Ca' Foscari University of Venice

Department of Economics

Working Paper

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ISSN: 1827-3580
No. 06/WP/2021

Working Papers
Department of Economics
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This paper presents the structure, data sources, assumptions and simulation methods of the Modelo de Equilibrio General para Andalusia_ (MEGA), a regional CGE model that has been designed for the analysis of the Andalusian economic structure, but which could also be applied to other regional economies. The document is intended to be a reference for simulation and assessment exercises based on this model.

Keywords
Computable General Equilibrium Models, Regional Economics, Numerical Simulations, Computational Economics

JEL Codes
C51, C68, D58, R13, R15

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The MEGA Regional General Equilibrium Model

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This paper presents the structure, data sources, assumptions and simulation methods of the "Modelo de Equilibrio General para Andalusia" (MEGA), a regional CGE model that has been designed for the analysis of the Andalusian economic structure, but which could also be applied to other regional economies. The document is intended to be a reference for simulation and assessment exercises based on this model.

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1 Introduction

Computable (or applied) general equilibrium models (CGE) are widely applied numerical simulation tools, mostly employed for the analysis of policy or external shocks on the structure of an economic system (for an introduction see, e.g., Burfisher [2016]). These multi-sectoral macroeconomic models provide a level of detail comparable to that of input-output models and, similarly to the latter, are calibrated on the basis of a Social Accounting Matrix (SAM), which depicts income circulation flows between different agents in the system.

CGE models are usually developed at the country level, either as a single- or as a multi-country formulation. Sub-national regional CGE models are much more rare, essentially for two reasons: (1) the underlying regional SAM is expensive to estimate and difficult to achieve when there are no data sources comparable to official national accounts; (2) behavioral assumptions and closure rules take some special forms at the regional model (for instance, because of the absence of an external trade balance condition, and of an exchange rate).

However, Roson and van der Vorst [2020] estimated a SAM for the Spanish region of Andalusia, based on the input-output tables provided by the Institute of Statistics and Cartography of Andalusia (IECA) for the year 2016. The MEGA CGE model is built over this source of data, which allows calibrating its structural parameters. MEGA has a relatively standard structure, mostly based on nested CES/CET functions, as well as on accounting identities. Its formulation, which is described in detail in section 3, depends on how data in the SAM is organized (section 2). Section 4 in this paper illustrates how non-calibrated parameters (e.g., elasticities) have been set, and the functioning of the software that can be employed to conduct numerical simulations. Some conclusions follow.

2 The Social Accounting Matrix

A Social Accounting Matrix (SAM) is an accounting framework, formulated as a matrix, depicting the circulation of income flows among various agents inside an economic system (Pyatt and Round [1985]). It usually takes the form of a square matrix, where each entity is associated with a row (identifying sources of income) and a column (showing expenditure outlets), such that the two vectors correspond to a double-entry bookkeeping balance. Since the monetary circulation of income is mirrored by a counter-cyclical circulation of real services, it is also possible to interpret entries in a row as sales and entries in a column as purchases.

In our Andalusian SAM, "Commodities" refer to physical products of good and services, distinguished by place of origin: Andalusia (A), Rest of Spain (E), Rest of World (W). There are 87 categories of commodities in the

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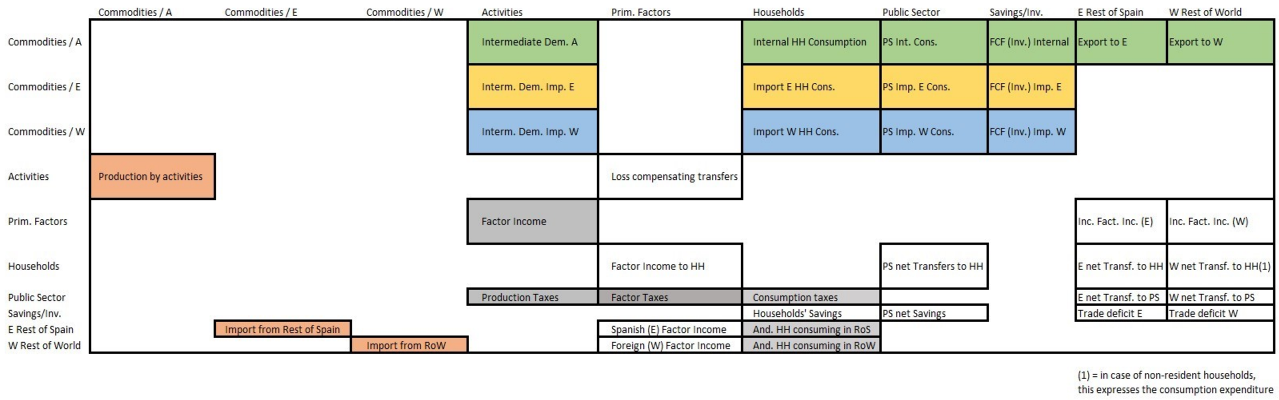
Figure 1: Structure of the 2016 Andalusian SAM



SAM. "Activities" are production activities taking place in Andalusia. An activity could be associated with multiple outputs, whereas one product may be realized by several activities. There are 84 activities in the SAM. "Primary Factors" refer non-produced production factors: employees, service and shop workers, agriculture and unskilled workers, technicians, land, managers, physical capital and infrastructure, natural resources. There are two categories of "Households": residents and non-residents (mainly tourists). "Public sector" accounts for all public administrations, state or local. There is a special account for "Savings and Investments", to consider the purchases of goods and services for investment purposes, and how they are funded. Finally, "Rest of Spain" and "Rest of World" are specific accounts associated with Andalusian external trade, transfers, and payments to non-Andalusian factors.

Figure 1 displays the structure of the SAM, which is a sparse matrix, where some non-empty blocks or sub-matrices can be identified. These are presented in the following, from left to right and from top to bottom.

In the "Production by activities" block, we identify which activities are producing which goods in Andalusia. The row vectors of imports from Rest of Spain and from Rest of World provide total values of imported goods and services in Andalusia. "Intermediate demand" blocks, distinguished by market source (A, E, W) and commodity, inform about the supply of intermediate production factors employed by Andalusian activities. Households, both residents and non-residents, as owners of primary factors receive income flows from the various activities, as shown in the "Factor Income" block. However, income is first assigned to primary factors, and then transferred to households, depending on their ownership shares.

The various activities also pay some taxes directly, mainly related to output levels, and those are accounted for in the "Production Taxes" block. The "loss compensating transfers" sub-matrix is introduced to correct for the possible existence of negative components of value added in some production activities.

The matrix below ("Factor Income to HH") indicates the flow of money received by Andalusian households, because of their ownership of primary factors. There are also two other similar blocks ("Spanish/Foreign Factor Income"), referring to households in the Rest of Spain and in the Rest of World. In the "Factor Taxes" block we can find the taxes paid on factor income to the public sector: taxes on the different types of labor, taxes on capital (profits), on land and on natural resources rents.

The "Households" columns illustrate the expenditure structure of residents and non-residents. Households spend money to buy goods and services produced in Andalusia ("Internal HH Consumption"), in the Rest of Spain ("Import E HH Cons.") and in the Rest of World ("Import W HH Cons."). Consumption of Andalusian products by non-Andalusian households, realized outside the region, is considered as export. Households pay consumption taxes ("Consumption taxes"), save ("Households' Savings"), and spend money when traveling outside the region ("And. HH consuming in RoS/RoW"). The latter does not apply to non-resident households.

Similarly, the public sector generates a demand for commodities needed to provide public goods and services ("PS Int./Imp. E/W Cons."). It also makes transfers to households ("PS net Transfers to HH"), and savings ("PS net Savings") if in the base year 2016 it was running a primary public surplus. Conversely, in case of primary deficit (not accounting for interest payments on accumulated debt) it would absorb savings, so that the entry would be negative, which does not pose any conceptual problem in terms of modelling.

Rest of Spain and Rest of World appear in the rightmost columns of the SAM as entities generating exports ("Export to E/W"), no matter whether their demand is final or intermediate, paying for Andalusian resources ("Inc. Fact. Inc. (E/W)"), assigning transfers to regional households ("E/W net Transf. to HH") or the public

Table 1: Sets in the MEGA model

Name	Description
a	activities
f	factors
g	goods
h	households
k	investments
l	locations
p	public sector
e (⊂l)	exterior

sector ("E/W net Transf. to PS"), for example when the central Spanish government give funds to the Andalusian government. The difference between income received by the external entities RoS and RoW, and their expenses directed to Andalusia ("Trade deficit E/W") can be interpreted as external or foreign savings. Alternatively, because of the accounting identity between net internal savings and the trade balance, the gap could be interpreted as trade deficit (if positive) or trade surplus (if negative).

3 Mathematical Structure of the Model

Table 1 shows the set indices employed in the MEGA model. The model considers 84 production activities a (industries), 87 produced or imported goods and services g, two representative households h, one capital goods productive sector k, three regions l (Andalusia A, Rest of Spain S, Rest of World W), one public sector p.

Model parameters, presented in Table 2, include calibrated CES/CET shares, elasticity values, productivity shifters and tax rates. Calibration ensures that, without modifications in the exogenous variables and parameters, the model reproduces the original SAM.

Equations in the model can be grouped in a series of functional blocks, as described in the following.

3.1 Production

Each industry in the model is associated with a nested CES homothetic production function. Output is produced by combining bundles of value added and intermediate factors, and inside each bundle the various components are also aggregated, by means of a second layer CES. For instance, equation (1) defines the value added aggregate qva, which depends on the overall output volume qa, price indexes pa and pva, and productivity parameters ao and ava.

$$\mathrm{qva}_a = \mathrm{shva}_a \cdot \mathrm{qa}_a \cdot \frac{\mathrm{pa}_a}{\mathrm{pva}_a}^{\mathrm{sigmap}_a} \cdot (\mathrm{ao}_a \cdot \mathrm{ava}_a)^{(\mathrm{sigmap}_a - 1)} \qquad \forall a \tag{1}$$

As it is customary in CGE models, all prices and productivity parameters are set to one at calibration stage, Therefore, the value added bundle is initially expressed as a share of the output level. This value may change during the simulation phase, after an exogenous shock, which may affect relative prices and/or productivities.

$$\mathrm{qnd}_a = (1 - \mathrm{shva}_a) \cdot \mathrm{qa}_a \cdot \frac{\mathrm{pa}_a}{\mathrm{pnd}_a}^{\mathrm{sigmap}_a} \cdot (\mathrm{ao}_a \cdot \mathrm{ain}_a)^{(\mathrm{sigmap}_a - 1)} \qquad \forall a \tag{2}$$

Equations (2), (3) and (5) work exactly the same way in the definition of, respectively: intermediate bundle, primary and intermediate factors. Demand for intermediate factors is not distinguished by origin source at this stage.

$$\mathrm{qfa}_{a,f} = \mathrm{shf}_{a,f} \cdot \mathrm{qva}_a \cdot \frac{\mathrm{pva}_a}{\mathrm{pfa}_{a,f}}^{\mathrm{sigmav}_a} \cdot \mathrm{af}_{a,f}^{(\mathrm{sigmav}_a - 1)} \qquad \forall a, f \tag{3}$$

Equations (4) and (6) express the price index for the value added and intermediate inputs composites, respectively. The price index formulation can be derived from the standard CES constrained cost-minimizing optimization.

$$\mathrm{pva}_a = \sum_f (\mathrm{shf}_{a,f} \cdot \frac{\mathrm{pfa}_{a,f}}{\mathrm{af}_{a,f}}^{(1-\mathrm{sigmav}_a)})^{\frac{1}{1-\mathrm{sigmav}_a}} \qquad \forall a \tag{4}$$

Table 2: Parameters in the MEGA model

Name	Domains	Description
shva	a	share of value added in activity a
shf	a, f	share of factor f in value added a
shfa	a, f	share of factor f employed in activity a
shn	a, g	share of intermediate factor g in intermediate bundle of a
shga	a, g	share of good g produced by a
shfh	f, h	share of factor income f going to household h
shofi	e, f	share of outgoing factor income
shpk	p, k	share of investment type k by public sector (normally one)
shhk	h, k	share of investment type k by household h (normally one)
shek	e, k	share of investment type k by foreign agent (normally one)
shgk	g, k	share of good g in investment demand k
impc	h	internal marginal propensity to consume by household h
empc	h	external marginal propensity to consume by household h
sheh	e, h	share of external consumption e in external consumption demand h
sharm	g, l	total Armington shares
ini_nx		initial level of trade balance
ao	a	output productivity shifter in activity a
ava	a	value added productivity shifter in activity a
ain	a	intermediate productivity shifter in activity a
af	a, f	productivity shifter of factor f in activity a
an	a, g	productivity shifter of intermediate factor g in activity a
aarm	g, l	productivity shifter for Armigton demand
to	a, p	output tax rate
tf	f, p	factor income tax rate
tc	h, p	consumption tax rate
c0	g, h	committed consumption
c1	g, h	marginal consumption parameter
lct	a, f	activity production losses
sigmap	a	CES elasticity of substitution in upper level nest production activity a
sigmav	a	CES elasticity of substitution in value added a
sigman	a	CES elasticity of substitution in intermediate bundle a
omegas	a	CET elasticity of transformation for activity a
sigmaa	g	CES elasticity of Armington nest for good g
omegaf	f	CET elasticity of transformation for factor f

$$\text{qna}_{a,g} = \text{shn}_{a,g} \cdot \text{qnd}_a \cdot \frac{\text{pnd}_a}{\text{pg}_g}^{\text{sigman}_a} \cdot \text{an}_{a,g}{}^{(\text{sigman}_a - 1)} \qquad \forall a, g \tag{5}$$

$$\text{pnd}_a = \sum_g (\text{shn}_{a,g} \cdot \frac{\text{pg}_g}{\text{an}_{a,g}}^{(1-\text{sigman}_a)})^{\frac{1}{1-\text{sigman}_a}} \qquad \forall a \tag{6}$$

The output level of each activity (industry) is allocated to a number of locally produced goods, since each sector in the model can produce multiple goods. Normally however, it is possible identify one main product and a number of secondary by-products. The allocation between the various products is driven by equation (8), stating that a larger share of output will be assigned to those products which command a higher market price (CET). For a few industries, more precisely those having negative entries in the value added of the SAM, the activity output is adjusted (to make it consistent with the SAM) and reduced before the product allocation stage (7).

$$\text{qan}_a = \text{qa}_a - \sum_f \text{lct}_{a,f} \qquad \forall a \tag{7}$$

$$\text{qga}_{a,g} = \text{shga}_{a,g} \cdot \text{qan}_a \cdot \frac{\text{pgl}_{g,\texttt{A}}}{\text{pa}_a}^{\text{omegas}_a} \qquad \forall a, g \tag{8}$$

$$\text{pa}_a = \sum_g (\text{shga}_{a,g} \cdot \text{pgl}_{g,\texttt{A}}{}^{(1+\text{omegas}_a)})^{\frac{1}{1+\text{omegas}_a}} \qquad \forall a \tag{9}$$

Total supply of a given good or service by Andalusian firms is given by the sum of supply levels across producing industries (equation (10)).

$$\text{qgl}_{g,\texttt{A}} = \sum_a \text{qga}_{a,g} \qquad \forall g \tag{10}$$

3.2 Public Sector

The public sector receives tax revenue from three types of taxes: on production output (11), with ad valorem tax rate *to*; on households consumption (12), with tax rate *tc*; on factor income (13), with tax rate *tf*. The model accommodates for the existence of multiple public sectors, as well as for multiple investment categories, but the sets p and k currently include only one element.

$$\text{tra}_{a,p} = \text{pa}_a \cdot \text{qa}_a \cdot \text{to}_{a,p} \qquad \forall a, p \tag{11}$$

Total aggregate consumption is locally taxed. Because of data available in the regional SAM, taxes are not differentiated by good. Notice that households generate a demand for goods and services, in which shares *esharm* determine the origin of the product (so that different prices *pgl* apply):

$$\text{trh}_p = \sum_{g,h,l} (\text{qgh}_{g,h} \cdot \text{esharm}_{g,l} \cdot \text{pgl}_{g,l} \cdot \text{tc}_{h,p}) \qquad \forall p \tag{12}$$

Different tax rates apply to different income sources. However, only average tax rates are considered, which excludes fiscal progressiveness. This is a standard assumption in CGE models:

$$\text{trf}_p = \sum_f (\text{tif}_f \cdot \text{tf}_{f,p}) \qquad \forall p \tag{13}$$

Total public sector revenues (14) are composed of local tax revenues and net incoming transfers (e.g., from central government):

$$\text{ip}_p = \sum_a \text{tra}_{a,p} + \text{trh}_p + \text{trf}_p + \sum_e \text{trep}_{e,p} \qquad \forall p \tag{14}$$

On the other hand, expenses by the public sector include purchases of good and services and net income transfers to households. The difference between revenue and expenses defines public savings (15), which may well be negative in the case of public sector deficit:

$$\text{psav}_{k,p} = \text{shpk}_{p,k} \cdot (\text{ip}_p - \sum_{g,l} (\text{qgp}_{g,p} \cdot \text{esharm}_{g,l} \cdot \text{pgl}_{g,l}) - \sum_h \text{trhp}_{h,p}) \qquad \forall k, p \tag{15}$$

3.3 Households

Households receive income because of their ownership of primary resources. In the MEGA model, two categories of households are considered in terms of consumption: residents and non residents. Only resident households own primary resources in Andalusia, getting income that can be spent (in the region or outside), saved or transferred. Non residents, mainly tourists, finance their expenses through income generated abroad, which is interpreted here as an incoming transfer. On the other hand, primary factors can well be owned by non-Andalusian citizens or foreigners, and the corresponding payment for factor services is seen here as an outgoing transfer.

Equation (16) defines income associated to one primary factor in the set *f*. Income is expressed as the sum of three components, and it is affected by taxes on factor income and activity output. The latter is assigned to the various factors in proportion to their share in the corresponding industrial value added (17). The main component of factor income is of course the market value of resources, where the quantity is adjusted as in (7). This value is augmented by incoming factor income (*ifi*), and diminished by outgoing factor income (*ofi*). Income is incoming when Andalusian factors (including labour) are employed abroad. Outgoing income (18) flows to the opposite direction and depends on the ownership share of non-Andalusian (*shofi*).

$$\mathrm{tif}_f \cdot (1 + \sum_p \mathrm{tf}_{f,p}) = \mathrm{pf}_f \cdot (\mathrm{qf}_f - \sum_a \mathrm{lct}_{a,f}) + \sum_e \mathrm{ifi}_{e,f} - \sum_e \mathrm{ofi}_{e,f} - \sum_{a,p} (\mathrm{tra}_{a,p} \cdot \mathrm{evash}_{a,f}) \qquad \forall f \tag{16}$$

$$\mathrm{evash}_{a,f} = \frac{\mathrm{pfa}_{a,f} \cdot \mathrm{qfa}_{a,f}}{\mathrm{qva}_a \cdot \mathrm{pva}_a} \qquad \forall a, f \tag{17}$$

$$\mathrm{ofi}_{e,f} = \mathrm{shofi}_{e,f} \cdot (\mathrm{pf}_f \cdot (\mathrm{qf}_f - \sum_a \mathrm{lct}_{a,f}) - \sum_{a,p} (\mathrm{tra}_{a,p} \cdot \mathrm{evash}_{a,f})) \qquad \forall e, f \tag{18}$$

Private sector income is obtained from factor income, plus net transfers from the public sector and abroad:

$$\mathrm{ih}_h = \sum_f (\mathrm{shfh}_{f,h} \cdot \mathrm{tif}_f) + \sum_p \mathrm{trhp}_{h,p} + \sum_e \mathrm{treh}_{e,h} \qquad \forall h \tag{19}$$

Demand for domestic private consumption is modelled in MEGA through a Linear Expenditure System (LES, see e.g. Pollak and Wales [1969]). In a LES demand system, demand for each good is expressed as the sum of two components: a fixed, income and prices independent one (*c0*), and a variable one. Available income is first employed to buy the committed consumption. The rest of income is allocated to purchase more goods and services, where the allocation is determined by fixed value shares *c1*. Consequently, the LES converges under progressively higher income levels to a homothetic, unitary income elasticity, Cobb-Douglas function. If endogenous variations in purchasing power are not very large, a LES well approximates a demand system with differentiated income elasticities.

The MEGA model considers two sets of households: residents and non residents. Parameters for the residents LES function are calibrated on the basis of given income elasticity values, combined with consumption levels as they appear in the SAM. For non residents (tourists), constancy of value shares in their expenditure budget is considered to be a reasonable hypothesis, implemented in the model by setting all committed consumption parameters to zero. Therefore, for both categories of households, consumption demand can be formulated as:

$$(\mathrm{qgh}_{g,h} - \mathrm{c0}_{g,h}) \cdot \mathrm{pg}_g \cdot (1 + \sum_p \mathrm{tc}_{h,p}) = \mathrm{c1}_{g,h} \cdot (\mathrm{impc}_h \cdot \mathrm{ih}_h - \sum_{gg} (\mathrm{c0}_{gg,h} \cdot \mathrm{pg}_{gg} \cdot (1 + \sum_p \mathrm{tc}_{h,p}))) \qquad \forall g, h \tag{20}$$

Not all consumption demand is generated internally. By this, we do not refer to the fact that some goods or services are imported, but to the possibility that Andalusian residents physically move and consume outside the regional boundaries, for example during vacations. This is accounted for in the model through a fixed parameter "external marginal propensity to consume" *empc*, then with a share *sheh*, splitting the external consumption between that directed to the rest of Spain and that directed to the rest of the world:

$$\mathrm{veh}_{e,h} = \mathrm{sheh}_{e,h} \cdot \mathrm{empc}_h \cdot \mathrm{ih}_h \qquad \forall e, h \tag{21}$$

Available income, which is not consumed internally (marginal propensity *impc*) or externally (*empc*), feeds private savings. The model allows for multiple investment types, corresponding to the share parameters *shhk* (currently one):

$$\mathrm{hsav}_{h,k} = \mathrm{shhk}_{h,k} \cdot (1 - \mathrm{impc}_h - \mathrm{empc}_h) \cdot \mathrm{ih}_h \qquad \forall h, k \tag{22}$$

3.4 Investment Demand

Income flowing from Andalusia to RoS or RoW, the two elements in the subset e of locations l, is the sum of imports (their exports), outgoing factor income and value of households consumption spent abroad:

$$\mathrm{ie}_e = \sum_g (\mathrm{pgl}_{g,e} \cdot \mathrm{qgl}_{g,e}) + \sum_f \mathrm{ofi}_{e,f} + \sum_h \mathrm{veh}_{e,h} \qquad \forall e \tag{23}$$

Foreign "non-Andalusian" savings can be defined as the difference between outgoing and incoming income flows (24). Please notice that an external trade deficit contributes to foreign savings.

$$\mathrm{esav}_{e,k} = \mathrm{shek}_{e,k} \cdot (\mathrm{ie}_e - \sum_g (\mathrm{qge}_{g,e} \cdot \mathrm{pgl}_{g,\mathtt{A}}) - \sum_f \mathrm{ifi}_{e,f} - \sum_h \mathrm{treh}_{e,h} - \sum_p \mathrm{trep}_{e,p}) \qquad \forall e, k \tag{24}$$

In a closed economy equilibrium, total savings match investment expenditure. In an open regional economy, investment is financed by private, public and external *net* savings:

$$\mathrm{ik}_k = \sum_{h|\mathrm{hsav.L}_{h,k}} \mathrm{hsav}_{h,k} + \sum_p \mathrm{psav}_{k,p} + \sum_e \mathrm{esav}_{e,k} \qquad \forall k \tag{25}$$

In the MEGA model, investment demand for specific products is obtained by imposing the condition that the value share of each item in the total investment expenditure is constant. In other words, investment demand is formulated as a Cobb-Douglas function:

$$\mathrm{qgk}_{g,k} = \frac{\mathrm{shgk}_{g,k} \cdot \mathrm{ik}_k}{\mathrm{pg}_g} \qquad \forall g, k \tag{26}$$

3.5 Trade

The "Armington assumption" from (Armington [1969]) is central in most CGE models. It states that goods and services belonging to the same industry but produced in different locations should be regarded as imperfect substitutes. This assumption is accommodated in MEGA through the endogenous determination, driven by a CES function, of the share of demand for good g served by location l:

$$\mathrm{esharm}_{g,l} = \mathrm{sharm}_{g,l} \cdot \frac{\mathrm{pg}_g}{\mathrm{pgl}_{g,l}}^{\mathrm{sigmaa}_g} \cdot \mathrm{aarm}_{g,l}{}^{(\mathrm{sigmaa}_g - 1)} \qquad \forall g, l \tag{27}$$

Where the aggregate Armington price index is formulated the usual way:

$$\mathrm{pg}_g = \sum_l (\mathrm{sharm}_{g,l} \cdot \frac{\mathrm{pgl}_{g,l}}{\mathrm{aarm}_{g,l}}^{(1-\mathrm{sigmaa}_g)})^{\frac{1}{1-\mathrm{sigmaa}_g}} \qquad \forall g \tag{28}$$

The Armington split of demand is applied, with the same share, to all the different components of demand: intermediate, household and public consumption, investments:

$$\mathrm{armDem}_g = \sum_a \mathrm{qna}_{a,g} + \sum_h \mathrm{qgh}_{g,h} + \sum_p \mathrm{qgp}_{g,p} + \sum_k \mathrm{qgk}_{g,k} \qquad \forall g \tag{29}$$

$$\mathrm{qgl}_{g,e} = \mathrm{esharm}_{g,e} \cdot \mathrm{armDem}_g \qquad \forall g, e \tag{30}$$

However, demand for Andalusian products also includes exports, which are only supplied locally:

$$\mathrm{qgl}_{g,\mathtt{A}} = \mathrm{esharm}_{g,\mathtt{A}} \cdot \mathrm{armDem}_g + \sum_e \mathrm{qge}_{g,e} \qquad \forall g \tag{31}$$

Equation (32) defines the regional trade balance. The normal closure of the model assumes this balance as exogenous and fixed.

$$\mathrm{ini\_nx} = \sum_{g,e} (\mathrm{qge}_{g,e} \cdot \mathrm{pgl}_{g,\mathtt{A}}) - \sum_{g,e} (\mathrm{pgl}_{g,e} \cdot \mathrm{qgl}_{g,e}) \tag{32}$$

3.6 Primary Resources

Contrary to many CGE models, primary factors in MEGA are neither perfectly mobile nor immobile (industry-specific). Instead, households provide an aggregate supply, which is allocated to the different sectors, on the basis of relative prices. For example, more labour is assigned to those industries where relative wages are higher. This is accomplished in the model by a Constant Elasticity of Transformation function, dual of the CES (33). Therefore, in this setting, the elasticity parameter *omegaf* defines the degree of factor mobility:

$$\mathrm{qfa}_{a,f} = \mathrm{shfa}_{a,f} \cdot \mathrm{qf}_f \cdot \frac{\mathrm{pfa}_{a,f}}{\mathrm{pf}_f}^{\mathrm{omegaf}_f} \qquad \forall a, f \tag{33}$$

Where the factor price index is:

$$\mathrm{pf}_f = \sum_a \left(\mathrm{shfa}_{a,f} \cdot \mathrm{pfa}_{a,f}{}^{(1+\mathrm{omegaf}_f)}\right)^{\frac{1}{1+\mathrm{omegaf}_f}} \qquad \forall f \tag{34}$$

Regional GDP can be defined as the sum of all sectoral values added. Consequently, a GDP deflator is readily computed as a quantity-weighted average of value added price indexes:

$$\mathrm{defl} \cdot \sum_a \mathrm{qva}_a = \sum_a (\mathrm{pva}_a \cdot \mathrm{qva}_a) \tag{35}$$

4 Implementation

The set of equations (1)-(35) constitutes a non-linear system, which can be solved numerically. To this end, MEGA has been coded in GAMS (Meeraus et al. [1988]), as a pseudo constrained optimization problem (that is, a problem where the admissible domain is restricted to a single, multidimensional point) and solved with the package CONOPT. Alternative formulations, like in terms of mixed complementarity problem, or solvers, like PATH, are also possible and were successfully tested.

The number of equations in the system determines the number of endogenous variables that can be determined, once values for parameters and exogenous variables have been set. Because of Walras law, however, equations are not independent and the general equilibrium can only be defined in terms of relative prices. This requires the definition of a pegged price *numeraire*. In the MEGA model, the numeraire is the whole set of external prices, corresponding to a virtual exchange rate. In other words, if the price of all non-Andalusian prices are scaled up or down (which may affect income transfers as well), then all endogenous prices will be scaled by the same proportion, whereas all real quantities in equilibrium will be unaffected.

The partition between endogenous and exogenous variables is not fixed, but can be changed at will, thereby defining the chosen model closure. MEGA comes with a proposed closure, where prices (wages) of all labour factors are fixed, so that employment/unemployment (by labour category) is endogenously determined. This differs from the traditional closure of basic CGE models, where all factors (including labour) are fully employed and prices (rather than quantities) are endogenous. Often, this hypothesis is associated with the one of perfect domestic mobility, meaning that there is a single market price for each factor, whereas MEGA is characterized by multiple factor prices *pfa*.

The numerical search for a solution of the system (1)-(35) starts with prices normalized at one and endogenous quantity variables initialized at the corresponding values in the SAM. If exogenous variables are also set at their corresponding SAM figures, then the initial point already provides the solution and the model simply replicates the SAM data. If, instead, some parameters or exogenous variables are varied, a counter-factual general equilibrium is generated. After each run, the GAMS code “dumps” all results into a binary GDX file, which could be later used to contrast different equilibrium states, like in any comparative static exercise. In addition, the code automatically computes some descriptive auxiliary variables, including changes (w.r.t. calibration) in real and nominal regional GDP, as well as the equivalent variation EV (a money-metric measure of welfare).

Results from the MEGA model are, like in every CGE model, highly sensitive and dependent on the values assigned to elasticity parameters in CES and CET functions. This is understandable as, for example, Armington elasticities determine how easy it could be substituting imports with domestic products (thereby influencing market power and the terms of trade). Analogously, CET elasticities drive the allocation of factors among industries (therefore, aggregate productivity).

Values for elasticity parameters cannot be calibrated, as they refer to *changes,* possibly occurring over time, whereas a SAM only provides a snapshot of data referring to *one* point in time. Like in other CGE models, values

Table 3: CES elasticity values

parameter	domain	minimum	maximum
sigmap	a	0.48	1.12
sigmav	a	0.2	1.68
sigman	a	0.25	0.25
sigmaa	g	1.15	4.49

Table 4: Elasticity of transformation for primary factors

Category	Value
Employees	1.5
Service and shop workers	2
Agriculture and unskilled workers	1.2
Technicians	0.8
Land	0.25
Managers, self-employed	0
Physical capital and infrastructure	0.5
Natural resources	0

are assigned in MEGA on the basis of a mix of econometric estimates, consensus (values adopted in other models) or educated guesses. Table 3 presents some information about the elasticity values initially set in the model (they could be changed) for the CES functions.

Sigmap is the elasticity in the CES nest between the aggregate of intermediate factors ("materials") and value added. Many CGE models adopt an identical value (sometimes zero or one) for all industries, but in MEGA this elasticity ranges for a minimum of 0.52, in extraction industries, to a maximum of 1.12, in sanitary activities.

Sigmav affects the subsitutability among primary factors inside the value added bundle. It goes from a minimum of 0.2 (forestry, fishery, extraction) to a maximum of 1.68 (domestic reparations). *Sigman* is the elasticity among intermediate factors. Most CGE models do not consider substitution at this level, imposing a zero elasticity, corresponding to a Leontief production function. In all industries inside MEGA, instead, we set a small but sightly positive value of 0.25.

Finally, *sigmaa* is the key Armington elasticity. Considering what is typically assumed in other models, we set all values larger than one. This means that the substitution effect prevails over the income effect, such that the value share increases when the corresponding relative price decreases. Geographical substitutability is lowest (1.15) for drinks, highest (4.49) for textile products.

CET transformation functions are employed in the model for two tasks. First, a CET function drives the allocation of activity levels among various product lines. Unfortunately, no empirical studies are available to lead the choice of a value for the corresponding elasticity, *omegas*. We opted for a low value in all sectors (0.2), as we believe that multiple lines are mostly a consequence of the existence of by-products, which offer a limited degree of flexibility in terms of production patterns.

Another CET function determines the degree of inter-sectoral mobility of primary resources. This kind of mobility can be affected by various factors. In the case of land, for example, it may depend on soil characteristics, affecting the possibility of conversion between crops and fertility. For labour, one can think about education level, skill specificity, access to information. All in all, after considering various aspects, econometric estimates and choices made in comparable models, we selected for the parameter *omegaf* the values displayed in Table 4.

There is another set of elasticities playing an important role in the MEGA model, despite the fact that they do not appear in the system of equations. Indeed, income elasticities were used to calibrate parameters of the LES demand system for resident households. Each LES demand function includes two parameters for each item: a fixed consumption level (*c0*) and a marginal propensity to consume (*c1*). The two parameters are estimated by imposing two specific conditions: (a) that income elasticity computed around the initial calibration point equals some pre-specified value; (b) that consumption demand generated at baseline prices and income corresponds to data in the SAM.

Fortunately, estimation of income elasticity are widely available in the literature, even if some caution is in order when applying them to industrial classifications (rather than consumption categories), like those of CGE models. We used in MEGA a combination of econometric estimates and values adopted in the standard GTAP model [Hertel and Tsigas, 1997]. These values range from those of income inelastic basic food (0.14) to those of superior services, like sea and air (passenger) transportation (5.82).

5 Concluding Remarks

MEGA is an open-source, freely accessible conventional CGE model, applied at the sub-national level to the Spanish region of Andalusia. In its current formulation it can be directly employed to conduct a wide range of simulation and policy assessment exercises. These include, for instance: effects of changing tourism flows, impact of trade wars, varying productivity of labour, and many others. Some of these exercises take a special meaning at the regional level.

Like other CGEs, MEGA can be interfaced with other models, possibly providing more detail at the sectoral level (e.g., for the energy system), or involving non purely economic variables (as in integrated assessment IAM models). The model could also be extended if more data would be available. One especially interesting direction of development would be the disaggregation of resident households, making it possible to analyze distributional impacts. MEGA could also be considered as a building block in the construction of a dynamic model, possibly expressed as a chain of temporal general equilibrium states.

The whole process of assembling the SAM data base, formulating. coding and testing the model is a long and expensive one, especially when data is scarce, as it is normally the case for the regional economies. As such, it imposes a non negligible fixed cost to the modellers, possibly preventing the realization of numerical simulations of the general equilibrium type. To overcome this problem, MEGA is being made freely available and accessible to the modelling community. Hopefully, the model could also become a common ground to facilitate the exchange and comparison of research findings, related to the Andalusian economy.

Access to Model Data and Software

The MEGA model is accessible at the web page: http://mizar.unive.it/roson/mega.html.

Acknowledgments

Wolfgang Britz contributes in various ways to the development of the MEGA model. Camille van der Vorst helped me in the construction of the Andalusian SAM, and in the choice of values for elasticity parameters, together with Olexandr Nekhay.

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