model-drives the variance of expected growth, which satisfies:

$$
\mathbb{V}_{t}\left[d\left(\mu+\bar{x}-x_{t}-\lambda_{z} z_{t}\right)\right]=\left(\sigma_{x}^{2} x_{t}+\lambda_{z}^{2} \sigma_{z}^{2}\right) d t
$$

The conditional variances of consumption and payout growth rates across the horizon are respectively given by:

$$
\begin{aligned}
& \sigma_{C}^{2}(t, \tau)=\frac{1}{\tau} \log \frac{\mathbb{E}_{t}\left[C_{t+\tau}^{2}\right]}{\mathbb{E}_{t}\left[C_{t+\tau}\right]^{2}}=s_{c 0}(\tau)+s_{c x}(\tau) x_{t} \\
& \sigma_{D}^{2}(t, \tau)=\frac{1}{\tau} \log \frac{\mathbb{E}_{t}\left[D_{t+\tau}^{2}\right]}{\mathbb{E}_{t}\left[D_{t+\tau}\right]^{2}}=s_{d 0}(\tau)+s_{d x}(\tau) x_{t}
\end{aligned}
$$

where both the terms $s_{c x}(\tau)$ and $s_{d x}(\tau)$ increase with the horizon $\tau$. As we illustrate in the following, the shock $x_{t}$ - the model counterpart of EGV - is key to rationalize the main empirical properties of the term structure of equity and its dynamics in general equilibrium.

## State-Price Density and Equity Returns

Recursive preferences lead to a non-affine state-price density. Therefore, to solve for prices and preserve analytic tractability, we follow the methodology presented by Eraker and Shaliastovich (2008), which is based on the Campbell and Shiller (1988)'s log-linearization. ${ }^{13}$ The continuous time (continuously compounded) log-return on equity - e.g., the claim on the shareholders' consumption $D_{t}$-can be expressed as

$$
d \log R_{t}=k_{0} d t+k_{1} d\left(p d_{t}\right)-\left(1-k_{1}\right) p d_{t} d t+d \log D_{t}
$$

[^12]where $p d_{t}=\log \left(P_{t} / D_{t}\right)$ and the endogenous constants $k_{0}$ and $k_{1}$ satisfy
$$
k_{0}=-\log \left(\left(1-k_{1}\right)^{1-k_{1}} k_{1}^{k_{1}}\right) \quad \text { and } \quad k_{1}=e^{\mathbb{E}\left(p d_{t}\right)} /\left(1+e^{\mathbb{E}\left(p d_{t}\right)}\right)
$$

Recursive preferences lead to the following Euler equation, which allows to characterize the state-price density, $M_{t}$, used to price any asset in the economy:

$$
\begin{equation*}
\mathbb{E}_{t}\left[\exp \left(\log \frac{M_{t+\tau}}{M_{t}}+\int_{t}^{t+\tau} d \log R_{s}\right)\right]=1 \tag{8}
\end{equation*}
$$

The state-price density satisfies

$$
\begin{equation*}
d \log M_{t}=\theta \log \beta d t-\frac{\theta}{\psi} d \log D_{t}-(1-\theta) d \log R_{t} . \tag{9}
\end{equation*}
$$

To solve for the return on equity and, in turn, the state-price density, we conjecture that $p d_{t}$ is affine in the vector of state variables. Then, the Euler equation is used to solve for the coefficients. In turn, the state-price density has dynamics:

$$
\begin{equation*}
\frac{d M_{t}}{M_{t}}=-r_{t} d t-\Omega_{x}\left(x_{t}\right) d B_{x, t}-\Omega_{z} d B_{z, t} \tag{10}
\end{equation*}
$$

In this equation, the risk-free rate is affine in the shocks $x_{t}$ and $z_{t}$, as follows:

$$
r_{t}=r_{0}+r_{x} x_{t}+r_{z} z_{t}
$$

where the coefficients $r_{x}$ and $r_{z}$ satisfy:

$$
\begin{aligned}
& r_{x}=-\gamma-\frac{A_{x}(\gamma \psi-1)\left(1-k_{1}\left(1-\lambda_{x}\right)\right)}{\psi-1}-\frac{A_{x}^{2} k_{1}^{2} \sigma_{x}^{2}(\gamma \psi-1)^{2}}{2(\psi-1)^{2}} \\
& r_{z}=-\frac{\lambda_{z} \phi}{\psi}
\end{aligned}
$$

Moreover, the two equilibrium prices of risk are given by

$$
\Omega_{x}\left(x_{t}\right)=\sigma_{x} \sqrt{x_{t}} \frac{k_{1} A_{x}(\gamma-1 / \psi)}{1-1 / \psi} \quad \text { and } \quad \Omega_{z}=\sigma_{z} \frac{k_{1} A_{z}(\gamma-1 / \psi)}{1-1 / \psi}+\sigma_{z} \gamma \phi,
$$

where the price elasticities $A_{x}$ and $A_{z}$ are defined below. The equity price is given by

$$
\begin{equation*}
P_{t}=\int_{0}^{\infty} \mathbb{E}_{t}\left[\frac{M_{t+\tau}}{M_{t}} D_{t+\tau}\right] d \tau=D_{t} \exp \left(A_{0}+A_{x} x_{t}+A_{z} z_{t}\right) \tag{11}
\end{equation*}
$$

where

$$
A_{x}=-\frac{2(1-1 / \psi)}{1-k_{1}\left(1-\lambda_{x}\right)+\Phi} \quad \text { and } \quad A_{z}=-\frac{\phi \lambda_{z}(1-1 / \psi)}{1-k_{1}\left(1-\lambda_{z}\right)}
$$

with $\Phi=\sqrt{1-k_{1}\left(2\left(1-\lambda_{x}\right)+k_{1}\left(2(\gamma-1) \sigma_{x}^{2}-\left(1-\lambda_{x}\right)^{2}\right)\right)}$. Under plausible assumptions about preferences $(\gamma>1 / \psi, \psi>1)$, we have that the market prices of permanent and transitory risk satisfy $\Omega_{x}<0, \Omega_{z}<0$, and $A_{x}<0$, and $-\phi<A_{z}<0$. Thus, prices relative to payouts decrease with both expected growth volatility and short-term shocks.

An application of Itô's Lemma provides the return variance:

$$
\sigma_{R}^{2}\left(x_{t}\right)=x_{t} \sigma_{x}^{2} A_{x}^{2}+\sigma_{z}^{2}\left(\phi+A_{z}\right)^{2}
$$

and the equity premium:

$$
R P\left(x_{t}\right)=x_{t} \sigma_{x}^{2}\left(\frac{\gamma-1 / \psi}{1-1 / \psi} A_{x}^{2} k_{1}\right)+\sigma_{z}^{2}\left(\phi+A_{z}\right)\left(\gamma \phi+\frac{\gamma-1 / \psi}{1-1 / \psi} A_{z} k_{1}\right)
$$

Return variance and equity premium increase with expected growth volatility, under plausible preferences. Return variance moves negatively with prices in accord with the volatility feedback (Campbell and Hentschel, 1992). The equity premium increases with expected growth volatility, in accord with the long-run risk literature (Bansal and Yaron, 2004).

## Term Structures of Equity and Bond

The price of the dividend strip with maturity $\tau$ is defined as the integrand of Eq. (11) and has exponential affine solution:

$$
P_{t, \tau}=\mathbb{E}_{t}\left[\frac{M_{t+\tau}}{M_{t}} D_{t+\tau}\right]=D_{t} \exp \left(a_{0}(\tau)+a_{x}(\tau) x_{t}+\left(a_{z}(\tau)-\phi\right) z_{t}\right)
$$

The deterministic functions $a_{0}(\tau), a_{x}(\tau)$, and $a_{z}(\tau)$ solve a system of ordinary differential equations. The price elasticities to the permanent and short-run shocks are given by:

$$
\begin{aligned}
& a_{x}(\tau)=\frac{2 \Psi_{0}}{\sqrt{\Psi_{1}^{2}-4 \Psi_{0} \Psi_{2}} \operatorname{coth}\left(\frac{\tau}{2} \sqrt{\Psi_{1}^{2}-4 \Psi_{0} \Psi_{2}}\right)-\Psi_{1}} \\
& a_{z}(\tau)=\frac{\phi}{\psi}+\phi(1-1 / \psi) e^{-\lambda_{z} \tau}
\end{aligned}
$$

where

$$
\begin{aligned}
& \Psi_{0}=\gamma-1+A_{x}\left(1-k_{1}\left(1-\lambda_{x}\right)\right) \frac{\gamma-1 / \psi}{1-1 / \psi}+A_{x}^{2} \frac{k_{1}^{2} \sigma_{x}^{2}(\gamma-1 / \psi)^{2}}{2(1-1 / \psi)^{2}} \\
& \Psi_{1}=-k_{1} \sigma_{x}^{2} A_{x} \frac{\gamma-1 / \psi}{1-1 / \psi}-\lambda_{x}
\end{aligned}
$$

$$
\Psi_{2}=\frac{\sigma_{x}^{2}}{2}
$$

Using the dividend strip price, we can compute the term structure of the dividend strip risk premium, $R P_{D S}\left(x_{t}, \tau\right)$, which is a function of only $x_{t}$ and the maturity $\tau$ :

$$
\begin{equation*}
R P_{D S}\left(x_{t}, \tau\right)=x_{t} \sigma_{x}^{2} a_{x}(\tau)\left(\frac{k_{1} A_{x}(\gamma-1 / \psi)}{1-1 / \psi}\right)+\sigma_{z}^{2} a_{z}(\tau)\left(\frac{k_{1} A_{z}(\gamma-1 / \psi)}{1-1 / \psi}+\gamma \phi\right) . \tag{12}
\end{equation*}
$$

The dividend strip risk premium is increasing in the conditional volatility of expected growth. Moreover, for $\gamma>\psi>1$, the first and the second term of Eq. (12) imply respectively that the permanent shock (and, thus, expected growth volatility) induces an upward-sloping effect, and the short-term shocks induce a downward-sloping effect on the term structure. When the conditional volatility of expected growth is large, the upward sloping effect dominates and the price-payout ratio declines. Conversely, when the conditional volatility of expected growth is small, the downward-sloping effect dominates and the price-payout ratio rises. In turn, the equity term premium - that is, the slope of the dividend strip risk premium - is counter-cyclical as documented by Gormsen (2020).

Similarly, we compute the term structure of the forward equity yields. The forward equity yield with maturity $\tau$ is the difference between the equity yield and the risk-less bond yield:

$$
\begin{equation*}
f e y(t, \tau)=e y(t, \tau)-b y(t, \tau) \tag{13}
\end{equation*}
$$

where

$$
e y(t, \tau)=-\frac{1}{\tau} \log \left(P(t, \tau) / D_{t}\right), \quad \text { and } \quad b y(t, \tau)=-\frac{1}{\tau} \log B(t, \tau)
$$

The price of the risk-less bond with maturity $\tau$ is given by

$$
B(t, \tau)=\mathbb{E}_{t}\left[\frac{M_{t+\tau}}{M_{t}}\right]=\exp \left(b_{0}(\tau)+b_{x}(\tau) x_{t}+b_{z}(\tau) z_{t}\right)
$$

The deterministic functions $b_{0}(\tau), b_{x}(\tau)$, and $b_{z}(\tau)$ solve a system of ordinary differential equations. The price elasticities are given by:

$$
b_{x}(\tau)=\frac{2 \Phi_{0}}{\sqrt{\Phi_{1}^{2}-4 \Phi_{0} \Phi_{2}} \operatorname{coth}\left(\frac{\tau}{2} \sqrt{\Phi_{1}^{2}-4 \Phi_{0} \Phi_{2}}\right)-\Phi_{1}}
$$

$$
b_{z}(\tau)=\frac{\phi}{\psi}\left(1-e^{-\lambda_{z} \tau}\right) .
$$

where $\Phi_{0}=\Psi_{0}+1, \Phi_{1}=\Psi_{1}$, and $\Phi_{2}=\Psi_{2}$.

## Cross-Sectional Equity Returns

We introduce a cross-section of payout streams, which can be interpreted as the payout of firms or the payout of portfolios of stocks. Specifically, we define the cross-sectional payout stream, $D_{t}^{\varphi}$, by the following dynamics:

$$
\begin{equation*}
d \log D_{t}^{\varphi}=d \log D_{t}+\varphi\left(\bar{x}-x_{t}\right) d t+\sigma_{\varphi} d B_{\varphi, t} . \tag{14}
\end{equation*}
$$

The loading $\varphi$ captures the heterogeneous additional exposure to $x_{t}$ in the cross-section. ${ }^{14}$ Furthermore, the volatility parameter $\sigma_{\varphi} \neq 0$ allows for idiosyncratic risk. Following Eraker and Shaliastovich (2008), the $\log$ return on the stock paying out $D_{t}^{\varphi}$ evolves as

$$
d \log R_{t}^{\varphi}=k_{0}^{\varphi} d t+k_{1}^{\varphi} d\left(p d_{t}^{\varphi}\right)-\left(1-k_{1}^{\varphi}\right) p d_{t}^{\varphi} d t+d \log D_{t}^{\varphi}
$$

where $k_{0}^{\varphi}$ and $k_{1}^{\varphi}$ are endogenous constants. The price of this stock then can be written as

$$
\begin{equation*}
P_{t}^{\varphi}=\int_{0}^{\infty} \mathbb{E}_{t}\left[\frac{M_{t+\tau}}{M_{t}} D_{t+\tau}^{\varphi}\right] d \tau=D_{t}^{\varphi} \exp \left(A_{0}^{\varphi}+A_{x}^{\varphi} x_{t}+A_{z}^{\varphi} z_{t}\right) \tag{15}
\end{equation*}
$$

where

$$
A_{x}^{\varphi}=-\frac{\sqrt{\eta_{1}^{2}-4 \eta_{0}(\varphi) \eta_{2}}+\eta_{1}}{2 \eta_{2}} \quad \text { and } \quad A_{z}^{\varphi}=-\frac{\phi \lambda_{z}(1-1 / \psi)}{1-k_{1}^{\varphi}\left(1-\lambda_{z}\right)}
$$

with $\eta_{0}(\varphi), \eta_{1}$, and $\eta_{2}$ derived in Appendix A. The price elasticity $A_{x}^{\varphi}$ with respect to $x_{t}$ is larger (smaller) in magnitude than the market price elasticity $A_{x}$ if $\varphi$ is larger (smaller) than zero. Thus, the larger the payout loading on expected growth volatility, the more pro-cyclical the valuation ratio and, hence, the more counter-cyclical the risk premium. Applying Itô's Lemma to Eq. (15), the corresponding risk premium is given by

$$
\begin{equation*}
R P^{\varphi}\left(x_{t}\right)=x_{t} \sigma_{x}^{2} A_{x}^{\varphi}\left(\frac{k_{1} A_{x}(\gamma-1 / \psi)}{1-1 / \psi}\right)+\sigma_{z}^{2}\left(A_{z}^{\varphi}+\phi\right)\left(\frac{k_{1} A_{z}(\gamma-1 / \psi)}{1-1 / \psi}+\gamma \phi\right) . \tag{16}
\end{equation*}
$$

The risk premium is increasing in the conditional volatility of expected growth.

[^13]Consider the payout streams associated with two loadings $\varphi_{V}>\varphi_{G}$. The valuation ratio associated to the payout with the higher exposure to $x_{t}$ (i.e., $\varphi_{V}$ ) is lower and more cyclical than the valuation ratio associated to the payout with the lower exposure (i.e., $\varphi_{G}$ ). Thus, for $\varphi_{V} \gg \varphi_{G}$, we can interpret the former and the latter as the payout streams of value and growth firms respectively. Therefore, the model-implied value premium is given by

$$
V P\left(x_{t}\right)=R P^{\varphi_{V}}\left(x_{t}\right)-R P^{\varphi_{G}}\left(x_{t}\right)>0 .
$$

Since the coefficient of $x_{t}$ in Eq. (16) is positive and increasing in $\varphi$, then the value premium is positive and counter-cyclical, in accord with the empirical evidence. In particular, $x_{t}$ is a driver of the value premium consistently with the predictive regressions in Section I.

To better connect this prediction with the model predictions about the term structure of equity, we look at the term structures of risk premia in the cross section. The price of the claim that pays out $D_{t+\tau}^{\varphi}$ at maturity $\tau$ is given by

$$
P_{t, \tau}^{\varphi}=\mathbb{E}_{t}\left[\frac{M_{t+\tau}}{M_{t}} D_{t+\tau}^{\varphi}\right]=D_{t}^{\varphi} \exp \left(a_{0}^{\varphi}(\tau)+a_{x}^{\varphi}(\tau) x_{t}+\left(a_{z}^{\varphi}(\tau)-\phi\right) z_{t}\right)
$$

The deterministic functions $a_{0}^{\varphi}(\tau), a_{x}^{\varphi}(\tau)$, and $a_{z}^{\varphi}(\tau)$ solve a system of ordinary differential equations. The price elasticities are given by:

$$
\begin{aligned}
& a_{x}^{\varphi}(\tau)=\frac{2 \Theta_{0}}{\sqrt{\Theta_{1}^{2}-4 \Theta_{0} \Theta_{2}} \operatorname{coth}\left(\frac{\tau}{2} \sqrt{\Theta_{1}^{2}-4 \Theta_{0} \Theta_{2}}\right)-\Theta_{1}}, \\
& a_{z}^{\varphi}(\tau)=\frac{\phi}{\psi}+\phi(1-1 / \psi) e^{-\lambda_{z} \tau}
\end{aligned}
$$

where $\Theta_{0}=\Psi_{0}-\varphi, \Theta_{1}=\Psi_{1}$, and $\Theta_{2}=\Psi_{2}$. Applying Itô's Lemma to this strip price, we compute the term structure of the strip risk premium, which depends on $x_{t}$ and the maturity:

$$
R P_{D S}^{\varphi}\left(x_{t}, \tau\right)=x_{t} \sigma_{x}^{2} a_{x}^{\varphi}(\tau)\left(\frac{k_{1} A_{x}(\gamma-1 / \psi)}{1-1 / \psi}\right)+\sigma_{z}^{2} a_{z}^{\varphi}(\tau)\left(\frac{k_{1} A_{z}(\gamma-1 / \psi)}{1-1 / \psi}+\gamma \phi\right) .
$$

The strip risk premium is increasing in the conditional volatility of expected growth. The larger the payout loading $\varphi$ on expected growth volatility, the steeper the (unconditional) strip risk premium. In turn, the unconditional slope of the strip risk premium is larger for value firms than growth firms.

## III Model Analysis

The next proposition summarizes the main model predictions about equilibrium dynamics, as a function of $x_{t}$, i.e., EGV, which are consistent with the empirical analysis of Section I.

Proposition. Under plausible parameters, the model predicts that:

1. The price-payout ratio decreases with $E G V$ :

$$
\frac{\partial}{\partial x} \log P_{t} / D_{t}<0
$$

2. The slope of the equity yields decreases with $E G V$ :

$$
\frac{\partial^{2}}{\partial \tau \partial x} e y(t, \tau)<0
$$

3. The slope of the dividend strip risk premium increases with $E G V$ :

$$
\frac{\partial^{2}}{\partial \tau \partial x} R P_{D S}\left(x_{t}, \tau\right)>0
$$

4. The value premium increases with $E G V$ :

$$
\frac{\partial}{\partial x}\left(R P^{\varphi_{V}}\left(x_{t}\right)-R P^{\varphi_{G}}\left(x_{t}\right)\right)>0 .
$$

In the following, we analyze these predictions in detail. We present the model calibration and the predictions about standard moments and price dynamics. Then, we discuss the term structure of equity and its dynamics, which is our main focus. We also explore the predictions about cross-sectional returns and the value premium. Finally, we show that the dynamics of equity slope hold irrespective of the sign of the unconditional average.

## III.A Calibration and Standard Moments

Table 3 provides our baseline setting. Economic fundamentals are described by long-run growth $(\mu)$, the parameters describing expected growth volatility ( $\bar{x}, \sigma_{x}$, and $\lambda_{x}$ ), short-run shocks ( $\sigma_{z}$ and $\lambda_{z}$ ), and the leverage effect on payouts $(\phi)$. These parameters are set to match a wide array of data moments from the time-series of fundamentals and financial returns. The table also displays the time discount factor $(\beta)$, risk aversion $(\gamma)$, and elasticity of intertemporal substitution $(\psi)$-that assume standard values in the literature.

Table 3: Model Parameters

| Fundamentals | Symbol | Value |
| :--- | :---: | ---: |
| Long-run expected growth | $\mu$ | 0.025 |
| Expected growth volatility |  |  |
| location | $\bar{x}$ | 0.08 |
| scale | $\sigma_{x}$ | 0.04 |
| reversion | $\lambda_{x}$ | 0.15 |
| Short-run shock |  |  |
| scale | $\sigma_{z}$ | 0.025 |
| reversion | $\lambda_{z}$ | 0.15 |
| Payout leverage | $\phi$ | 7.5 |
| Preferences | Symbol | Value |
| Time discount factor | $\beta$ | 0.96 |
| Risk aversion | $\gamma$ | 7.5 |
| Elasticity of intertemporal substitution | $\psi$ | 1.5 |

Note. Parameter values of the baseline calibration.

Table 4 displays selected moments about consumption, payout, and financial returns from both the data and the simulated model. The model is simulated at monthly frequency, and each simulation covers a time period comparable to the postwar experience. Simulated time-series are then aggregated at yearly frequency. For each moment, we report selected percentiles from the distribution of our model simulations as well as the population statistic.

Notably, the model matches standard moments considered in the literature quite well. Consumption and corporate payouts are cointegrated and, thus, their growth rates have similar sample averages (about 2.5\%), which in turn are close to their empirical counterparts. Moreover, consumption growth volatility is modest (about $3 \%$ in the model vs $1.8 \%$ in the data). Notably, our model captures the 1-year excess volatility of payout growth (about $18 \%$ in the model vs $15 \%$ in the data), ${ }^{15}$ and the decline in payout growth volatility at long horizons (20-year volatility is about $10.5 \%$ in the model vs $8.5 \%$ in the data). Cointegration implies the stationarity of the payout to consumption ratio, whose volatility is captured quite

[^14]Table 4: Standard Moments

|  |  |  | Model |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Moment | Data |  |  | $50 \%$ | $95 \%$ | $97.5 \%$ | Pop. |  |  |
|  |  | $2.5 \%$ | $5 \%$ | $50 \%$ |  |  |  |  |  |
| Avg consumption growth | 0.021 | 0.007 | 0.010 | 0.025 | 0.038 | 0.040 | 0.025 |  |  |
| Std consumption growth | 0.018 | 0.024 | 0.025 | 0.030 | 0.036 | 0.037 | 0.031 |  |  |
| Avg payout growth |  |  |  |  |  |  |  |  |  |
| Std payout growth | 0.030 | 0.005 | 0.008 | 0.025 | 0.041 | 0.043 | 0.025 |  |  |
| 20-year Std payout growth | 0.148 | 0.152 | 0.157 | 0.182 | 0.208 | 0.212 | 0.183 |  |  |
|  | 0.083 | 0.019 | 0.028 | 0.105 | 0.222 | 0.249 | 0.121 |  |  |
| Std log payout-consumption ratio | 0.228 | 0.177 | 0.186 | 0.255 | 0.355 | 0.378 | 0.296 |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Avg risk-free rate | 0.007 | -0.022 | -0.017 | 0.008 | 0.032 | 0.037 | 0.008 |  |  |
| Std risk-free rate | 0.025 | 0.021 | 0.022 | 0.031 | 0.045 | 0.048 | 0.036 |  |  |
| Avg excess equity return | 0.068 | 0.025 | 0.031 | 0.057 | 0.085 | 0.090 | 0.057 |  |  |
| Std excess equity return | 0.175 | 0.116 | 0.120 | 0.139 | 0.159 | 0.163 | 0.140 |  |  |
| Avg Sharpe ratio | 0.388 | 0.178 | 0.216 | 0.409 | 0.625 | 0.666 | 0.403 |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Avg log price-dividend ratio | 3.435 | 3.121 | 3.135 | 3.204 | 3.271 | 3.284 | 3.203 |  |  |
| Std log price-dividend ratio | 0.443 | 0.061 | 0.065 | 0.090 | 0.125 | 0.134 | 0.104 |  |  |
| Avg excess high-minus-low return | 0.035 | 0.001 | 0.008 | 0.037 | 0.066 | 0.072 | 0.036 |  |  |
| Std excess high-minus-low return | 0.129 | 0.106 | 0.109 | 0.127 | 0.146 | 0.149 | 0.127 |  |  |

Note. This table reports moment statistics from both data and model simulations. Model-implied statistics are either moment quantiles from short-sample ( 72 years) simulations or population moments. The model is simulated at monthly frequency. Statistics are yearly moments if not stated otherwise. Consumption and payout data are from NIPA tables. Returns are from K. French webpage. The price-dividend ratio is from R. Shiller webpage.
well by the model (about $25 \%$ in the model vs $23 \%$ in the data).
Consistent with the data, the risk-free rate implied by the model is low (about $0.8 \%$ vs $0.7 \%$ in the data) and smooth (about $3 \%$ volatility vs $2.5 \%$ in the data). Moreover, the model predicts a sizeable equity premium that compares well with its empirical estimates (about $6 \%$ vs $7 \%$ in the data). The excess return volatility in the model is somewhat smaller than in the actual data (about $14 \%$ in the model vs $17.5 \%$ in the data). In turn, the model Sharpe ratio is slightly larger than in the data (about $41 \%$ in the model vs $39 \%$ in the data). The model predicts a realistic value for the average $\log$ price-payout ratio ( 3.2 vs 3.4 in the
data) but its volatility is smaller than in the data ( $9 \%$ vs $44 \%$ in the data). Moreover, the model predicts a positive and sizeable value premium that compares well with the historical average return on the high-minus-low (HML) portfolio (3.7\% in the model vs $3.5 \%$ in the data). The HML return volatility in the model is close to its empirical counterpart (12.7\% vs $12.9 \%$ in the data). ${ }^{16}$ Overall, whereas our focus is the term structure of equity and its dynamics, Table 4 suggests that the model performs well in describing the main properties of financial markets - then proposing a solution to the challenge posed by van Binsbergen et al. (2012) to leading models in the literature. For instance, the model explains well the risk-free rate and equity premium puzzles under a reasonable preference setting.

The quantitative analysis of the model predictions, especially those regarding the term structure of equity, crucially depends on the balance between long-term shocks-which induce upward-sloping risk with the horizon - and short-term shocks-which induce downwardsloping risk. We devise three exercises to verify that the timing of risk and its equilibrium pricing are consistent with the data.

First, we compare the term structure of the volatility of payout growth rates from the model simulation with that in the actual data. The left panel of Figure 7 displays the modelimplied term structure at any horizon up to 20 years. Volatility monotonically decreases from about $20 \%$ to $10 \%$. The plot also shows the empirical volatility of growth rates computed from either corporate profits or dividend plus net repurchases. The former decreases from about $14.8 \%$ to $8.3 \%$, and the latter decreases from about $26.6 \%$ to $11.8 \%$. Thus, the model volatility matches well the level and the timing of fundamental risk.

Second, we exploit cointegration and consider the model predictability of payout growth through the logarithm of the payout-consumption ratio:

$$
\left(\log D_{t+\tau}-\log D_{t}\right) / \tau=\alpha+\beta \log D_{t} / C_{t}+\varepsilon_{t}, \quad \tau \in(0.25,10)
$$

The middle panel of Figure 7 shows the median as well as the $2.5 \%$ and $97.5 \%$ percentiles of the predictive slope at any horizon between 1-quarter and 10 years. Slopes are negative

[^15]

Figure 7: Cointegration and the Timing of Payout Risk. The left panel displays the payout growth rates volatility as a function of the horizon in the model and in the data (corporate profits and net dividends). The middle and the right panels show the predictive coefficient from the regression of cumulative payout growth on the payout to consumption ratio across the horizon in the model and in the data respectively.
and statistically different from zero. This pattern arises because the model assumes that the ratio is positively driven by the stationary short-run shock $z_{t}$ of consumption and payout. The right panel of Figure 7 displays the same regression estimates from actual data. Slopes are negative, and the Hansen-Hodrick $95 \%$ confidence interval documents that they are statistically different from zero across the horizon. The model well matches the sign, the magnitude, and the slope across the horizons of payout growth predictability.

Third, we verify that our state-price density satisfies the bound introduced by Alvarez and Jermann (2005). They decompose the state-price density into a permanent and transitory component and estimate a lower bound on the fraction of growth rates volatility due to the permanent component. The empirical estimates of the bound are close to and bounded above by unity. Following Hansen and Scheinkman (2009), we compute the volatility ratio in our model and find values between 1.03 and 1.05 depending on the horizon. Thus, the bound is satisfied. Therefore, the way our model transmits permanent and transitory risks from fundamentals to the endogenous state-price density is consistent with the empirical evidence: Sizable short-term compensations are not due to over-weighing short-term risk.


Figure 8: Price Dynamics. The figure reports the model log price to payout ratio (left panel) and its instantaneous volatility (right panel) as a function of EGV $\left(x_{t}\right)$. The (standardized) unconditional density of EGV is superimposed.

Overall, the model captures well the timing of fundamental risk and how it gets priced in equilibrium. We believe that, although simple and parsimonious, our model represents a well-suited laboratory to understand the term structure of equity in equilibrium. ${ }^{17}$

We conclude this section by showing the dynamics of the logarithm of the price-payout ratio and its volatility. Figure 8 shows that prices decline when EGV $\left(x_{t}\right)$ increases, whereas their volatility rises. These model results (claim 1 of our Proposition) are consistent with the empirical evidence in Figure 3. In the following, we study the dynamics of the equity slope and refer to countercyclical behavior in terms of either a positive correlation with EGV or, equivalently, a negative correlation with the price-payout ratio (see Gormsen, 2020).

## III.B The Term Structure of Equity

We now study the term structure of equity and its dynamics, which are the key focus of our paper. In particular, we analyze the slope dynamics of forward equity yields and dividend strip risk premia (claim 2 and 3 of our Proposition), which have attracted substantial attention by the recent literature. To the best of our knowledge, our framework is the first

[^16]general equilibrium model that jointly reproduces and rationalizes the empirical patterns documented by van Binsbergen et al. (2012), van Binsbergen et al. (2013), and Gormsen (2020), as we illustrate in the following.

In our model, permament shocks induce upward-sloping risk. This upward-sloping risk exhibits the time-varying and counter-cyclical properties of EGV. Conversely, transitory (short-term) shocks induce downward-sloping risk. This downward-sloping risk is constant over time, because transitory shocks are homoskedastic. As a result, the equity term premium is positive in bad times (in which EGV is high), and negative in good times (in which EGV is low)-i.e., the countercyclical dynamics of the equity term premium depend on the heteroskedastic nature of the main source of risk affecting long-term payouts.

The upper panels of Figure 9 show the term structures of the forward equity yield and dividend strip risk premium in economic expansion, recession, and in the steady state. In the steady state $\left(x_{t}=\bar{x}\right)$, forward equity yields are about flat across the horizon. When economic conditions deteriorate (EGV rises), short-term forward equity yields substantially rise, whereas long-term ones increase by a smaller amount. As a result, the slope of forward equity yields becomes negative during economic recessions. Conversely, when economic conditions improve (EGV falls), short-term forward equity yields substantially decrease (and become negative), whereas long-term ones decrease by a smaller amount. Thus, the slope becomes positive during expansions. Overall, the slope dynamics of forward equity yields are procyclical, as documented by van Binsbergen et al. (2013).

Consider now the dividend-strip risk premia. In the steady-state $\left(x_{t}=\bar{x}\right)$, the risk premium is slightly downward-sloping (van Binsbergen et al., 2012). When economic conditions deteriorate (EGV rises), long-term risk premia rise substantially, whereas short-term ones increase only slightly. Thus, the slope of risk premia is positive during economic recessions. Conversely, when economic conditions improve (EGV falls), long-term risk premia substantially decrease, whereas short-term ones decrease by a smaller amount. Thus, the slope is negative during economic booms. Overall, the equity term premium dynamics are counter-


Figure 9: Equity Slope Dynamics. The upper panels of the figure report the model forward equity yield (left) and the model dividend strip risk premium (right) as a function of the horizon for several values of EGV $\left(x_{t}\right)$. The lower panels of the figure show the model forward equity yield spread (left) and the model equity term premium (right) as a function of EGV $\left(x_{t}\right)$. The (standardized) unconditional density of EGV is superimposed.
cyclical (Gormsen, 2020). The slope dynamics of the forward equity yield and the dividend strip premium can also be inspected through the behavior of their infinite-horizon spreads:

$$
\lim _{\tau \rightarrow \infty} f e y(t, \tau)-\lim _{\tau \rightarrow 0} f e y(t, \tau) \quad \text { and } \quad \lim _{\tau \rightarrow \infty} R P_{D S}\left(x_{t}, \tau\right)-\lim _{\tau \rightarrow 0} R P_{D S}\left(x_{t}, \tau\right)
$$

The lower panels of Figure 9 report these spreads as functions of EGV. The forward equity yield spread is decreasing with EGV, positive in good states, and negative in bad states. Thus, forward equity yields feature procyclical dynamics. Conversely, the spread of the
dividend strip risk premium is increasing with EGV, being negative in expansion and positive in recession. Thus, the equity term premium features countercyclical dynamics. These slopes switch sign across economic conditions.


Figure 10: Equity Yield Decomposition. The panels report the model equity yield (upper, left), the model payout expected growth (upper, right), the model bond yield (lower, left), and the model equity yield premium (lower, right) as a function of the horizon for several values of EGV $\left(x_{t}\right)$.

To better inspect the model mechanism, we exploit the equity yield decomposition:

$$
e y(t, \tau)=b y(t, \tau)-g_{D}(t, \tau)+\vartheta(t, \tau)
$$

The equity yield is the difference between the risk-free bond yield, $b y(t, \tau)$, and expected payout growth, $g_{D}(t, \tau)$, plus the equity yield premium, $\vartheta(t, \tau)$. In turn, the forward equity
yield can be written either as the difference between the equity yield and the risk-free bond yield or as the difference between the equity yield premium and expected payout growth:

$$
f e y(t, \tau)=e y(t, \tau)-b y(t, \tau)=\vartheta(t, \tau)-g_{D}(t, \tau)
$$

Figure 10 studies the dynamics of the equity yield and its three components as a function of the horizon in expansion, recession, and the steady state. First, the premium component features countercyclicality in level and slope, because it represents a time-varying compensation for the exposure to EGV-affecting long-term payouts more heavily than short-term ones. Second, the bond is a hedge instrument against equity risk and, thus, the risk-free bond yield features procyclical level. In turn, the slope of bond yields inherits the countercyclical dynamics of the risk premium slope. Third, expected payout growth is procyclical in level and has countercyclical slope because of the mean-reverting dynamics of both EGV and short-run shocks. As a result of these three forces, the equity yield is countercyclical in level-because the joint effect of expected growth and risk premium dominates the effect of the bond yield-but features procyclical slope-because the effect of expected growth dominates the joint effect of the bond yield and risk premium. In turn, the forward equity yield shows slightly sharper cyclicality of both level and slope than the equity yield, because it does not account for the opposite bond yield cyclicality of both level and slope. ${ }^{18}$

## III.C Cross-Sectional Returns and Value Premium

We now study the model predictions for the cross-section of equity returns. We set $\varphi_{V}=$ $-\varphi_{G}=0.40$ and assume the idiosyncratic risk $\sigma_{\varphi}=7.5 \%$, see Eq. (14).

The upper left panel of Figure 11 shows valuation ratios as a function of EGV for both value and growth firms. The price of value firms is lower in level and more sensitive to expected growth volatility. The upper right panel of the figure displays the premium of the high-minus-low portfolio - that is, the value premium - as a function of EGV. The value

[^17]premium is positive, sizeable (about $3.7 \%$ ), and countercyclical. These three results are consistent with the empirical findings in the literature and with our results in Section I: EGV is a driver of the value premium dynamics (claim 4 of our Proposition).

To better inspect the model mechanism, we consider the premium on the claim of value and growth payouts across the horizons. The long-run impact of EGV on payouts suggests that long-term claims to value-type payouts should feature a larger and more volatile premium than short-term claims. Instead, the lower loading of growth-type payouts to EGV suggests that both long-term claims and short-term claims should command a similar risk premium and feature little sensitivity to EGV. The lower panels of Figure 11 display such heterogeneity in the dynamics of the premia to claims of value-type and growth-type payouts across the horizons. This result is consistent with the long-run predictability of value returns by EGV and the lack of predictability of growth returns documented in Section I.

The model mechanism leading to the value premium dynamics is interesting for three reasons. First, Giglio et al. (2020) document that the risk premia to claims of value- and growth-type payouts respectively increase and decrease with the horizon. To the best of our knowledge, our framework is the first general equilibrium that explains this stylized fact and reconciles it with the counter-cyclical dynamics of both the value premium and the equity term premium. ${ }^{19}$ Second, our results share with Bansal, Dittmar, and Lundblad (2005) the idea that the value premium arises from the excess loading of value firms on long-term fundamental risk over growth firms. However, differently from their approach, our model does not lead to negligible short-term compensations-in contrast with the empirical evidence. Instead, we explain in general equilibrium sizeable short-term risk premia, similar to the partial equilibrium model of Lettau and Wachter (2007). Differently from their durationbased interpretation of the value premium but consistent with recent empirical findings, our model does not predict downward-sloping (respectively, upward-sloping) compensations to value (growth) firms. Third, the close connection between the model predictions about cross-

[^18]

Figure 11: Value and Growth Price Dynamics and Term Structures. The upper panels of the figure report the model log price-payout ratio of value and growth firms (left), the model value premium (right) as a function of EGV $\left(x_{t}\right)$. The lower panels of the figure show the model dividend strip risk premium for value (left) and growth firms (right) as a function of the horizon for several values of EGV $\left(x_{t}\right)$.
sectional returns and term structures and their empirical counterparts strongly corroborates the main model mechanism on the dynamics of the equity term premium.

## III.D Robustness: Unconditional Equity Term Premium

We now study the main model predictions in light of the recent debate regarding the measurement of the unconditional equity term premium. van Binsbergen et al. (2012), Gormsen (2020), and Giglio et al. (2020) provide evidence that the unconditional equity compensa-


Figure 12: Unconditional Equity Term Premium. The figure reports the dividend strip risk premium as a function of the horizon for several values of EGV $\left(x_{t}\right)$ under our alternative calibration. The shaded area denote the difference with respect to the baseline calibration in the steady-state.
tions are downward-sloping. Recently, however, Bansal et al. (2020) call into question this result as small samples can over-represent economic conditions in which the slope is negative (e.g., recessions) and lead to a wrong assessment about the unconditional slope.

While we are agnostic about the resolution of this empirical issue, we verify whether our economic mechanism is robust to the sign of the unconditional slope. In particular, we verify whether (i) the model dynamics of equity slope are affected by the sign of the unconditional slope, and (ii) the model can reconcile standard asset pricing moments with the dynamics of the equity term structure under either increasing or decreasing unconditional term premium.

Our model allows to easily verify that the slope dynamics are robust to the sign of the unconditional slope. The dividend strip risk premium in Eq. (12) has two components: a permanent and heteroskedastic component driven by EGV (and commanding upward-sloping compensations) and a transitory and homoskedastic component due to short-run shocks (and commanding downward-sloping compensations). The unconditional slope depends on the relative strength of the two components. However, the conditional slope moves with EGV and, so, features counter-cyclical dynamics and can switch sign over time.

We consider an alternative calibration where we increase the persistence of EGV, $x_{t}$ and
decrease the persistence of the short-run shock, $z_{t}$. Namely, we change $\lambda_{x}$ from 0.15 to 0.10 and $\lambda_{z}$ from 0.15 to 0.20 . We keep all the other parameters unchanged. Under this alternative calibration, standard asset pricing moments are still reasonable and similar to those under our baseline calibration. Appendix Table B5 reports several moment statistics from model simulations. We observe a slightly higher equity premium and a lower risk-free rate as a result of preferences for the early resolution of uncertainty. Importantly, the change in the relative strength of the two shocks affects the risk premium across the horizon. Figure 12 shows the dividend strip risk premium at the average state under both the alternative and the baseline calibrations. We observe that a more persistent EGV leads to higher long-term premia, and a less persistent short-run shock leads to lower short-term premia. Thus, under this alternative calibration, the unconditional equity term premium shifts from negative to positive. Figure 12 also shows the dividend strip risk premium in good and bad states. As under the baseline calibration (see Figure 9) and in accord with the empirical evidence, compensations increase with the horizon in bad states (recessions) and decrease with the horizon in good states (expansions). In turn, the equity term premium has counter-cyclical dynamics as documented by Gormsen (2020).

Overall, our model reconciles standard asset pricing models with the empirically consistent dynamics of the equity term premium. The economic mechanism is robust to the sign of its unconditional average. Moreover, in accord with the empirical evidence, short-term assets command a high compensation even if the unconditional term premium is positive, as shown in Figure 12. Thus, our model provides a solution to the theoretical challenge posed by the empirical findings in van Binsbergen et al. (2012) to leading models, such as Campbell and Cochrane (1999), Bansal and Yaron (2004), and Wachter (2013) - that cannot explain sizeable short-term risk premia, independently of the sign of the unconditional slope.

## IV Conclusion

Using a parsimonious general equilibrium framework, we provide a comprehensive understanding of the recent empirical findings regarding the term structure of equity, its dynamics, and its implications for the cross section of returns. The economic mechanism relies on the interaction between two risks affecting economic fundamentals: permanent shocks driven by expected growth volatility - which induce upward-sloping risk with the horizon-and transitory shocks capturing stationary fluctuations-which induce downward-sloping risk. In equilibrium, the interaction and the relative magnitude of these two risks determine the slope dynamics of equity compensations. We provide empirical evidence supportive of the model assumptions, economic mechanism, and predictions.

Our model jointly generates the observed features of the term structure of equity, including the unconditional negative slope of term premia, counter-cyclical variation of term premia, and pro-cyclical variation of equity yields. The model also explains premia to value (respectively, growth) stocks, which are increasing (decreasing) with the horizon. At the same time, our model also perform well in capturing standard asset pricing moments (e.g., risk-free rate and equity premium puzzles) under realistic assumptions about the economic environment (e.g., consumption and dividend co-integration and limited market participation) and standard preferences.

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## A Model Derivation and Proofs

## Affine Notation

The vector $X_{t}=\left(y_{t}, x_{t}, z_{t}, w_{t}\right)^{\top}$ collects the two state variables of our model $x_{t}$ and $z_{t}$ as well as accumulated expected growth $y_{t}=\mu t+\int_{0}^{t}\left(\bar{x}-x_{s}\right) d s$ and the specific component of cross-sectional payouts $w_{t}=\varphi \int_{0}^{t}\left(\bar{x}-x_{s}\right) d s+\sigma_{\varphi} B_{\varphi, t}$. The vector belongs to the affine class and has dynamics:

$$
\begin{aligned}
d X_{t} & =\mu\left(X_{t}\right) d t+\Sigma\left(X_{t}\right) d \mathcal{B}_{t}, \\
\mu\left(X_{t}\right) & =\mathcal{M}+\mathcal{K} X_{t}, \\
\Sigma\left(X_{t}\right) \Sigma\left(X_{t}\right)^{\top} & =h+\sum_{i \in\{y, x, z, w\}} H_{i} X_{i, t},
\end{aligned}
$$

with Brownian motions $\mathcal{B}_{t}=\left(B_{y, t}, B_{x, t}, B_{z, t}, B_{w, t}\right)^{\top}$ and the following coefficients:

$$
\begin{gathered}
\mathcal{M}=\left(\begin{array}{c}
\mu+\bar{x} \\
\lambda_{x} \bar{x} \\
0 \\
\varphi \bar{x}
\end{array}\right), \quad \mathcal{K}=\left(\begin{array}{ccc}
0 & -1 & 0 \\
0 & -\lambda_{x} & 0 \\
0 \\
0 & 0 & -\lambda_{z} \\
0 \\
0 & -\varphi & 0
\end{array}\right), \\
h=\left(\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \sigma_{z}^{2} & 0 \\
0 & 0 & 0 & \sigma_{\varphi}^{2}
\end{array}\right), \quad H_{y}=H_{z}=H_{w}=\left(\begin{array}{llll}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right), \quad H_{x}=\left(\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & \sigma_{x}^{2} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right) .
\end{gathered}
$$

The following selection vectors allow to recover consumption, aggregate payouts, and crosssectional payouts:

$$
\begin{aligned}
v_{C}=(1,0,1,0)^{\top} & \Rightarrow \quad v_{C}^{\top} X_{t}=\log C_{t} \\
v_{D}=(1,0, \phi, 0)^{\top} & \Rightarrow \quad v_{D}^{\top} X_{t}=\log D_{t} \\
v_{\varphi}=(1,0, \phi, 1)^{\top} & \Rightarrow \quad v_{\varphi}^{\top} X_{t}=\log D_{t}^{\varphi}
\end{aligned}
$$

## Moment Generating Function

The following conditional expectation allows to compute the moment generating function for the logarithm of $C, D$, and $D^{\varphi}$ at any future horizon $\tau$ :

$$
\begin{equation*}
\mathbb{E}_{t}\left[\exp \left(\mathbf{u}^{\top} X_{t+\tau}\right)\right]=\exp \left(\bar{b}_{0}(\tau)+\bar{b}(\tau)^{\top} X_{t}\right) \tag{A1}
\end{equation*}
$$

As shown in Duffie, Pan, and Singleton (2000), the functions $\bar{b}_{0}(\tau)$ and $\bar{b}(\tau)=\left(\bar{b}_{y}(\tau), \bar{b}_{x}(\tau)\right.$, $\left.\bar{b}_{z}(\tau), \bar{b}_{w}(\tau)\right)^{\top}$ solve the following system of ODE's:

$$
\begin{aligned}
\bar{b}_{0}^{\prime}(\tau) & =\mathcal{M}^{\top} \bar{b}(\tau)+\frac{1}{2} \bar{b}(\tau)^{\top} h \bar{b}(\tau) \\
\bar{b}^{\prime}(\tau) & =\mathcal{K}^{\top} \bar{b}(\tau)+\frac{1}{2} \bar{b}(\tau)^{\top} H \bar{b}(\tau)
\end{aligned}
$$

By setting the initial conditions $\bar{b}_{0}(0)=0$ and either $\bar{b}(0)=u v_{C}, \bar{b}(0)=u v_{D}$, or $\bar{b}(0)=u v_{\varphi}$, Eq. (A1) computes the time- $t$ conditional expectation of either $C_{t+\tau}, D_{t+\tau}$, or $D_{t+\tau}^{\varphi}$ with power $u$ respectively. These expectations can be used to build the term structure of growth rates volatility.

## Equity and State-Price Density

We follow Eraker and Shaliastovich (2008) and the state-price density based on recursive preferences of Epstein and Zin (1989) type. To do so, we use the Campbell and Shiller (1988) approximation to log-linearize the return $R_{t}$ on the wealth of market participants (that in our economy corresponds to the equity market and pays-out $D_{t}$ ):

$$
\begin{equation*}
d \log R_{t}=k_{0} d t+k_{1} d p d_{t}-\left(1-k_{1}\right) p d_{t} d t+d \log D_{t} \tag{A2}
\end{equation*}
$$

Where $k_{0}$ and $k_{1}$ are endogenous constants to be determined. We conjecture that the log price-payout ratio (which is also the log wealth-consumption ratio of market participants) is an affine function of $X_{t}: p d_{t}=A_{0}+A^{\top} X_{t}$.

We use Eq. (A2) to rewrite the state-price density dynamics as Follows:

$$
\begin{align*}
d \log M_{t} & =\theta \log \beta d t-\frac{\theta}{\psi} d \log D_{t}-(1-\theta) d \log R_{t} \\
& =\left(\theta \log \beta-(\theta-1) \log k_{1}+(\theta-1)\left(k_{1}-1\right) A^{\top}\left(X_{t}-\mu_{X}\right)\right) d t-\lambda^{\top} d X_{t} \tag{A3}
\end{align*}
$$

where $\lambda=\gamma v_{D}+(1-\theta) k_{1} A$ and $\mu_{X}=(0, \bar{x}, 0,0)^{\top}$. Then, the Euler equation can be written as:

$$
1=\mathbb{E}_{t}\left[\frac{M_{t+\tau}}{M_{t}} e^{\int_{0}^{\tau} d \log R_{t+s}}\right], \quad \forall \tau
$$

Since the term in the conditional expectation has to be a martingale, we apply Itô's lemma to compute its drift that we set equal to zero:

$$
\begin{equation*}
0=\theta \log \beta+\chi^{\top}\left(\mathcal{M}+\mathcal{K} X_{t}\right)+\theta k_{0}-\theta\left(1-k_{1}\right)\left(A_{0}+A^{\top} X_{t}\right)+\frac{1}{2} \chi \Sigma\left(X_{t}\right)^{\top} \Sigma\left(X_{t}\right) \chi^{\top} X_{t} \tag{A4}
\end{equation*}
$$

where $\chi=\theta\left(\left(1-\frac{1}{\psi}\right) v_{D}+k_{1} A\right)$. Since Eq. (A4) holds for all $X_{t}$ and we set the coefficients on $X_{t}$ and the residual constant equal to zero. The endogenous coefficients $k_{1}, A_{0}$ and $A=\left(A_{y}, A_{x}, A_{z}, A_{w}\right)^{\top}$ are obtained by solving the following system:

$$
\begin{aligned}
0 & =\mathcal{K}^{\top} \chi-\theta\left(1-k_{1}\right) A+\frac{1}{2} \chi^{\top} H \chi \\
0 & =\theta\left(\log \beta+k_{0}-\left(1-k_{1}\right) A_{0}\right)+\mathcal{M}^{\top} \chi+\frac{1}{2} \chi^{\top} h \chi \\
\theta \log k_{1} & =\theta\left(\log \beta+\left(1-k_{1}\right) A^{\top} \mu_{X}\right)+\mathcal{M}^{\top} \chi+\frac{1}{2} \chi^{\top} h \chi
\end{aligned}
$$

The solution coefficients should be inserted into (A3) to obtain the equilibrium state price
density. ${ }^{20}$ The equity price is then given by $P_{t}=D_{t} \exp \left(A_{0}+A^{\top} X_{t}\right)$, where $A_{y}=A_{w}=0$.
Applying Itô's Lemma to (A3) yields:

$$
\begin{align*}
\frac{d M_{t}}{M_{t}}= & \left(\theta \log \beta-(\theta-1) \log k_{1}+(\theta-1)\left(k_{1}-1\right) A^{\top}\left(X_{t}-\mu_{X}\right)+\mu\left(X_{t}\right)^{\top} \lambda\right) d t \\
& +\frac{1}{2} \lambda^{\top} \Sigma\left(X_{t}\right) \lambda d t-\lambda^{\top} \Sigma\left(X_{t}\right) d \mathcal{B}_{t} \\
= & -\left(\Phi_{0}+\Phi^{\top} X_{t}\right) d t-\lambda^{\top} \Sigma\left(X_{t}\right) d \mathcal{B}_{t} \tag{A5}
\end{align*}
$$

where the coefficients $\Phi_{0}$ and $\Phi=\left(\Phi_{y}, \Phi_{x}, \Phi_{z}, \Phi_{w}\right)^{\top}$ are:

$$
\begin{aligned}
\Phi_{0} & =-\theta \log \beta+(\theta-1)\left(\log k_{1}+\left(k_{1}-1\right) A^{\top} \mu_{X}\right)+\mathcal{M}^{\top} \lambda-\frac{1}{2} \lambda^{\top} h \lambda \\
\Phi & =(1-\theta)\left(k_{1}-1\right) A+\mathcal{K}^{\top} \lambda-\frac{1}{2} \lambda^{\top} H \lambda
\end{aligned}
$$

Therefore, the risk-free rate is given by $r_{t}=\Phi_{0}+\Phi^{\top} X_{t}$ and the vector of risk prices is given by $\Omega\left(X_{t}\right)=\left(\Omega_{y}, \Omega_{x}, \Omega_{z}, \Omega_{w}\right)^{\top}=\Sigma\left(X_{t}\right)^{\top} \lambda$, where it turns out that $\Phi_{y}=\Phi_{w}=0$ and $\Omega_{y}=\Omega_{w}=$ 0 . Consequently, the risk premium on equity is equal to $R P\left(x_{t}\right)=\left(\left(A+v_{D}\right)^{\top} \Sigma\left(X_{t}\right)\right) \Omega\left(X_{t}\right)$, which is an affine function of $x_{t}$ only.

## Term Structures

Following Duffie et al. (2000), the risk-neutral dynamics of $X_{t}$ are given by:

$$
\begin{aligned}
d X_{t} & =\left(\mathcal{M}^{\mathcal{Q}}+\mathcal{K}^{\mathcal{Q}} X_{t}\right) d t+\Sigma\left(X_{t}\right) d \mathcal{B}_{t}^{\mathcal{Q}} \\
\mathcal{M}^{\mathcal{Q}} & =\mathcal{M}-h \lambda \\
\mathcal{K}^{\mathcal{Q}} & =\mathcal{K}-H \lambda \\
d \mathcal{B}_{t}^{\mathcal{Q}} & =d \mathcal{B}_{t}+\Sigma\left(X_{t}\right)^{\top} \lambda d t
\end{aligned}
$$

Then, we can compute the discounted value of several payouts, such as $D_{t+\tau}, D_{t+\tau}^{\varphi}$, and the unitary payout of a risk-less bond:

$$
\mathbb{E}_{t}\left[\frac{M_{t+\tau}}{M_{t}} \exp \left(v^{\top} X_{t+\tau}\right)\right]=\mathbb{E}^{\mathcal{Q}}\left[\exp \left(-\int_{0}^{\tau} r_{t+s} d s+v^{\top} X_{t+\tau}\right)\right]=\exp \left(q_{0}(\tau)+q(\tau)^{\top} X_{t}\right)
$$

where $v \in\left\{v_{D}, v_{\varphi},(0,0,0,0)^{\top}\right\}$. The deterministic function $q_{o}(\tau)$ and $q(\tau)=\left(q_{y}(\tau), q_{x}(\tau)\right.$, $\left.q_{z}(\tau), q_{w}(\tau)\right)$ solve the following system of ODE's:

$$
\begin{gathered}
q_{0}^{\prime}(\tau)=-\Phi_{0}+\left(\mathcal{M}^{\mathcal{Q}}\right)^{\top} q(\tau)+\frac{1}{2} q(\tau)^{\top} h q(\tau) \\
q^{\prime}(\tau)=-\Phi+\left(\mathcal{K}^{\mathcal{Q}}\right)^{\top} q(\tau)+\frac{1}{2} q(\tau)^{\top} H q(\tau)
\end{gathered}
$$

[^19]with initial conditions $q_{0}(0)=0$ and $q(0)=v$.
Therefore, the risk-less bond price, the strip price of the aggregate payout and the strip price of the cross-sectional payout are given by
\[

$$
\begin{array}{ll}
B_{t, \tau}=\exp \left(q_{0}(\tau)+q(\tau)^{\top} X_{t}\right), & \text { with } \quad v=(0,0,0,0)^{\top}, \\
P_{t, \tau}=\exp \left(q_{0}(\tau)+q(\tau)^{\top} X_{t}\right), & \text { with } \quad v=v_{D} \\
P_{t, \tau}^{\varphi}=\exp \left(q_{0}(\tau)+q(\tau)^{\top} X_{t}\right), & \text { with } \quad v=v_{\varphi}
\end{array}
$$
\]

## Cross-Sectional Equity

The price of the stock paying out the stream $D_{t}^{\varphi}$ can be computed either as the time integral of the corresponding strip price over any maturity or via an exponential affine approximation. Such exponential affine approximation is given by

$$
P_{t}^{\varphi}=D_{t}^{\varphi} \exp \left(A_{0}^{\varphi}+\left(A^{\varphi}\right)^{\top} X_{t}\right),
$$

where the coefficients $A_{0}^{\varphi}, A^{\varphi}=\left(A_{y}^{\varphi}, A_{x}^{\varphi}, A_{z}^{\varphi}, A_{w}^{\varphi}\right)^{\top}$, and the endogenous constant $k_{1}^{\varphi}$ solve the following system:

$$
\begin{aligned}
0= & (\theta-1)\left(k_{1}-1\right) A+\left(k_{1}^{\varphi}-1\right) A^{\varphi}+\mathcal{K}^{\top} \chi_{\varphi}+(1 / 2) \chi_{\varphi}^{\top} H \chi_{\varphi}, \\
0= & \theta \log \beta-(\theta-1)\left(\log k_{1}+\left(k_{1}-1\right) A^{\top} \mu_{X}\right)-\left(\log k_{1}^{\varphi}+\left(k_{1}^{\varphi}-1\right)\left(A^{\varphi}\right)^{\top} \mu_{X}\right)+\mathcal{M}^{\top} \chi_{\varphi} \\
& +(1 / 2) \chi_{\varphi}^{\top} h \chi_{\varphi}, \\
0= & A_{0}^{\varphi}+\left(A^{\varphi}\right)^{\top} \mu_{X}-\log k_{1}^{\varphi}+\log \left(1-k_{1}^{\varphi}\right),
\end{aligned}
$$

where $\chi_{\varphi}=v_{\varphi}+k_{1}^{\varphi} A^{\varphi}-\lambda$. It turns out that $A_{y}^{\varphi}=A_{w}^{\varphi}=0$. Therefore, the risk premium on the cross-sectional stock is equal to $R P^{\varphi}\left(x_{t}\right)=\left(\left(A^{\varphi}+v_{\varphi}\right)^{\top} \Sigma\left(X_{t}\right)\right) \Omega\left(X_{t}\right)$, which is an affine function of $x_{t}$ only.

## State-Price Density Decomposition

We follow Alvarez and Jermann (2005) and Hansen and Scheinkman (2009) and decompose the equilibrium state-price density in its permanent (martingale) component and its transitory component. The logarithm of the state-price density equals:

$$
\log M_{t}=-\int_{0}^{t}\left(r_{s}+\frac{1}{2}\left(\Omega_{x}^{2}\left(x_{s}\right)+\Omega_{z}^{2}\right)\right) d s-\int_{0}^{t} \Omega_{x}\left(x_{s}\right) d B_{x, s}-\Omega_{z} B_{z, t}
$$

with $\log M_{0}=0$.
Example 6.2 in Hansen and Scheinkman (2009) nests the above functional form. The
permanent (martingale) component $\widehat{M}_{t}$ of the state-price density is given by

$$
\begin{aligned}
\log \widehat{M}_{t}= & -\frac{1}{2}\left(c_{x} \sigma_{x}-\Omega_{x}\right)^{2} \int_{0}^{t} x_{s} d s-\frac{1}{2}\left(c_{z} \sigma_{z}-\Omega_{z}\right)^{2} t \\
& -\left(c_{x} \sigma_{x}-\Omega_{x}\right) \int_{0}^{t} \sqrt{x_{s}} d B_{x, s}-\left(c_{z} \sigma_{z}-\Omega_{z}\right) B_{z, t}
\end{aligned}
$$

where

$$
\begin{aligned}
c_{x} & =\frac{\lambda_{x}+\sigma_{x} \Omega_{x}-\sqrt{2 r_{x} \sigma_{x}^{2}+\left(\lambda_{x}+\sigma_{x} \Omega_{x}\right)^{2}}}{\sigma_{x}^{2}} \\
c_{z} & =-\frac{r_{z}}{\lambda_{z}}
\end{aligned}
$$

with $\Omega_{x}=\Omega_{x}\left(x_{t}\right) / \sqrt{x_{t}}$. This decomposition allows us to verify that the state-price density satisfies the bound introduced by Alvarez and Jermann (2005), as reported in Section III.A.

## Proposition Proof

Our results are valid under the parametric restriction $\sigma_{x}^{2}<\bar{\sigma}_{x}^{2}$, where

$$
\begin{equation*}
\bar{\sigma}_{x}^{2}=\frac{\psi \lambda_{x}\left(2 \gamma \psi+k_{1}\left(-2 \gamma \psi+(\gamma \psi+\psi-2) \lambda_{x}+2\right)-2\right)}{2 k_{1}(\gamma \psi-1)^{2}} \tag{A6}
\end{equation*}
$$

In the steady-state, the equity premium equals $9.5 \%$ at the upper boundary $\sigma_{x}^{2}=\bar{\sigma}_{x}^{2}$ in our baseline calibration. Since the equity premium is much lower in the data, the above constraint is never binding in our analysis. Other parameter restrictions we impose are $0<k_{1}<1,0<\lambda_{x}<1, \gamma>\psi>1$.

Lemma 1. $\Psi_{0}<0$.
Proof. Note that $\Psi_{0}$ is strictly increasing in $\sigma_{x}^{2}$ :

$$
\begin{equation*}
\frac{\partial \Psi_{0}}{\partial \sigma_{x}^{2}}=\frac{2 k_{1}^{2}(\psi-1)(\gamma \psi-1)\left(k_{1} \lambda_{x}-k_{1}+1\right)}{\Psi \psi^{2}\left(k_{1}\left(\lambda_{x}-1\right)+\Psi+1\right)^{2}}>0 \tag{A7}
\end{equation*}
$$

Furthermore, setting $\sigma_{x}^{2}=\bar{\sigma}_{x}^{2}$ (i.e., to its highest value given our constraint) yields $\Psi_{0}<0$. Since $\Psi_{0}$ is strictly increasing in $\sigma_{x}^{2}$, this implies that for $\sigma_{x}^{2}<\bar{\sigma}_{x}^{2}, \Psi_{0}$ is negative.

Lemma 2. $\Psi_{1}<0$.
Proof. Note that $\Psi_{1}$ is strictly increasing in $\sigma_{x}^{2}$ :

$$
\begin{equation*}
\frac{\partial \Psi_{1}}{\partial \sigma_{x}^{2}}=\frac{k_{1}(\gamma \psi-1)}{\Phi \psi}>0 \tag{A8}
\end{equation*}
$$

Furthermore, setting $\sigma_{x}^{2}=\bar{\sigma}_{x}^{2}$ (i.e., to its highest value given our constraint) yields $\Psi_{1}=0$. Since $\Psi_{1}$ is strictly increasing in $\sigma_{x}^{2}$, this implies that for $\sigma_{x}^{2}<\bar{\sigma}_{x}^{2}, \Psi_{1}$ is negative.

Lemma 3. $A_{x}<0$.
Proof. Since $A_{x}=-\frac{2(1-1 / \psi)}{1-k_{1}\left(1-\lambda_{x}\right)+\Phi}$ and $\psi>1$, we simply need to verify that:

$$
\begin{equation*}
1-k_{1}\left(1-\lambda_{x}\right)+\Phi>0 \tag{A9}
\end{equation*}
$$

Since $\Phi$ is a square root term and non-negative and since $0<k_{1}<1$ and $0<\lambda_{x}<1$, this condition is satisfied.

## Part 1: Valuation ratios decrease with EGV:

We compute the derivative term:

$$
\frac{\partial}{\partial x_{t}} \log \frac{P_{t}}{D_{t}}=\frac{A_{x}}{D_{t}} \exp \left(A_{0}+A_{x} x_{t}+A_{z} z_{t}\right)
$$

The above term is negative since $A_{x}$ is negative as shown in Lemma 3.

## Part 2: The slope of equity yields decreases with EGV:

We compute the cross-derivative term:

$$
\begin{equation*}
\frac{\partial^{2}}{\partial \tau \partial x} e y(t, \tau)=\frac{a_{x}}{\tau^{2}}-\frac{\frac{\partial a_{x}}{\partial \tau}}{\tau}=-\frac{2 \Psi_{0}\left(\omega \operatorname{cosech} \omega^{2}-\operatorname{coth} \omega\right)\left(\sqrt{\Psi_{1}^{2}-4 \Psi_{0} \Psi_{2}}+\Psi_{1}\right)}{\tau^{2}\left(\Psi_{1}-\operatorname{coth} \omega \sqrt{\Psi_{1}^{2}-4 \Psi_{0} \Psi_{2}}\right)^{2}} . \tag{A10}
\end{equation*}
$$

To determine the sign of (A10), note that the denominator contains a squared (real) term, ans is therefore positive. We therefore focus on determining the sign of the numerator. The term $\omega=\frac{\tau}{2} \sqrt{\Psi_{1}^{2}-4 \Psi_{0} \Psi_{2}}$ is strictly positive since $\Psi_{0}<0, \Psi_{1}<0, \Psi_{2}>0$, and $\tau>0$, Consequently, the trigonometric expressions satisfy $\omega \operatorname{cosech} \omega^{2}-\operatorname{coth} \omega<0$ and $\operatorname{coth} \omega>1$. Additionally considering $\Psi_{0}<0$ and the minus sign in front of the expression, it suffices to show that the following term is positive:

$$
\sqrt{\Psi_{1}^{2}-4 \Psi_{0} \Psi_{2}}+\Psi_{1}
$$

Since $\Psi_{0}<0$ and $\Psi_{2}>0$, this condition is satisfied.

## Part 3: The slope of the dividend strip risk premium increases with EGV:

We compute the cross-derivative term:

$$
\begin{equation*}
\frac{\partial^{2}}{\partial \tau \partial x} R P_{D S}\left(x_{t}, \tau\right)=\frac{A_{x} k_{1} \sigma_{x}^{2}\left(\gamma-\frac{1}{\psi}\right)}{1-\frac{1}{\psi}} \frac{\partial}{\partial \tau} a_{x}(\tau) . \tag{A11}
\end{equation*}
$$

Given $0<k_{1}<1, \gamma>\psi>1$ and $A_{x}<0$, a sufficient condition for showing that (A11) is negative is that $\partial a_{x}(\tau) / \partial \tau$ is negative, where

$$
\frac{\partial}{\partial \tau} a_{x}(\tau)=\frac{\Psi_{0}\left(\Psi_{1}^{2}-4 \Psi_{0} \Psi_{2}\right)(\operatorname{cosech} \omega)^{2}}{\left(\Psi_{1}-\operatorname{coth} \omega \sqrt{\Psi_{1}^{2}-4 \Psi_{0} \Psi_{2}}\right)^{2}}
$$

Considering that the expression contains two squared and, hence positive terms, it suffices to show that the following term is negative:

$$
\Psi_{0}\left(\Psi_{1}^{2}-4 \Psi_{0} \Psi_{2}\right)
$$

Since, $\Psi_{0}<0$ and $\Psi_{2}>0$, this condition is satisfied.

## Part 4: The value premium increases with EGV:

It is sufficient to show the following cross-derivative is negative:

$$
\frac{\partial^{2}}{\partial \varphi \partial x} R P^{\varphi}\left(x_{t}\right)=\frac{2 k_{1}(\gamma \psi-1) \sigma_{x}^{2}}{\psi \sqrt{\eta_{1}^{2}-4 \eta_{0} \eta_{2}}\left(k_{1}\left(\lambda_{x}-1\right)+\Phi+1\right)}>0 .
$$

Using our parameter restrictions, it follows immediately that the numerator is positive. Additionally, the term $\psi\left(k_{1}\left(\lambda_{x}-1\right)+\Phi+1\right)$ positive as well. The denominator is therefore positive if and only if the square root term $\sqrt{\eta_{1}^{2}-4 \eta_{0} \eta_{2}}$ is positive. Since we assume our model is well-defined (no imaginary quantities), this condition is satisfied.

## B Additional Tables

Table B1: Definition of Variables

| Variable | Sources | Definition |
| :---: | :---: | :---: |
| Expected Growth Volatility |  |  |
| EGV | SPF | Expected volatility of growth based on mean growth forecasts of real GDP for the next quarter and on equations (1)-(2). |
| EGV(Dispersion) | SPF | The difference between the 75 th and the 25 th percentile of real GDP growth forecasts for the next quarter. Because SPF forecasts are disseminated at the end of the first month of the quarter (January, April, July, October), the conversion of monthly data to quarterly frequency is done accordingly throughout the paper. |
| $\operatorname{EGV}(\operatorname{AR}(\mathrm{MA})(1,1)-(\mathrm{G}) \mathrm{ARCH}(1,1))$ | SPF | Expected volatility of growth based on mean growth forecasts of real GDP for the next quarter and on a $\operatorname{AR}(\mathrm{MA})(1,(1))-(\mathrm{G}) \operatorname{ARCH}((1), 1)$ model for conditional volatility. |
| EGV(Cond. Macro) | SPF, FRED | Macroeconomic component of expected volatility of growth based on mean growth forecasts of real GDP. First, mean GDP growth forecasts are demeaned using equation 1. Second, residuals from such a model are regressed on the three-month moving average of the CFNAI, an indicator for NBER recessions, the inflation rate, and the one- and ten-year real Treasury rate. The final EGV measure is the time series of the fitted values from the last regression. |
| EGV(Cond. Macrofinance) | SPF, FRED, CRSP | Macrofinance component of expected volatility of growth based on mean growth forecasts of real GDP. First, mean GDP growth forecasts are demeaned using equation 1. Second, residuals from such a model are regressed on the three-month moving average of the CFNAI, an indicator for NBER recessions, the inflation rate, the one- and ten-year real Treasury rate, the default spread, and the real return and the logarithm of the price-dividend ratio of the CRSP value-weighted measure. The final EGV measure is the time series of the fitted values from the last regression. |
| EGV(Cond. GDP) | SPF, FRED | Expected volatility of growth based on filtered forecasts of GDP, i.e., residuals from a regression of mean growth forecasts of GDP for the next quarter on the (de-trended) labor share of the nonfinancial corporate sector. The econometric approach is the same as in equations (1)-(2). |
| EGV(IP) | SPF | Expected volatility of growth based on mean growth forecasts of IP for the next quarter. The econometric approach is the same as in equations (1)-(2). |
| EGV(PCE) | SPF | Expected volatility of growth based on mean growth forecasts of PCE for the next quarter. The econometric approach is the same as in equations (1)-(2). |
| Stock Market |  |  |
| $\ln (\mathrm{P} / \mathrm{D})$ | CRSP, Robert Shiller's Webpage | Natural logarithm of the quarterly price-dividend ratio of the CRSP value-weighted index or of the S\&P 500 index. The baseline analysis relies on the CRSP index except when using data on equity yields from van Binsbergen et al. (2012): because those yields are extracted from options on the S\&P 500 index, in that case we use information on the S\&P 500 index. To extract information on dividends on the CRSP index, we exploit differences between total and ex-dividend returns as in Eaton and Paye (2017). As in Eaton and Paye (2017), we then compute dividends in a given month as the moving average over the previous year to mitigate seasonal patterns. Available data both for the CRSP and the S\&P 500 index are monthly and are converted to quarterly frequency by taking the average of the ratio over the previous three months as of the first month of each quarter. |

Table B1: - Continued

Equity Term Premium
van Binsbergen et al. (2012, BBK12) GKK20)

Other macrofinance variables
CFNAI

## FRED

NBER Recession
FRED

Inflation
FRED
Treasury Rates
FRED

Default Spread
FRED

Difference between the S\&P 500 dividend yield (ey(t,long)) and short-maturity equity yields (ey(t, short)), whose maturity ranges between 0.5 and two years. Short-maturity equity yields are extracted from data on dividend prices and current dividends by BBK12 by means of the formula $e y(t$, short $)=-\frac{1}{n} \ln \left(\frac{P_{n, t}}{n D_{t}}\right)$, where $P_{n, t}$ denotes the price of dividends up to maturity $n$ at time $t, D_{t}$ is the current annual dividend at time $t$, and $n=0.5,1,2$ years. Note that, differently from the standard formula to extract equity yields from dividend futures (e.g., Bansal et al., 2020), $P_{n, t}$ is obtained from option prices and needs to be divided by $n$. Indeed, $P_{n, t}$ in BBK12 is the price of a claim on all dividends paid up to maturity $n$ (i.e., not on the single dividend paid at date $n$ ). Available data are monthly and are converted to quarterly frequency by taking the average of the yields over the previous three months as of the first month of each quarter.
Difference between a long-maturity yield (ey(t,long)) and the two-year equity yield (ey(t,short)), where yields are model-implied measures. The maturiy of the long-maturity yield ranges between ten and 100 years. For consistency with equity yield slopes based on BBK12, a measure relying on the dividend yield of the CRSP value-weighted index as ey $(t$, long $)$ is also computed. Available data are monthly and are converted to quarterly frequency by taking the average of the yields over the previous three months as of the first month of each quarter.
Difference between the CRSP value-weighted index logarithmic real return (long-maturity claim) and the logarithmic real return on the two-year dividend strip based on the corresponding model-implied equity yield by GKK20. CRSP index returns are monthly and are converted to quarterly frequency by summing them over the previous three months as of the first month of each quarter. The quarterly return on two-year dividend strip at time $t$ is computed as $-1.75 e y_{1.75, t}+2 e y_{2, t-0.25}+\ln \left(\frac{D_{t}}{D_{t-0.25}}\right)$, where the 1.75 -year equity yield is obtained by interpolating the oneand the two-year yields. Finally, the one to ten-year ahead cumulative equity term premia are computed.
Real returns on the value (growth) returns correspond to the top (bottom) decile of stocks sorted on the book-to-market ratio and their difference (value-growth), where portfolio construction follows Fama and French (1992). Portfolio returns are monthly and are converted to quarterly frequency by summing them over the previous three months as of the first month of each quarter. Finally, the one to ten-year ahead cumulative returns for each of the three portfolio strategies are computed.

The three-month moving average of the CFNAI as of the end of the first month of each quarter.
Indicator equal to one if at least one month in a given quarter is classified as a recession by the NBER. Each observation corresponds to the first month of the quarter.
Quarterly logarithmic inflation rate computed from the seasonally adjusted Consumer Price Index for All Urban Consumers (CPI). The conversion of other variables in real terms is based on this CPI measure.
Real one- and ten-year constant maturity rates. Available data are monthly and are converted to quarterly frequency by taking the average of the spread over the previous three months as of the first month of each quarter. Rates are expressed in quarterly terms.
The difference between the yield to maturity of Aaa- and Baa-rated corporate bonds. Available data are monthly and are converted to quarterly frequency by taking the average of the spread over the previous three months as of the first month of each quarter.

Table B2: Goodness of Fit of Alternative EGV Specifications

| Specification | GDP |  | Conditional GDP |  |
| :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) |
|  | RMSE | Correlation with baseline | RMSE | Correlation with baseline |
| EGV - Baseline | 0.0021 | 1.0000 | 0.0020 | 0.9932 |
| EGV (AR(1)-ARCH(1)) | 0.0029 | 0.6451 | 0.0027 | 0.6454 |
| $\operatorname{EGV}(\operatorname{AR}(1)-\operatorname{GARCH}(1,1))$ | 0.0030 | 0.8904 | 0.0028 | 0.8812 |
| EGV (ARMA(1,1)-ARCH(1)) | 0.0030 | 0.6568 | 0.0028 | 0.6572 |
| EGV (ARMA(1,1)-GARCH $(1,1)$ ) | 0.0030 | 0.8841 | 0.0029 | 0.8746 |

Note. This table reports the root-mean-square error (RMSE)

$$
\operatorname{RMSE}=\left((1 / T) \sum_{t=0}^{T-1}(|\epsilon(t+1)|-\sigma(t, t+1))^{2}\right)^{1 / 2}
$$

from several specifications at quarterly frequency of conditional and volatility models as well as the correlation of the EGV measure they generate with the baseline EGV measure (i.e., based on equations (1)-(2)) for the period 1968-2019. In columns 1 and 2, the EGV measures are estimated from GDP growth forecasts. In columns 3 and 4 , the EGV measures are estimated after conditioning growth forecasts on the (de-trended) labor share of the corporate sector. Detailed variable definitions are provided in Appendix Table B1.
Table B3: The Cyclicality of the Equity Yield Slope and of the Equity Term Premium

|  | Equity Yield Slope (MKT-2Y, GKK20) |  |  |  |  | Equity Term Premium (10Y) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| Constant | $\begin{gathered} 0.022^{* * *} \\ (8.12) \end{gathered}$ | $\begin{gathered} 0.011^{* * *} \\ (4.53) \end{gathered}$ | $\begin{gathered} 0.013^{* * *} \\ (6.10) \end{gathered}$ | $\begin{gathered} 0.034^{* * *} \\ (8.29) \end{gathered}$ | $\begin{gathered} -0.096^{* * *} \\ (-11.26) \end{gathered}$ | $\begin{gathered} 0.293^{*} \\ (1.88) \end{gathered}$ | $\begin{gathered} 0.506^{* * *} \\ (5.62) \end{gathered}$ | $\begin{gathered} 0.475^{* * *} \\ (5.46) \end{gathered}$ | $\begin{aligned} & 0.201 \\ & (1.18) \end{aligned}$ | $\underset{(7.94)}{3.195^{* * *}}$ |
| EGV (GDP) | $\begin{gathered} -5.276^{* * *} \\ (-5.40) \end{gathered}$ |  |  |  |  | $\begin{gathered} 87.145^{*} \\ (1.89) \end{gathered}$ |  |  |  |  |
| CFNAI |  | $\begin{gathered} 0.005^{*} \\ (1.69) \end{gathered}$ |  |  |  |  | $\begin{gathered} -0.042 \\ (-0.51) \end{gathered}$ |  |  |  |
| NBER Recession |  |  | $\begin{gathered} -0.018^{* *} \\ (-2.43) \end{gathered}$ |  |  |  |  | $\begin{gathered} 0.215 \\ (1.15) \end{gathered}$ |  |  |
| Default Spread |  |  |  | $\begin{gathered} -0.022^{* * *} \\ (-6.95) \end{gathered}$ |  |  |  |  | $\begin{gathered} 0.280^{* * *} \\ (2.72) \end{gathered}$ |  |
| $\ln (\mathrm{P} / \mathrm{D})$ |  |  |  |  | $\begin{gathered} 0.029^{* * *} \\ (12.71) \end{gathered}$ |  |  |  |  | $\begin{gathered} -0.744^{* * *} \\ (-7.24) \end{gathered}$ |
| Observations | 165 | 165 | 165 | 165 | 165 | 124 | 124 | 124 | 124 | 124 |
| Adj. $R^{2}$ | 0.28 | 0.08 | 0.15 | 0.41 | 0.62 | 0.06 | -0.00 | 0.02 | 0.06 | 0.59 |

Note. This table reports estimates from regressions at quarterly frequency of the equity yield slope (columns 1 to 5) and the equity term premium (columns 6 to 10) on several measures proxying for the state of the business cycle. Such measures comprise the EGV of GDP, three-month moving average of the CFNAI, an indicator for NBER recessions, the default spread, and the logarithm of the pricedividend ratio of the CRSP value-weighted index. Both the equity yield slope and the equity term premium are obtained from data by Giglio et al. (2020, GKK20) on model-implied equity yields for the period 1975:2016. The equity yield slope is the difference between the CRSP index dividend yield and the two-year equity yield. The equity term premium is the ten-year ahead cumulative long-minus-short equity return over the next 10 years, where the long leg is the CRSP index return and the short one

 ${ }^{*},{ }^{* *},{ }^{* * *}$, respectively. Detailed variable definitions are provided in Appendix Table B1.
Table B4: Alternative EGV Measures and Asset Prices

|  | Price Dynamics |  | Equity Yield Slope (MKT-2Y) |  | Equity Term Premium |  | Cross-Section |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} (1) \\ \ln (\mathrm{P} / \mathrm{D}) \end{gathered}$ | $\begin{gathered} (2) \\ \sigma_{\ln (\mathrm{P} / \mathrm{D})}^{(2)} \end{gathered}$ | (3) <br> BBK12 | (4) GKK20 | $\begin{aligned} & \text { (5) } \\ & 5 \mathrm{Y} \end{aligned}$ | $\begin{aligned} & (6) \\ & 10 \mathrm{Y} \end{aligned}$ | (7) <br> Value 5 Y | (8) <br> Growth 5Y | (9) <br> Value-Growth 5Y |
| EGV (Dispersion) | $\begin{gathered} -116.190^{* * *} \\ (-9.727) \end{gathered}$ | $\begin{aligned} & \hline 3.906^{* *} \\ & (2.026) \end{aligned}$ | $\begin{gathered} \hline-35.546^{* * *} \\ (-4.674) \end{gathered}$ | $\begin{gathered} -4.337^{* * *} \\ (-6.977) \end{gathered}$ | $\begin{aligned} & 19.295 \\ & (0.935) \end{aligned}$ | $\begin{gathered} 58.373^{* *} \\ (2.355) \end{gathered}$ | $\begin{gathered} 32.314 \\ (1.135) \end{gathered}$ | $\begin{gathered} -57.322^{* * *} \\ (-2.656) \end{gathered}$ | $\begin{gathered} 89.636^{* * *} \\ (4.355) \end{gathered}$ |
| $\operatorname{EGV}(\operatorname{AR}(1)-\mathrm{ARCH}(1))$ | $\begin{gathered} -102.419^{* * *} \\ (-4.056) \end{gathered}$ | $\begin{gathered} 8.867^{* * *} \\ (4.026) \end{gathered}$ | $\begin{gathered} -29.603^{* * *} \\ -3.880) \end{gathered}$ | $\begin{gathered} -5.460^{* * *} \\ (-3.098) \end{gathered}$ | $\begin{aligned} & 39.401 \\ & (1.589) \end{aligned}$ | $\begin{aligned} & 60.052 \\ & (1.308) \end{aligned}$ | $\begin{aligned} & 59.989^{*} \\ & (1.912) \end{aligned}$ | $\begin{aligned} & -36.955 \\ & (-1.282) \end{aligned}$ | $\begin{gathered} 96.943^{* * *} \\ (3.362) \end{gathered}$ |
| EGV (Cond. Macro) | $\begin{gathered} -198.239^{* * *} \\ (-10.542) \end{gathered}$ | $\begin{aligned} & 5.663^{*} \\ & (1.874) \end{aligned}$ | $\begin{gathered} -42.005^{* * *} \\ (-2.822) \end{gathered}$ | $\begin{gathered} -7.821^{* * *} \\ (-5.221) \end{gathered}$ | $\begin{gathered} 86.030^{* * *} \\ (3.159) \end{gathered}$ | $\begin{gathered} 165.316^{* * *} \\ (3.188) \end{gathered}$ | $\begin{gathered} 150.461^{* * *} \\ (4.187) \end{gathered}$ | $\begin{aligned} & 18.619 \\ & (0.439) \end{aligned}$ | $\begin{gathered} 131.842^{* * *} \\ (3.303) \end{gathered}$ |
| EGV (Cond. Macrofinance) | $\begin{gathered} -227.682^{* * *} \\ (-9.799) \end{gathered}$ | $\begin{aligned} & 6.629^{* *} \\ & (2.240) \end{aligned}$ | $\begin{gathered} -19.786^{* * *} \\ (-2.725) \end{gathered}$ | $\begin{gathered} -8.942^{* * *} \\ (-6.746) \end{gathered}$ | $\begin{gathered} 70.158^{* *} \\ (2.516) \end{gathered}$ | $\begin{gathered} 168.970^{* * *} \\ (3.618) \end{gathered}$ | $\begin{gathered} 129.645^{* * *} \\ (3.845) \end{gathered}$ | $\begin{array}{r} 27.851 \\ (0.656) \end{array}$ | $\begin{gathered} 101.793^{* *} \\ (2.209) \end{gathered}$ |
| EGV (Cond. GDP) | $\begin{gathered} -119.915^{* * *} \\ (-5.351) \end{gathered}$ | $\begin{gathered} 10.108^{* * *} \\ (5.109) \end{gathered}$ | $\begin{gathered} -19.507^{* * *} \\ (-4.413) \end{gathered}$ | $\underset{(-5.071)}{-5.299^{* * *}}$ | $\begin{aligned} & 39.093 \\ & (1.546) \end{aligned}$ | $\begin{aligned} & 78.936^{*} \\ & (1.687) \end{aligned}$ | $\begin{gathered} 70.558^{* *} \\ (2.159) \end{gathered}$ | $\begin{aligned} & -42.190 \\ & (-1.319) \end{aligned}$ | $\begin{gathered} 112.748^{* * *} \\ (3.368) \end{gathered}$ |
| EGV (IP) | $\begin{gathered} -48.900^{* * *} \\ (-6.735) \end{gathered}$ | $\begin{gathered} 3.513^{* * *} \\ (3.349) \end{gathered}$ | $\begin{gathered} -9.677^{* * *} \\ (-3.313) \end{gathered}$ | $\begin{gathered} -1.703^{* * *} \\ (-6.293) \end{gathered}$ | $\begin{aligned} & 13.792 \\ & (1.495) \end{aligned}$ | $\begin{aligned} & 24.612 \\ & (1.537) \end{aligned}$ | $\begin{gathered} 35.092^{* * *} \\ (3.040) \end{gathered}$ | $\begin{aligned} & -10.752 \\ & (-0.954) \end{aligned}$ | $\begin{gathered} 45.845^{* * *} \\ (4.412) \end{gathered}$ |
| EGV (PCE) | $\begin{gathered} -140.709^{* * *} \\ (-4.213) \end{gathered}$ | $\begin{gathered} 10.977^{* * *} \\ (4.066) \end{gathered}$ | $\begin{gathered} -20.684^{* *} \\ (-2.214) \end{gathered}$ | $\underset{(-4.360)}{\substack{-6.096^{* * *} \\( }}$ | $\begin{gathered} 24.909 \\ (0.570) \end{gathered}$ | $\begin{gathered} 144.396^{* *} \\ (2.290) \end{gathered}$ | $\begin{gathered} 70.726 \\ (1.455) \end{gathered}$ | $\begin{gathered} 8.768 \\ (0.202) \end{gathered}$ | $\begin{aligned} & 61.958 \\ & (1.344) \end{aligned}$ |

Note. This table reports slope estimates from regressions at quarterly frequency of several of asset pricing quantities on alternatives measures of the EGV of GDP. The dependent variables are the logarithm of the price-dividend rat of the CRSP value-weighted index, its conditional volatility computed following equation (2), the equity yield slope computed as the difference between the CRSP index dividend yield and the two-year equity yield (the latter being either from van Binsbergen et al. (2012, BBK12) or Giglio et al. (2020, GKK20)), the five- and ten-year ahead cumulative equity term premium (where the long leg is the CRSP index return and the short one is the return on two-year dividend strip from Giglio et al. (2020, GKK20)), the five-year ahead cumulative return on value (firms belonging to the top decide of the book-to-market ratio), growth (firms belonging to the bottom decide of the book-to-market ratio) and value-minus-growth portfolios. The EGV measures include the cross-sectional dispersion of real GDP growth forecasts, the $\mathrm{AR}(1)-\mathrm{ARCH}(1)$ specification of EGV, two regression-based measures capturing respectively the macroeconomic and the macrofinance components of EGV, and finally EGV of alternative macroeconomic aggregates. The latter comprise GDP growth forecasts filtered from the (de-trended) labor share of the corporate sector, IP growth forecasts, and PCE growth forecasts. The $t$-statistics are reported in parentheses and are based on Newey-West standard errors with four lags. Significance at the $10 \%, 5 \%$, and $1 \%$ levels is indicated by ${ }^{*},{ }^{* *},{ }^{* * *}$, respectively. Detailed variable definitions are provided in Appendix Table B1.

Table B5: Alternative Calibration: Standard Moments

|  |  |  | Model |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Moment | Data |  |  |  |  |  |  |  |  |
|  |  | $2.5 \%$ | $5 \%$ | $50 \%$ | $95 \%$ | $97.5 \%$ | Pop. |  |  |
| Avg consumption growth | 0.021 | -0.001 | 0.004 | 0.026 | 0.043 | 0.046 | 0.025 |  |  |
| Std consumption growth | 0.018 | 0.025 | 0.026 | 0.031 | 0.040 | 0.042 | 0.034 |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Avg payout growth | 0.030 | -0.002 | 0.003 | 0.026 | 0.044 | 0.047 | 0.025 |  |  |
| Std payout growth | 0.148 | 0.151 | 0.156 | 0.181 | 0.207 | 0.212 | 0.181 |  |  |
| 20-year Std payout growth | 0.083 | 0.019 | 0.028 | 0.101 | 0.216 | 0.240 | 0.125 |  |  |
| Std log payout-consumption ratio | 0.228 | 0.163 | 0.172 | 0.230 | 0.309 | 0.327 | 0.259 |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Avg risk-free rate | 0.007 | -0.034 | -0.028 | 0.001 | 0.028 | 0.033 | 0.000 |  |  |
| Std risk-free rate | 0.025 | 0.026 | 0.027 | 0.038 | 0.053 | 0.057 | 0.044 |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Avg excess equity return | 0.068 | 0.042 | 0.047 | 0.074 | 0.100 | 0.106 | 0.073 |  |  |
| Std excess equity return | 0.175 | 0.119 | 0.123 | 0.142 | 0.162 | 0.166 | 0.143 |  |  |
| Avg Sharpe ratio | 0.388 | 0.292 | 0.326 | 0.519 | 0.725 | 0.772 | 0.512 |  |  |
| Avg log price-dividend ratio | 3.435 | 2.884 | 2.902 | 2.994 | 3.073 | 3.087 | 2.993 |  |  |
| Std log price-dividend ratio | 0.443 | 0.071 | 0.075 | 0.104 | 0.150 | 0.161 | 0.124 |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Avg excess high-minus-low return | 0.035 | 0.033 | 0.037 | 0.059 | 0.080 | 0.085 | 0.059 |  |  |
| Std excess high-minus-low return | 0.129 | 0.114 | 0.118 | 0.137 | 0.158 | 0.163 | 0.137 |  |  |

Note. This table reports moment statistics from both data and model simulations. Model-implied statistics are either moment quantiles from short-sample ( 72 years) simulations or population moments. The model is simulated at monthly frequency. Statistics are yearly moments if not stated otherwise. Consumption and payout data are from NIPA tables. Returns are from K. French webpage. The price-dividend ratio is from R. Shiller webpage.


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[^1]:    *Research support from Long-Term Investors at the University of Torino is gratefully acknowledged. The views expressed are those of the authors and should not be interpreted as reflecting the Federal Reserve System or its staff. Matthijs Breugem: matthijs.breugem@carloalberto.org; Stefano Colonnello: stefano.colonnello@unive.it; Roberto Marfè: roberto.marfe@carloalberto.org; Francesca Zucchi: francesca.zucchi@frb.gov

[^2]:    ${ }^{1}$ For instance, Gormsen and Koijen (2020) investigate the impact of Covid-19 pandemic on economic growth expectations. Breugem, Marfè, and Zucchi (2020) study the effect of heterogeneity in the pricing and firm's exposure to risks of various persistence on corporate policies, showing how it affects the horizon of corporate policies.

[^3]:    ${ }^{2}$ Differently from the empirical findings of van Binsbergen et al. (2012) and Gormsen (2020), the unconditional negative slope estimated by Giglio et al. (2020) is less susceptible to this criticism, as well as the criticism concerning liquidity and bid-ask spreads, because their methodology exploits a broad cross-section of stock returns over more than forty years of data.
    ${ }^{3}$ All the results of the empirical analysis are robust to many alternative specifications of EGV.

[^4]:    ${ }^{4}$ Berk and Walden (2013) show that limited market participation arises endogenously because labor markets provide risk-sharing to workers. Consistently, a major fraction of workers does not invest in the financial markets and the consumption of market participants is more correlated with corporate payouts and equity returns than aggregate consumption (Mankiw and Zeldes, 1991; Guvenen, Schulhofer-Wohl, Song, and Yogo, 2017). However, market participants' consumption is subject to aggregate consumption long-run risk (Malloy, Moskowitz, and Vissing-Jørgensen, 2009), in accord with cointegration.

[^5]:    ${ }^{5}$ Marfè (2015) and Ai et al. (2018) also study the term structure of equity and the value premium in general equilibrium, explaining the link with cash-flow duration (Dechow, Sloan, and Soliman, 2004).

[^6]:    ${ }^{6}$ Technically, the model assumes that expected growth is driven by a heteroskedastic long-run factor and a homoskedastic short-run factor. To verify the validity of our measure of EGV in (2) in light of these assumptions, we thus conduct the following exercise. First, we condition expected growth on businesscycle by regressing GDP growth forecasts on the (de-trended) labor share of the corporate sector, finding a Newey-West $t$-statistic of -3.52 . Second, we apply the model of equations (1)-(2) to the residuals. The resulting conditional volatility $\sigma(t, t+1)$ is strongly correlated with the EGV computed without conditioning on business-cycle. Appendix Table B2 shows that the correlation is $99.32 \%$. This suggests that expected growth heteroskedasticity mostly concerns its long-run component. The same table also shows that our preferred EGV specification generally provides a better fit of residuals than ARCH or GARCH models.

[^7]:    ${ }^{7}$ In this case, we use the CRSP value-weighted index rather than the S\&P 500 index, because modelimplied equity yields by Giglio et al. (2020) are based on a wide cross-section of stocks corresponding to the former index. Results remain unscathed when using the S\&P 500 dividend yield.

[^8]:    ${ }^{8}$ To limit measurement error, we use spot equity yields $(e y(t, \tau))$ rather than forward ones $(f e y(t, \tau))$ to compute the equity yield slope. Because $f e y(t, \tau)=e y(t, \tau)-b y(t, \tau)$, using forward yields would require subtracting risk-free yields $(b y(t, \tau))$ of the appropriate maturity $\tau$ from each leg of the slope. This would be problematic in our case, because we also use the 100-year equity yield and the market dividend yield as the long maturity leg, and it is not obvious to find information on risk-free rates for maturities above 30 years. Nonetheless, using spot equity yields from Giglio et al. (2020), in untabulated tests we compute the equity yield slope with forward yields for the 5 -, 10 -, and 20 -year horizons (i.e., the maturities for which an appropriate risk-free rate is easily available). In each case, the correlation with the slope based on spot equity yields is above $98 \%$. Reassuringly, we also find that the coefficient estimates from regressions of these forward slopes on EGV are virtually the same as in Table 2.

[^9]:    ${ }^{9}$ An alternative approach to measure the equity term premium is to look at the so-called cash-flow duration premium (Weber, 2018). However, such an approach builds on the assumption of a constant discount rate across firms and maturities and is thus unsuitable to study the dynamics of the term structure of equity.

[^10]:    ${ }^{10}$ Recent asset pricing models assuming limited market participation are Marfè (2017) and Greenwald et al. (2014). Although unnecessary for the qualitative predictions of our model, the assumption of limited market participation helps generate sizeable risk premia for short-term assets. Moreover, it allows for tractability and for comparability with endowment economy asset pricing models.

[^11]:    ${ }^{11}$ A complementary channel is sticky financial leverage (Belo et al., 2015).
    ${ }^{12}$ The function $\omega\left(z_{t}\right)$ belongs to $(0,1)$ with probability very close to one because $\delta>0$ is small in the data (e.g., about 10\%).

[^12]:    ${ }^{13}$ Campbell, Lo, and MacKinlay (1997), Bansal, Kiku, and Yaron (2012), and Hasler and Marfè (2016) show the high accuracy of the return log-linearization, which we assume exact hereafter.

[^13]:    ${ }^{14}$ For the sake of simplicity and exposition, we do not assume other forms of heterogeneity.

[^14]:    ${ }^{15}$ Payout moments in Table 4 correspond to corporate profits data. Other measures of shareholders remuneration, such as dividends plus net repurchases, feature a 1-year growth rates volatility of about $26.6 \%$ (Belo et al., 2015). These measures can be viewed as bounds, with respect to which the model performs well.

[^15]:    ${ }^{16}$ We comment in Section III.C about the setting of cross-sectional heterogeneity.

[^16]:    ${ }^{17}$ In contrast, many models in the literature disregard co-integration and markedly overestimate longhorizon payout risk. Such a bias becomes even more relevant in combination with the preferences for the early resolution of uncertainty, that amplify the impact of long-horizon payout risk on asset prices.

[^17]:    ${ }^{18}$ Consistently, in Section I we document that forward equity yields and equity yields are very similar and their relation with EGV is indistinguishable.

[^18]:    ${ }^{19}$ Recently, Hasler, Khapko, and Marfè (2020) show that rational learning helps understand the unconditional term structures of value and growth risk premia but do not investigate dynamics.

[^19]:    ${ }^{20}$ Note that the above system of equations could yields multiple solutions. Tauchen (2011) proposes to select the root which ensures the non-explosiveness of the system. Alternatively, one could select an economically reasonable solution.

